# Structural Causality

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## Abstract

Using quantum entanglement as a simple model of correlations between two systems, one can show that a new causal category must be incorporated into any complete description of physical reality. Such "structural causality" is related to the structural properties of the product space underlying a functional description of a multipart entity. Aristotelian causal categories and the restricted derivative notion of causality prevalent in modern science necessarily involve an exchange of matter and energy between causally connected events if change is to occur. Structural causality is fundamentally different in that no matter-energy connectivity is required for events to be causally linked providing that the underlying entities are suitably entangled.

## Aristotelian Causal Categories

Aristotle recognized four causal categories: the material and the formal spring from, simply, matter and form. To these two obvious causes, Aristotle added efficient cause and final cause, but recognized that all four causes may coincide in particular instances. To know is to know by means of causes; causes are answers to "why?" questions. In a relational description of an organism, the organism is closed to efficient cause [Rosen, 1991] and the material cause can be identified with the final cause; that is, the answer to the question "why an organism?" is based in the physical properties of matter and energy and needs no other explanation. The ensuing questions considered relevant to science are descriptive and historical only: how matter and energy interact to give rise to complex forms and what particular historical path led from undifferentiated energy (Schopenhauer's Wille) to the manifest and overwhelming complexity of the observer and observed (Schopenhauer's "subject" and "object").

## 18th and 19th Century Causality

Humean empiricism, so prevalent in modern science, makes do with a single causal category, one that Hume identified with contiguity in space and succession in time, a property that we would call correlation. Hume's skepticism could find no demonstrable way to infer the existence of one object from that of another, leading to the conclusion that, beyond correlation, causality is a construction of the mind, a simple association of ideas. The inability of reductionist science to deal with living organisms is a direct result of this impoverished causal scheme.

Kant went beyond Hume to argue that we do not acquire our ideas of space and time by reflecting on the empirically given since the very description of such empirical situations presuppose familiarity with both space and time. Thus, the notions of space and time are a priori intuitions (a Kantian term), that are built in, as it were, by the evolutionary process, as one might say today. Schopenhauer, following Kant, made these arguments stronger and more forceful.

Schopenhauer's starting point was the simple observation taken from Kant [Schopenhauer, 1969] that "time and space, however, each by itself, can be represented in intuition even without matter; but matter cannot be so represented without time and space." Schopenhauer evidently gives primacy to the material cause when he states that "the subjective correlate of matter or of causality, for the two are one and the same, is the understanding, and it is nothing more that this." [Schopenhauer, p. 11]. He goes on to state that "the first, simplest, everpresent manifestation of understanding is perception of the actual world." Thus, causality, like time and space are a priori categories built in to human perception. This comprehensive view of causality subsumes Hume's skeptic approach while not ruling out intuitive and speculative insights.

## 20th Century Causality

In the 20th century, with the advent of quantum mechanics and relativity, the view of causal connections changed to a relationship between entities located in space-time. Entities may be taken to include matter and radiation, both forms of energy and manifestations of Schopenhauer's Wille. The last vestiges of Aristotle's causal categories was removed from science by black-listing the question: 'Why?' allowing scientists formulate answers only to 'How?' An example is Feynman's

famous remark on the understanding of quantum mechanics where he posed the rhetorical question, “But how can it be like that?” and answered, “Nobody knows how it can be like that.” Both the question and its answer are clear uses of “why” presented in the guise of “how.” For example, in plain English, one would simply say, “Nobody knows why [quantum entities behave as they do].”

Rosen [1987] recognized that the Aristotelian causal categories must be brought back into science if physics and biology were ever to deal consistently with organisms. The ability to ask ‘why’ must be reintroduced into scientific investigations. To be sure, the often contentious and pointless questions and arguments found in the past under the general label “teleology” must still be avoided. Such ‘why’ questions are still best left for poets to ponder, but Rosen [1987, 1991, 1999] pointed the way toward a rational and meaningful way to ask why within the framework of science.

**Einsteinian Causality**

Einsteinian causality restricts the Humean notion to events that can be connected by a ray of light (events lying within the so-called light cone). The prevalent view in modern physics harkens back to the naïve view of Humean correlation, but a correlation restricted to the light cone. It is tacitly assumed but seldom directly discussed, while maintaining consistency with the concept of the forward light cone, that two events must also actually be connected by matter or energy transfer to be considered causal; a direction is also imposed by the time asymmetry between the forward light cone (the future) and the backward one (the past). This notion of physical causality goes beyond the Hume’s idea and permits an extension of the causal beyond mere correlation. Einstein, in his famous 1935 paper [Einstein, Poldosky, and Rosen, 1935], opened the door to a new causal category based on quantum entanglement. Quantum entanglement is simply described on the tensor product of two or more Hilbert spaces and is presented in some detail in a following section.

