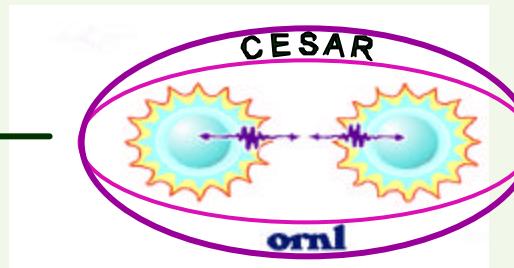


# *Coherence and Synchronization in Arrays of Class B Lasers*

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# *Motivation*

Ultra High Coherent Power (2-10W)

In-Phase Synchronization

Ultra Fast (10Gbps) Communication  
Rates

Attractor Switching

Desired Properties of Waveform

Control of Spatial and Temporal  
Dynamics

# *Laser Array as a Dynamical System*

- In -phase synchronization in a single array
- Chaotic synchronization of distinct laser arrays
- Suppression of chaos in laser arrays
- Control of transient behaviors
- Fast attractor switching

# Synchronization

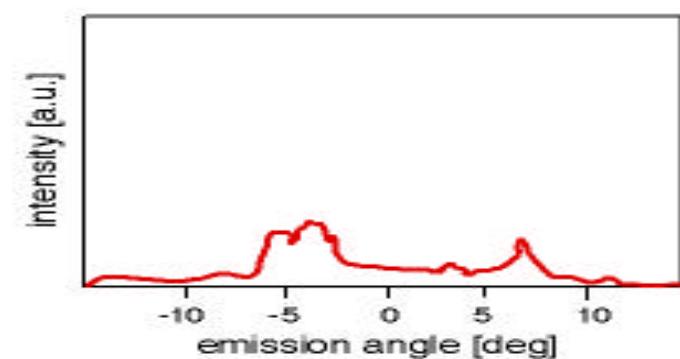
$$?_j(t) = E_j \exp(-i\omega_j t) + cc$$

