A new approach to deformed proton emitters: non-adiabatic coupled-channels

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1. Introduction

The field of proton radioactivity has experienced a new excitement in the past several years. Just beyond the proton drip line there are many nuclei where the unbound proton is trapped behind a large Coulomb-plus-centrifugal barrier leading to lifetimes ranging from microseconds to seconds [1,2]. New ground-state and isomeric proton emitters [3–6] are continually being discovered, and the first evidence for fine structure in proton decay [7] was recently announced. The focus of recent investigations has been on well-deformed nuclei which exhibit rotational motion. These are of particular interest due to the interplay between proton emission and angular momentum.

The theoretical description of long-lived proton emitters requires a detailed understanding of narrow resonances. Although proton radioactivity is a complicated A-body phenomenon, much insight may be gained by considering the simplified problem of a single proton penetrating the Coulomb barrier of the core consisting of the remaining A–1 nucleons. It has been found that this simple, one-body picture works surprisingly well. In particular, one has been able to determine the angular momentum content of the resonance and the associated spectroscopic factor for many spherical proton emitters [2,8].

The array of theoretical tools available for deformed emitters is not as well developed. The existing ones fall into three general categories. The first family of calculations [3,7,9] is based on the reaction-theoretical framework of Kandenskiĭ and collaborators [10]. The second suite uses the theory of Gamow (resonance) states [5,11–13]. Finally, an approach, based on the time-dependent Schrödinger equation, has been introduced in Ref. [14].

In all of these previous attempts, the strong coupling approximation of the particle-plus-rotor model has been used. The core is taken to be a perfect rotor with an infinite moment of inertia. This has the effect of (i) collapsing the rotational spectrum of the daughter nucleus to the ground state, and (ii) neglecting the Coriolis coupling. Recently we have introduced a technique based on the weak coupling scheme which is free from these deficiencies [15]. Within this method, partial proton widths from different states of the parent nucleus to various final states in the daughter system can be calculated in a straightforward and consistent manner. Currently, the weak coupling approach is being extended to spherical nuclei which are susceptible to vibrational excitations to study possible fine structure in these decays as well.

2. Coupled-Channel Formalism

From a theoretical point of view, proton radioactivity is an excellent example of three-dimensional, quantum-mechanical tunneling. As such, the understanding of proton emission is really a test of our knowledge of very narrow resonances. Since the lifetimes which can be seen experimentally range from microseconds to seconds, the corresponding widths are extremely small; they vary between 10^{-16} MeV and 10^{-22} MeV. A theoretical description of such small widths requires extraordinary numerical accuracy. In the following, the coupled-channel Schrödinger equation method with Gamow states is outlined, and the proton-plus-core Hamiltonian is defined.

The parent nucleus is described by the core-plus-proton Hamiltonian,

$$H = H_d + H_p + V, \tag{1}$$

where H_d is the Hamiltonian of the daughter nucleus, H_p is that of the proton, and V is the proton-daughter interaction. In the weak coupling scheme, the wave function of the parent nucleus is written as

$$\Psi_{JM} = \frac{1}{r} \sum_{J_d l_p j_p} u^J_{J_d l_p j_p}(r) \left(\mathcal{Y}_{l_p j_p} \otimes \Phi_{J_d}\right)_{JM}. \tag{2}$$

This wave function is labeled by parity, total angular momentum J, and its projection M. In Eq. (2), $u^J_\alpha(r)$ [where $\alpha \equiv (J_d l_p j_p)$ completely labels the channel quantum numbers] is the cluster radial wave function representing the relative radial motion of the proton and the core, and $\mathcal{Y}_{l_p j_p m_p}$ is the orbital-spin wave function of the proton. The daughter wave function, $\Phi_{J_d M_d}$, satisfies

$$H_d \Phi_{J_d M_d} = E_{J_d} \Phi_{J_d M_d}. \tag{3}$$

In the present formalism, the daughter spectrum enters through specifying the energies E_{J_d}. Where possible, the energies E_{J_d} are taken from experiment; otherwise, the spectrum is modeled theoretically. Figure 2 (left) shows a schematic diagram illustrating the energetics of proton emission from a J^π state of an odd-Z parent nucleus to the ground-state rotational band of the deformed daughter nucleus.