$$I(t) = |\sqrt{I_1} \exp(if_1) + \sqrt{I_2} \exp(if_2) + \dots|^2$$

$E_j$  - complex amplitude

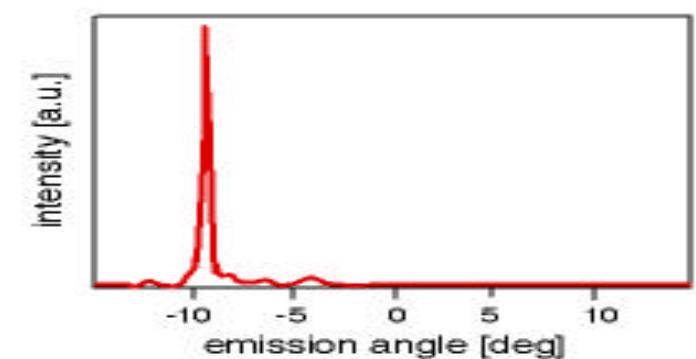
$\omega_j$  - the frequency

$$E_j = \sqrt{I_j} e^{if_j}$$



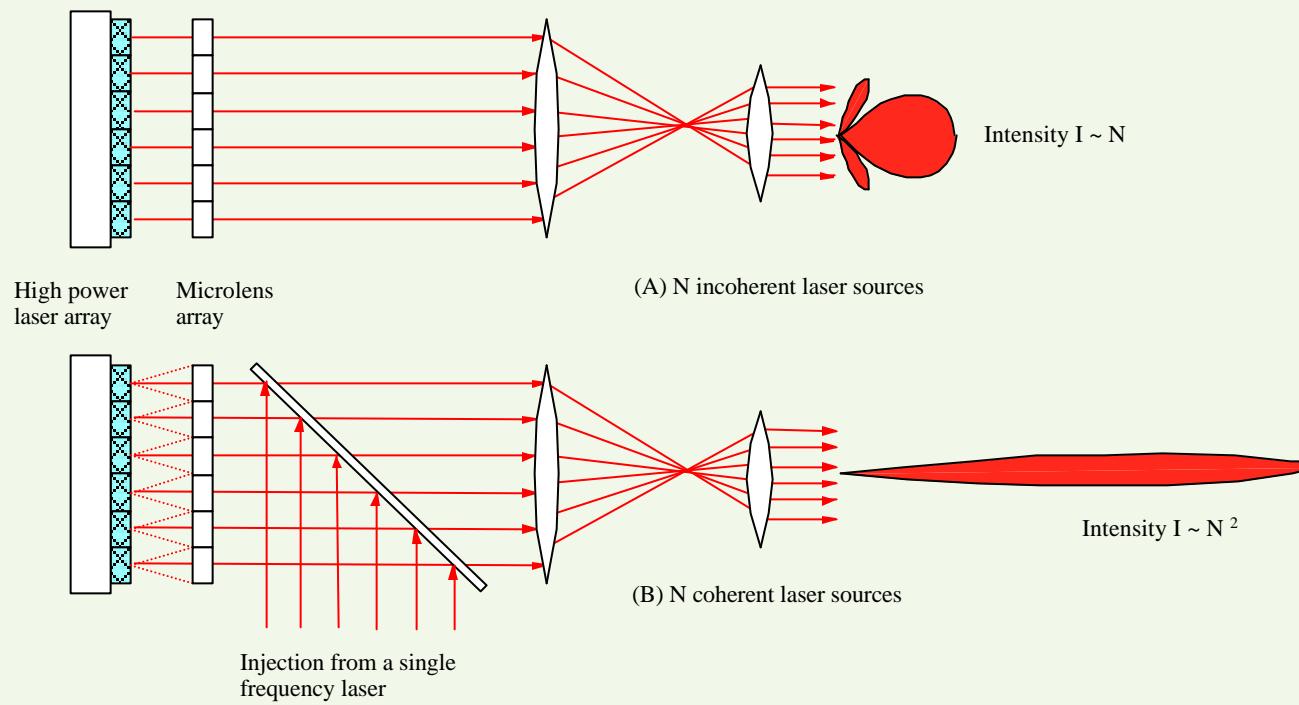
Noncoherent addition

$$\begin{aligned} \text{"in-phase"} \rightarrow \phi_1 &= \phi_2 = \dots = \phi_N \\ \Rightarrow I &\sim N^2 I_0 \end{aligned}$$



Coherent addition

# *High Coherent Power Generation Using Arrays of Semiconductor Lasers*



## *Attributes/Benefits of this Concept*

- Compact high intensity power source
- Potential of order of magnitude increase in power
- Cost effective - semiconductor lasers are not expensive

## *Additional (Potential) Applications*

- High Speed Image Transmission (in Gbps rate)
- Improved Space Communications  
(more power, larger distances)
- Pumping Solid State Lasers

# *Why has high coherent power not yet been achieved in lasers ?*

## Array Synchronization & Stability

- In-phase behavior of identical lasers is not stable for a broad range of parameters
- High power arrays are not stable
  - Optics + Lasers + Nonlinear Dynamics & Chaos
    - Nontrivial Combination !

## *What has already been achieved ?*

- Phase locking by injection of a single laser
- Synchronization of low power nonchaotic arrays of lasers
- Synchronization of two - three chaotic lasers

# *Basic Equations for Semiconductor Laser Array*

Evolution of mode amplitude  $E_j$  and population  $N_j$  in the  $j$ th laser

$$\frac{dE_j}{dt} = \frac{1}{2} [G(N_j) - \frac{1}{\tau_p}] (1 - i\alpha) E_j + ik(E_{j+1} + E_{j-1}) + id_j E_j + E_e$$

$$\frac{dN_j}{dt} = p - \frac{N_j}{\tau_s} - G(N_j) |E_j|^2$$

Electric field in the  $j$ th laser:  $E_j(t)\exp(-i\omega_0 t)$

$N_j$	inversion population
$G_j$	gain
$\tau_p$	photon lifetime ( $\sim 1$ ps)
$\tau_s$	lifetime of the active population ( $\sim 2$ ns)
$p$	pumping rate
$\kappa$	magnitude of the coupling strength between adjacent lasers
$\delta_j$	uncoupled laser frequencies
$a$	line-width enhancement factor

$$G(N_j) = G(N_{th}) + g(N_j - N_{th})$$

$$g = \partial G / \partial N - \text{the differential gain and } G(N_{th}) = 1/\tau_p$$

## *Equation of Motion for Semiconductor Laser Array*

$$\frac{dE_j}{dt} = [G_j - \mathbf{a}_j + i\mathbf{d}_j]E_j + \mathbf{k}(E_{j+1} + E_{j-1}) + E_e$$

$$\frac{dG_j}{dt} = \frac{\mathbf{t}_c}{\mathbf{t}_f} [ p_j - (1 + |E_j|^2)G_j ]$$

Electric field in the  $j$ th laser:  $E_j(t)\exp(-i\omega_0 t)$

$G_j$

the gain

$t_f$

fluorescence time ( $\sim 240\mu\text{s}$ )

$t_s$

cavity round trip time ( $\sim 2\text{ns}$ )

$p$

pumping rate

$\kappa$

magnitude of the coupling strength between adjacent lasers

$\delta_j$

detunings

$a$

losses

# *Equations of Motion*

Substitute

$$E_e(t) = \sqrt{I_e}$$

$$E_j(t) = \sqrt{I_j}(t) \exp(if_j(t))$$

To obtain

$$\dot{I}_j = 2(G_j - \mathbf{a})I_j + 2k\sqrt{I_1 I_2} \cos(f_2 - f_1) + 2\sqrt{I_e I_j} \cos f_j$$

$$\dot{f}_j = d_j + (-1)^j k \frac{\sqrt{I_1 I_2}}{I_j} \sin(f_1 - f_2) - \sqrt{\frac{I_e}{I_j}} \sin f_j$$

$$\dot{G}_j = \frac{t_c}{t_f} (p - G_j - G_j I_j)$$

# *Stability Analysis of the Phase Model*

$$\dot{\mathbf{f}}_j = \mathbf{d}_j - |\mathbf{k}| \{ \sin(\mathbf{f}_{j+1} - \mathbf{f}_j) + \sin(\mathbf{f}_{j-1} - \mathbf{f}_j) \} - A_e \sin \mathbf{f}_j$$

$$\mathbf{d}_j = 0 \Rightarrow \mathbf{f}_j^0 = 0 \quad \text{is a solution}$$

Stability ?

$$\mathbf{f}_j(t) = \mathbf{f}_j^0 + \mathbf{d}\mathbf{f}_j(t)$$

$$\dot{\mathbf{d}\mathbf{f}}_j = -|\mathbf{k}| [\mathbf{d}\mathbf{f}_{j+1} - 2\mathbf{d}\mathbf{f}_j + \mathbf{d}\mathbf{f}_{j-1}] - A_e \mathbf{d}\mathbf{f}_j$$

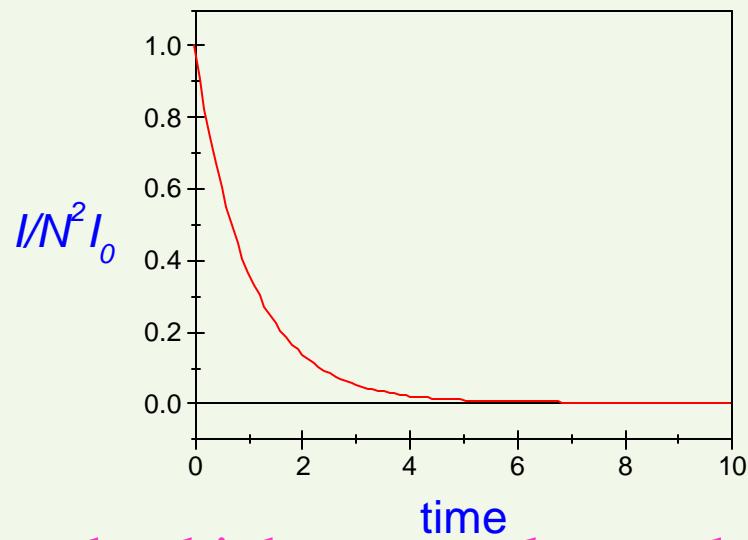
$$\mathbf{d}\mathbf{f}_j = x_{jm} \exp(I_m t) \quad \Rightarrow I_m = -A_e + 4 |\mathbf{k}| \sin^2 \frac{mp}{N}, \quad m = 0, 1, \dots, N-1$$

In-phase solution stable for

$$A_e > 4 |\mathbf{k}|$$

## *Stability of the “In-phase” Solution*

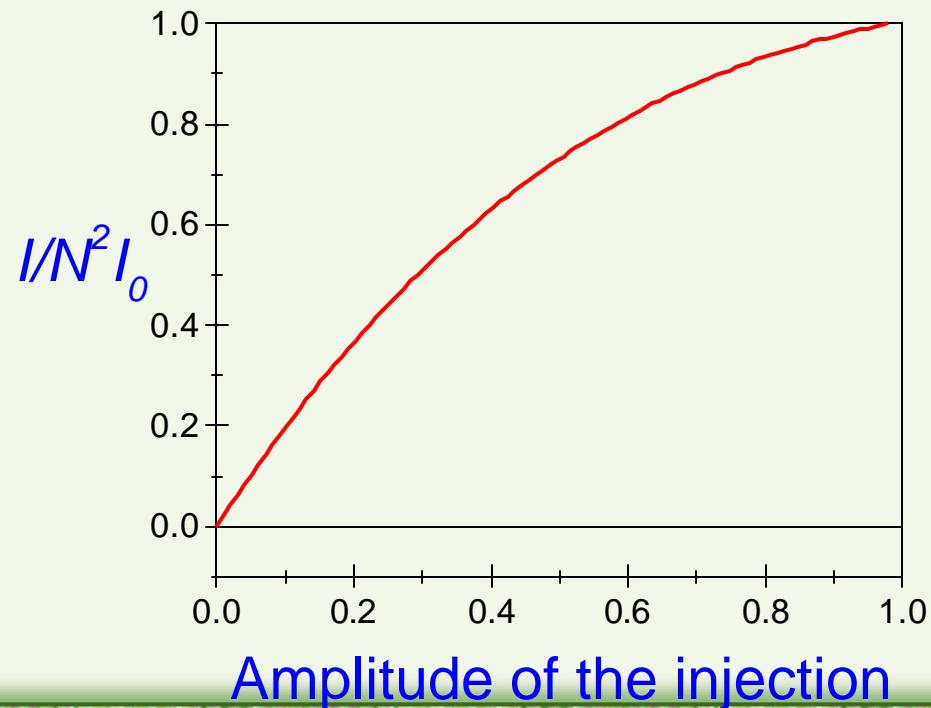
- \* In-phase solution is not stable for a wide range of parameters.
- \* Instead, the “Out-of-phase” solution ( $\phi_{j+1} - \phi_j = \pi$ ) is stable, leading to a destructive interference.



- \* When operated at high power - lasers show chaotic behavior.