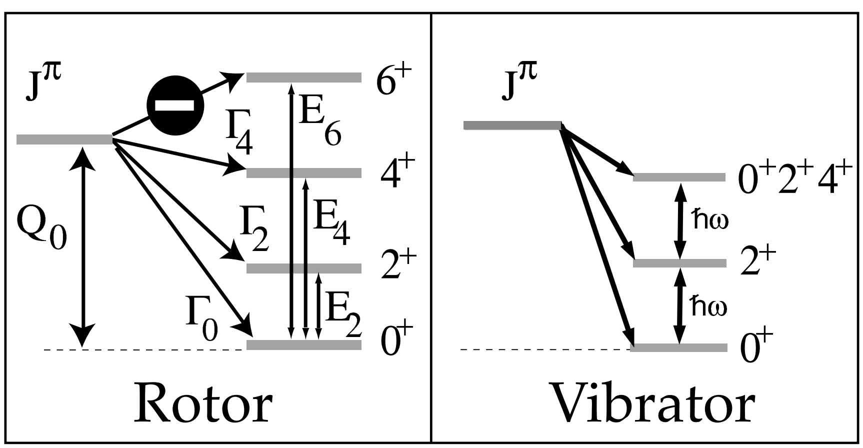

Figure 1. Left: Schematic diagram representing proton decay to a rotational ground-state band in the daughter nucleus. Q_0 is the energy of the resonance state referenced to the daughter's ground state. Right: Proton decay to (quadrupole) vibrational states.

As usual, the coupled-channel equations are obtained by inserting Eq. (2) into the Schrödinger equation and integrating over all coordinates except the radial variable r [9,16]:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l_p(l_p+1)}{2\mu r^2} + V_{\alpha\alpha}(r) - Q_{J_d}\right] u^J_\alpha(r) + \sum_{\alpha' \neq \alpha} V^J_{\alpha,\alpha'}(r)\, u^J_{\alpha'}(r) = 0. \tag{4}$$

In Eq. (4), $V_{\alpha\alpha}$ is the diagonal part of the proton-core potential, Q_{J_d} is the energy of the emitted proton leaving the daughter nucleus in the state J_d, and $V^J_{\alpha\alpha'}$ are the off-diagonal coupling terms. The Q_{J_d} values follow from the spectrum of the daughter nucleus, $Q_{J_d} = Q_0 - E_{J_d}$ where Q_0 is the Q_p value for the decay to the 0^+ ground state (see Fig. 2).

To illuminate the dynamics of the system, one can expand the proton-daughter potential in multipoles [16],

$$V = \sum_\lambda v_\lambda(r)\,(\hat{\mathcal{M}}_\lambda \otimes Y_\lambda)_{00}. \tag{5}$$

The matrix elements $V^J_{\alpha,\alpha'}(r)$ can then be written in the simple, yet generic, form

$$V^J_{\alpha,\alpha'}(r) = \sum_\lambda v_\lambda(r)\,\langle J_d||\hat{\mathcal{M}}_\lambda||J'_d\rangle\,\mathcal{A}(l_p j_p J_d, l'_p j'_p J'_d, \lambda J). \tag{6}$$

The factor $\mathcal{A}$ is purely geometric and comes from the proper coupling of angular momentum vectors. The reduced matrix elements of $\hat{\mathcal{M}}_\lambda$ contain all of the dynamics of the core.

To be a resonant state, the cluster radial wave function must vanish at the origin and behave as an outgoing Coulomb wave, $O_l = G_l + iF_l$, beyond the range of the nuclear interaction and the off-diagonal Coulomb interaction,

$$\begin{aligned} u^J_{J_d l_p j_p}(r) &\stackrel{\text{large } r}{\longrightarrow} O_{l_p}(\eta_{J_d}, rk_{J_d}) \\ &= G_{l_p}(\eta_{J_d}, rk_{J_d}) + iF_{l_p}(\eta_{J_d}, rk_{J_d}), \end{aligned} \tag{7}$$

where $k^2_{J_d} = 2\mu Q_{J_d}/\hbar^2$ and $\eta_{J_d}k_{J_d} = \mu Ze^2/\hbar^2$. These two conditions are only satisfied for a discrete set of complex wave numbers k. The generalized eigenvalues of Eq. (4) correspond to the poles of the scattering matrix [17,18]. The corresponding solutions are either bound or antibound states, $\mathcal{E} = E_b < 0$, with negative real energies and imaginary wave numbers $k = i\gamma$ ($\gamma > 0$ for bound and $\gamma < 0$ for antibound states), or resonance states, $\mathcal{E} = Q - i\frac{\Gamma}{2}$, with a nonzero imaginary part $\Gamma \neq 0$ and $k = \kappa - i\gamma$.