## *Entrainment by Injection of the Field*

Injection of the same electromagnetic field in to the cavity of each laser results in stabilization of the “in-phase behavior”.



## *Two Coupled Lasers*

$$\dot{f}_1 = d_1 + k (\sin(f_1 - f_2)) - A_e \sin f_1$$

$$\dot{f}_2 = d_2 + k (\sin(f_2 - f_1)) - A_e \sin f_2$$

Fixed Point Solutions

$$d_1 + k (\sin(f_1 - f_2)) - A_e \sin f_1 = 0$$

$$d_2 + k (\sin(f_2 - f_1)) - A_e \sin f_2 = 0$$

Injection Tuning

$$d_1 + d_2 \square 0$$

## *Analysis of the Phase Model*

$$\sin f_1 + \sin f_2 = 0$$

$$d_1 - d_2 + 2k(\sin(f_2 - f_1)) - A_e(\sin f_2 - \sin f_1) = 0$$

The first equation in (1.3) implies that either a):  $f_2 - f_1 = (2m + 1)p$  or b):  $f_1 + f_2 = 2pm$ , where  $m$  is an integer. Solutions of class (a) imply  $\sin(f_2 - f_1) = 0$ , yielding  $\sin f_1 = d_1/A_e$ ,  $\sin f_2 = d_2/A_e$  and  $\sin(f_1 - f_2) = \sin(\sin^{-1}(d_1/A_e) - \sin^{-1}(d_2/A_e)) \neq 0$ , i.e. inconsistency. Hence, the only possibility is the class (b) of solutions which, in turn, can be divided in two sub-classes:  $m$  even and  $m$  odd. For  $m$  even, the second equation in becomes:

$$f(x) \equiv -d - 2k \sin x - 2A_e \sin \frac{x}{2} = 0$$

## *Nonmonotonicity Transition Point*

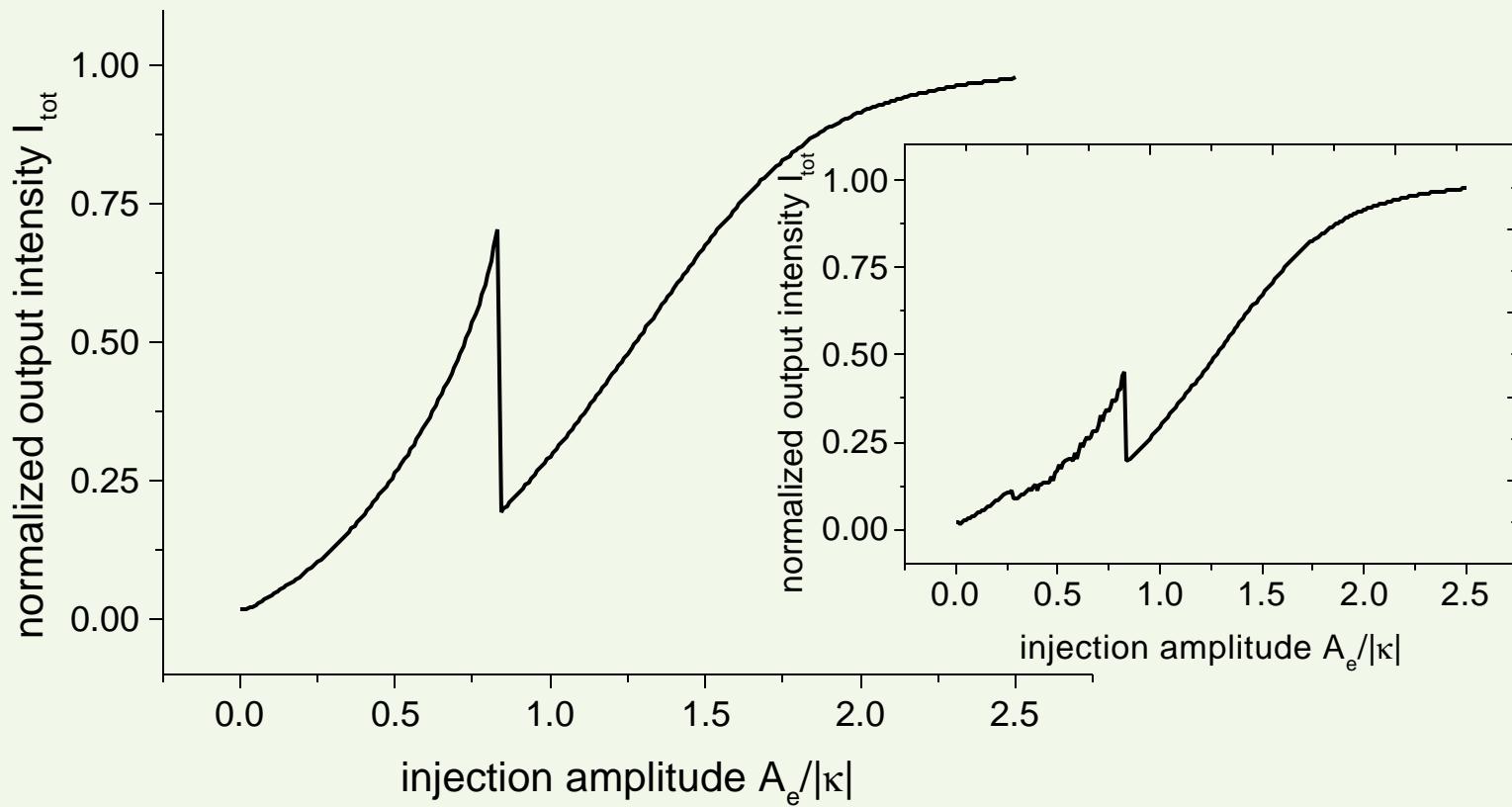
$$f(x) = -d - 2k \sin x_c - A_c \sin \frac{x_c}{2} = 0$$