The asymptotic behavior of these solutions is determined by k; at a very large distance the outgoing solution is proportional to e^{ikr}. For resonance states, $e^{ikr} = e^{i\kappa r}e^{\gamma r}$, i.e., the wave function diverges exponentially. As discussed in Refs. [17,18] this seemingly unphysical feature of Gamow wave functions has a natural explanation in the fact that Gamow states do not represent time-dependent wave packets, but static sources.

The nonadiabatic approach allows for a straightforward calculation of branching ratios. We can define the partial width associated with a given channel as [17],

$$\Gamma_\alpha(r) = i\frac{\hbar^2}{2\mu}\frac{u'^*_\alpha(r)u_\alpha(r) - u'_\alpha(r)u^*_\alpha(r)}{\sum_{\alpha'}\int_0^r |u_{\alpha'}(r')|^2 \mathrm{d}r'}. \tag{8}$$

Then the partial width corresponding to the decay to a core state J_d is given by

$$\Gamma_{J_d} = \sum_{\{lj\}} \Gamma_{J_d lj}. \tag{9}$$

Once the total width is known, the half-life for proton emission is

$$T_{\frac{1}{2}} = \frac{\hbar \ln 2}{\Gamma}. \tag{10}$$

The use of the weak coupling scheme represented by Eq. (2) has several advantages. First, excitations of the core are included in a straightforward manner. This enables us to study the proton decay from the rotational bands of the parent nucleus to the ground-state rotational band of the daughter nucleus. Furthermore, since the formalism is based on the laboratory-system description [Hamiltonian (1) is rotationally invariant and the wave function Ψ conserves angular momentum], the Coriolis coupling is automatically included.

3. Rotational Systems

For well-deformed nuclei where we are interested in the ground-state rotational band, the reduced matrix element in Eq. (6) has the simple expression [16]

$$\langle J_d || \hat{\mathcal{M}}_\lambda || J_d' \rangle = \sqrt{2J_d'+1}\, \langle J_d' \lambda\; K0 | J_d K \rangle . \tag{11}$$

Since the nuclei of interest are not well studied, the energy spectrum of the ground-state band is usually poorly known. For proper convergence in the calculation, we have found that the first six levels (up to 10^+) must be included, although the results are rather insensitive to the placement of the levels above the first excited state. A rigid rotor model works quite well to fill in the experimental gaps in the spectrum.

Now let us examine a few examples. Previously [19,20] we have done detailed studies of the highly deformed proton emitters, ^{131}Eu and ^{141gs,m}Ho. We offer just a brief recapitulation of our results here. When comparing lifetimes and branching ratios, we are able to find a unique Nilsson orbital which is consistent with the data. At large elongations, there is little sensitivity to the deformation. This is because the orbitals have taken on their asymptotic properties at this point. The wave function contains large contributions from channels which are energetically forbidden; but the width is dominated by the channel where the proton has the lowest orbital angular momentum. This can lead to cases where the dominant decay channel goes to the excited state and not the ground state, resulting in large branching ratios. We found that the unobserved $[532]\frac{5}{2}$ in ^{131}Eu would have a branching ratio above 50% due to this effect.

The nucleus ^{109}I offers an interesting case. It is expected to be modestly deformed [21], so it is unclear whether the rotor assumption is justified. However, let us stick with this picture for the time being. The predicted lifetimes for two orbitals are shown in Fig. 3 as a function of deformation. At small deformations a peak in both curves shows up as a result of a level crossing. (See Ref. [12] for a similar prediction for ^{113}Cs.) As a result of the configuration change, the dependence of lifetimes on deformation is fairly strong in this case.