$$f'(x) = -2k \cos x_c - A_c \cos \frac{x_c}{2} = 0$$

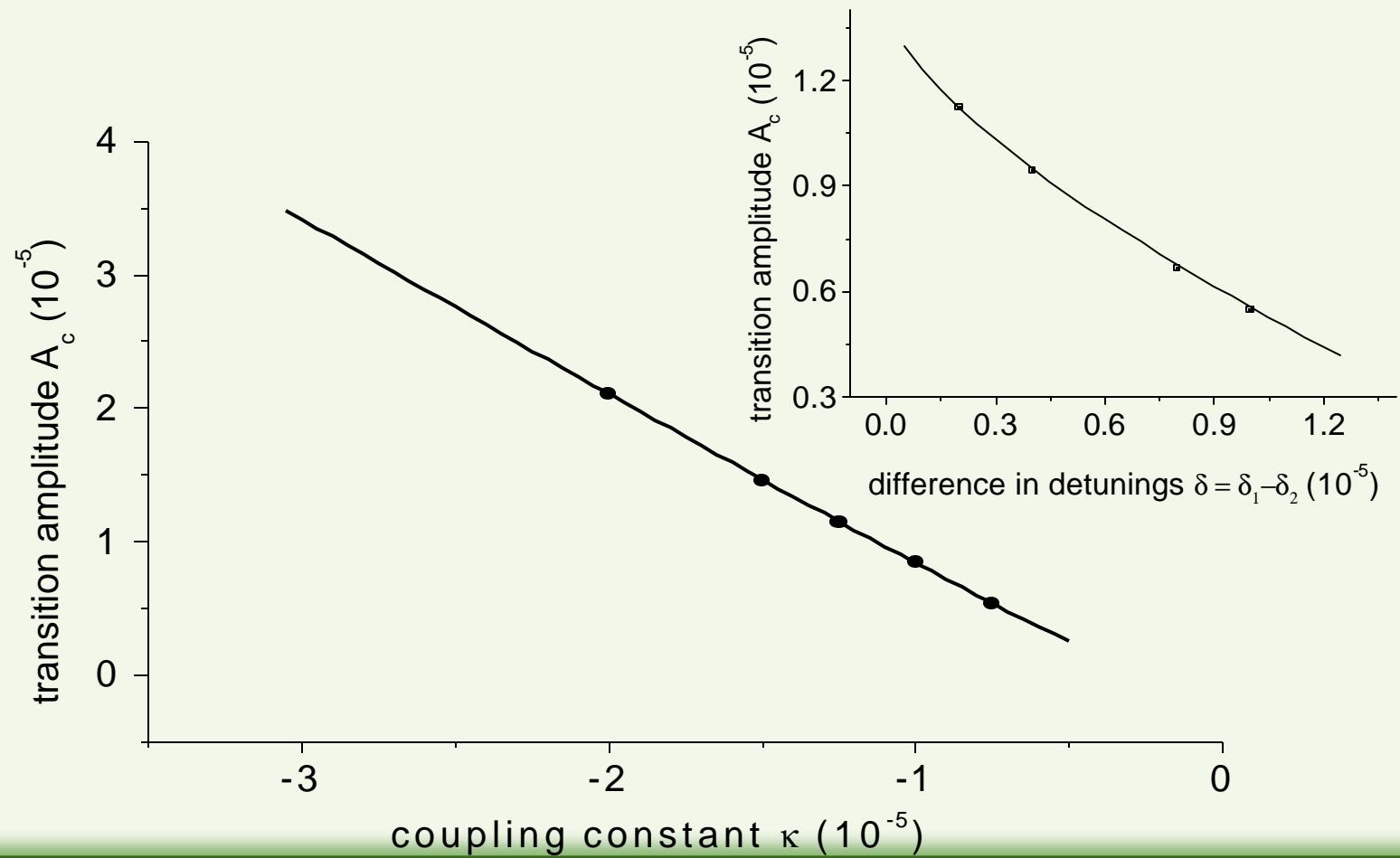
$$A_c = -2k \frac{1-z^2}{\sqrt{1+z^2}}.$$