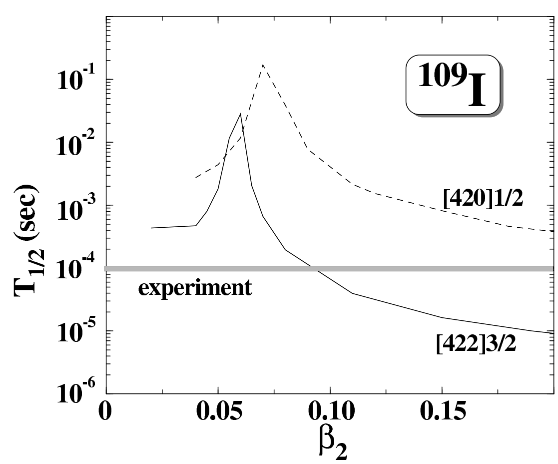

Figure 2. Predicted lifetimes for two orbitals in ^{109}I as a function of deformation. The maxima occur near level crossings. The horizontal line corresponds to the experimentally measured lifetime.

There is currently a proposal at the Holifield Radioactive Ion Beam Facility [22] to look for ^{137}Tb which is the important odd-Z, even-N nucleus lying between ^{131}Eu and ^{141}Ho. Mass formula predictions give a $Q_p \approx 800\,$keV. Figure 3 shows the proton partial lifetimes calculated for three Nilsson orbitals close to the Fermi level as a function of Q_p. Above the horizontal bar, β-decay will completely dominate the proton branch. These calculations show that with current detectors proton emission from ^{137}Tb would only be visible for Q-values above 850 keV in the best case. For these calculations we have assumed a deformation around β_2=0.28 and a rigid rotor ground-state band. Note that if the odd proton happens to reside in the negative parity orbital $[532]\frac{5}{2}$, that one would need an unexpectedly large Q_p-value for the proton branch to be seen. Finally, also note the strong dependence of the lifetime on Q_p falling seven orders of magnitude with a change of only 400 keV in Q_p.

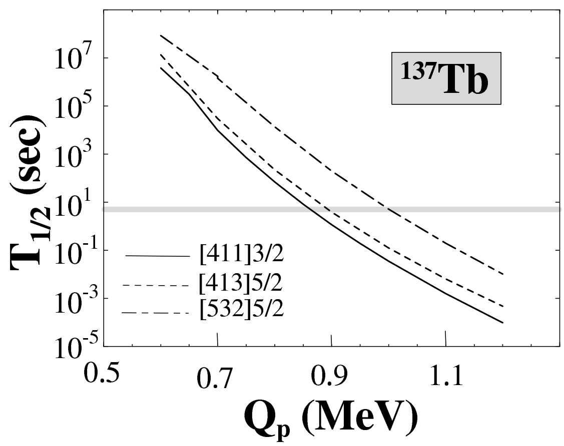

Figure 3. Predicted lifetimes for ^{137}Tb shown as a function of Q_p. A deformation of $\beta_2 \approx 0.28$ is assumed. Current experiments will not be able to see lifetimes above the horizontal line due to the dominance of the β-decay branch ($> 90\%$).

4. Vibrational Systems

Recently we have extended this work to include vibrational nuclei. This is particularly useful when discussing proton emitters near closed shells where the deformation can be rather small ($\beta_2 \leq 0.15$). To achieve this, only a few small conceptual points must be altered.

The first step is to replace the reduced matrix element of Eq. (6). For the rotational case, the operator $\hat{\mathcal{M}}_{\lambda\mu}$ is equivalent to $Y_{\lambda\mu}$. For the vibrational case, it is replaced by the one- and two-phonon excitation operators. These are given explicitly in Refs. [16,23]. In this work, only quadrupole phonons have been considered to this point. Regarding the daughter nucleus, we shall assume that it is a perfect harmonic vibrator so that there is a degenerate triplet of levels ($0^+, 2^+, 4^+$) at twice the energy of the first excited 2^+ state. This is shown schematically in Fig. 2, righthand side. This now completely defines our coupled-channel equations for a vibrational system. The solution method proceeds as before. A comparison of the vibrational and rotational approaches is currently under way and will be presented in the forthcoming publication.

5. Conclusions

We have developed a method to solve the full, non-adiabatic coupled-channel equations for narrow resonances seen in proton emission [15,20]. This has allowed us to include effects of excitations in the daughter nucleus during emission and to consistently calculate branching ratios to excited states. With these tools, we have been able to identify the deformed Nilsson resonances in a number of cases. An extension of these methods to vibrational excitations is currently under way and looks promising. This would allow us to get a detailed description of proton emitters for a wide variety of deformations.

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