$$\tan x_c / 2 = z$$

# *Nonmonotonicity*



# *Comparison of the Analysis with Numerical Simulations*



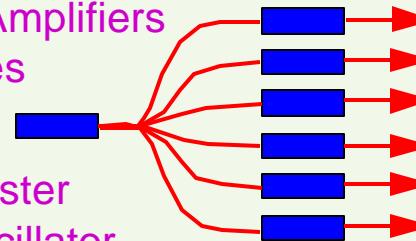
# *Commonly Used Array Architectures*

## MOPA/Injection Locking

Single, stable mode from master oscillator

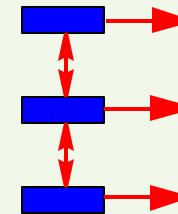
## Power Amplifiers or Slaves

## Master Oscillator

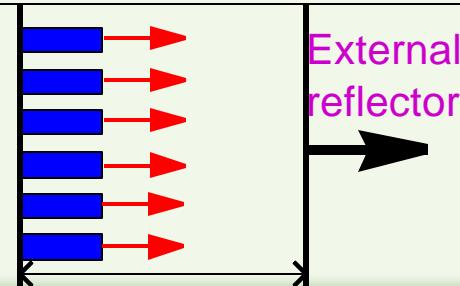


## Coupled Oscillators

Evanescence or Leaky Wave-coupled (side by side)  
Longitudinally coupled (end-to-end)



## External Cavities



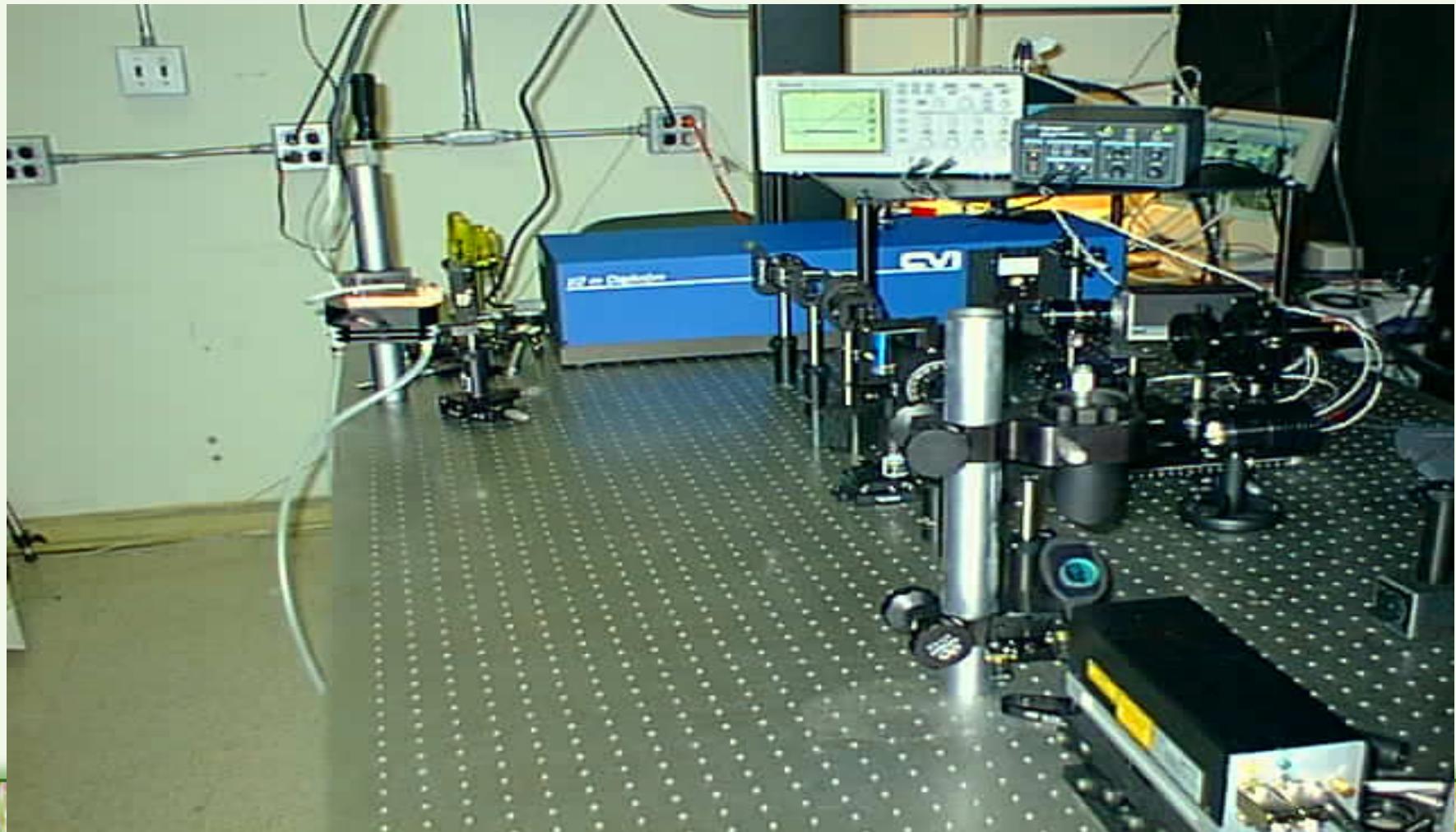
# *Challenges*

- Beam injection into each laser
  - achieve distribution controlled injection under micro-optic geometrical constraints
- Phase locking the array
  - though lasers are almost identical, the desired *in-phase state is unstable* for a broad range of parameters
- Beam collection
  - Fusing outcoming beams without losing phase coherence

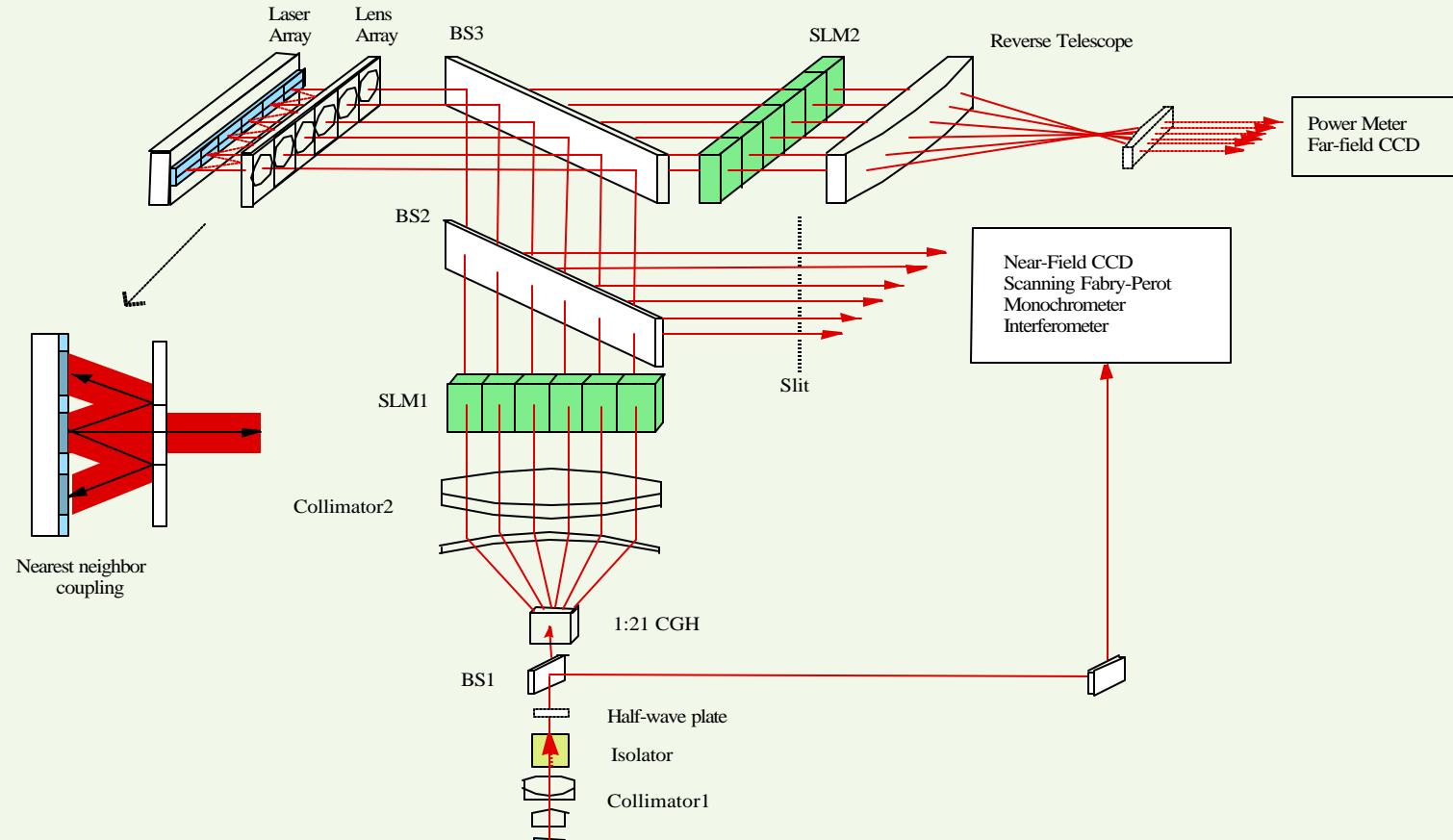
# *Approach*

- Control output power of array by changing the amplitude and the frequency of the injected field
- Modulate injected field to produce a high power modulated output field
- Achieve coupling by **feedback** from neighboring lasers
  - enables us to **control coupling strength**
- Our starting point is an **uncoupled** array
  - $\lambda = 810 \text{ nm}$     $d = 500 \text{ micron}$
  - Because of *large* separation between array elements, we can **control injection**

# *Experimental Apparatus*

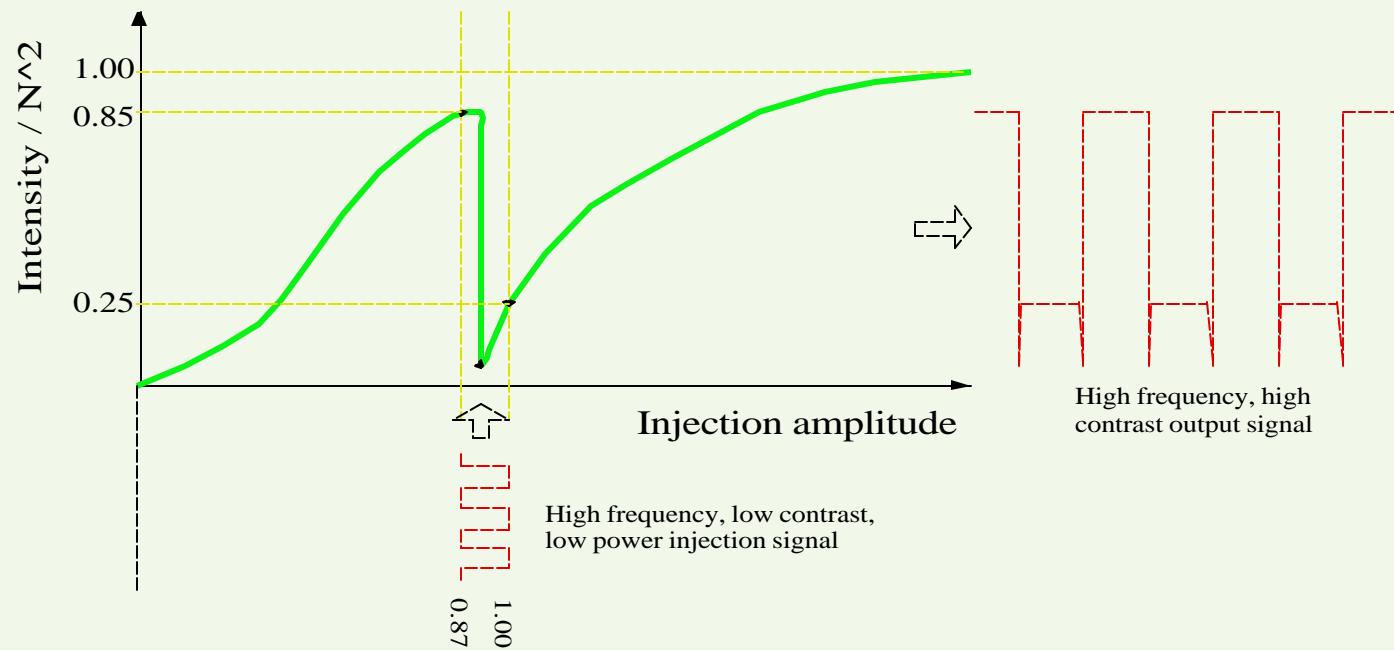
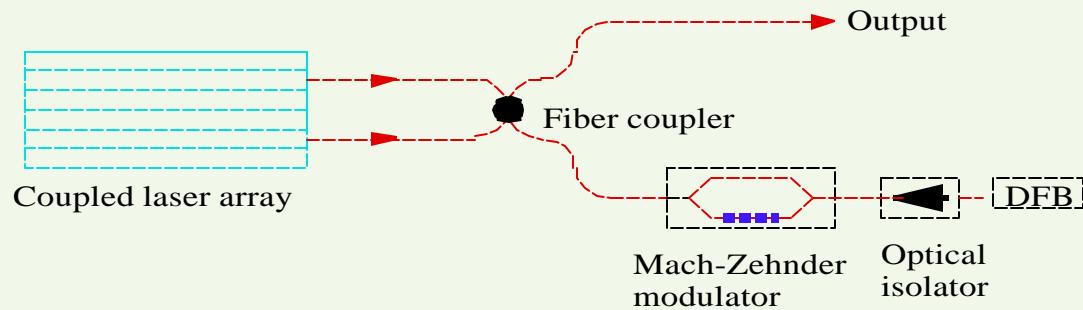


# *Schematic Diagram of Experiment*



CGH \_\_ Computer Generated Hologram,    SLM \_\_ Spatial Light Modulator,    BS \_\_ Beamsplitter

# *High Contrast Fast Modulation (Gbps)*



## *Proposed Design*

- control the injection amplitudes and phase distribution *to each laser separately*
- control the form and the strength of the coupling between lasers
- measure the amplitudes and the phases of *each laser separately*

# *Combining Experimental Measurements with Numerical/Theoretical Analysis*

- a) Computation and analysis of the equations of motion for a specific arrays of lasers.
- b) Estimation of the range of parameters where the experimental probability for obtaining high power output radiation from the array is maximized.

# *Publications*

- Y. BRAIMAN, T. A. B. KENNEDY, K. WIESENFELD, and A. I. KHIBNIK, *Entrainment of Solid-State Laser Arrays*, Phys. Rev. A **52**, 1500, (1995).
- A. I. KHIBNIK, Y. BRAIMAN, T. A. B. KENNEDY, and K. WIESENFELD, *Phase Model Analysis of Two Lasers with Injected Field*, Physica D **111**, 295 (1998).
- A. KHIBNIK, Y. BRAIMAN, V. PROTOPOPESCU, T. A. B. KENNEDY, and K. WIESENFELD, *Amplitude Dropout in Coupled Lasers*, submitted to Phys. Rev. A (May, 2000).