**Causality in Physics**

Since the classical view of causality presupposes the forbidden question: “Why?”, contemporary science limits use of the word and concept erstwhile found in “causality” to an impoverished version of Hume’s empirical view. Causal connections are viewed in much the same way as James Burke’s Connections: amusing and educational but hardly serious science. The notion of causes are replaced by phase-space trajectories and their temporal evolution; as a system evolves in time, its phase-space point moves along a trajectory. The ‘cause’ of the present state is nothing more than its preceding state and the equations of motion. Such a view is computationally fruitful and has lead to amazing advances in science and engineering over the past 300 years. However, this view of causality, while fruitful, presupposes that the system under consideration is described by a Newtonian view to sufficient accuracy to explain (understand?) the relevant behaviors. If one loses sight of this underlying assumption and blindly applies the Newtonian state-space “paradigm” to an arbitrary natural system, confusion and contradiction can result. An example is to view the energy-transfer mechanisms of a cell under the umbrella of equilibrium thermodynamics (the classic example of a state-space, system-trajectory model); a result is the conclusion that muscle efficiencies are greater than 100%, clearly an error if the Second Law (of thermodynamics) has any meaning.

Quite often, understanding is relegated to the realm of “physical intuition” and replaced with view that the differential equation defining the trajectory in phase space is somehow the primary ontological object. World-class scientists have both an extraordinary physical intuition or native understanding of how things work and a highly developed skill in state-space concepts and formalisms; they are able to avoid most of these conceptual traps. While understanding causal connections, they are able to work within the framework that restricts causality to correlations.

# Structural Causality

To formulate the ideas underlying structural causality and to support the assertion that it represents a separate and novel causal category, a few of the basic concepts of quantum mechanics and some of the mathematics of Hilbert spaces must be presented. The formalism presented below is an excerpt from a more extensive development [Dress, 1999], and contains just enough detail for the reader to grasp the essentials of entanglement from a mathematical and physical perspective.

The "essence" of quantum mechanics, that is where it differs from classical theories, lies in the behavior of amplitudes, how amplitudes are combined, and how to derive testable predictions, including statements of probability, from these amplitudes. Perhaps the most dramatic departure from any classical theory is to be found in the predicted and observed correlations between quantum entities such as electrons, photons, or atoms that are "entangled." In fact, quantum interference and uncontrollable disturbances described and predicted by the Uncertainty Principle pale beside the behavior of two entangled photons or electrons. Why? Simply because interference phenomena and the Uncertainty Principle have well-understood analogues in classical physics whereas the behavior of two entangled quantum particles has no counterpart—unless one is willing to admit magic or faster-than-light actions-at-a-distance.

**Entangled Dice—A model of entangled entities**

Steps sufficient to obtain entangled states involve simple linear combinations of eigenfunctions of operators on finite-dimensional Hilbert spaces. The interpretation of the square of the absolute value of the complex coefficients in such a linear expansion as a probability measure leads directly to a joint probability table of observing, or obtaining in an experiment, particular states or eigenvalues.

The example for this development will be "magic" dice that have from 1 to $k$ sides with each side distinctively marked. This heuristic allows one to maintain a concrete picture of discrete objects with discrete states. Each die, or coin when $k = 2$, is represented on a $k$-dimensional Hilbert space. The eigenvalues and functions belong to an operator on the product Hilbert space (one Hilbert space for each die) and may be identified with observing the 'up' faces of dice in a tossing

game. The behavior of entangled dice as well as quantum particles is simply and easily explained while their existence may be inferred from experimental observations on a dice game or experiments with photons or electrons.

### *Quantum mechanics on Hilbert space*

Define a Hilbert space $\mathcal{H}$ as a complete inner-product space over the complex numbers. The completeness property, which assures convergence in $\mathcal{H}$ of sequences of elements of $\mathcal{H}$, is required for dealing with infinite-dimensional spaces necessary for the complete development of quantum mechanics. Since the dice of interest have a finite number of sides, completeness will not be used in the following development and the proofs will be limited to the finite-dimensional case. The term "vector" will be used interchangeably with "function" when referring to elements of $\mathcal{H}$. The complex-number field is essential to describe interference phenomena; as the examples presented below do not require interference, the phases (arguments) of the complex numbers are generally ignored. Any vector in $\mathcal{H}$ may be expanded as a linear combination, with complex coefficients, of a suitable basis set of $\mathcal{H}$. Dirac notation is a convenient way to represent vectors and operators in a Hilbert space, for example, $|a\rangle$ represents a vector with label $a$ (usually an eigenvalue of some operator) while $\langle a|$ represents the corresponding covector. The inner product of two vectors is represented by $\langle a \mid b\rangle = \langle b \mid a\rangle^*$, where * is complex conjugation. If $\{|j\rangle\}$ is a basis of $\mathcal{H}$, then any vector in $\mathcal{H}$ may be represented by a linear expansion on that basis as $|\psi\rangle = \sum_j c_j \mid j\rangle$, where the $\{c_j\}$ are complex numbers. If $|\psi\rangle$ is normalized so that $\langle\psi \mid \psi\rangle = 1$, then $\sum_j c_j\, c_j^2 = 1$.

Define (Bayesian) probability as a mapping from a denumerable set of statements X to the interval [0,1] such that p(x) ≥ 0, p(∅) = 0, and p(X) = 1. The symbol, p(x), read "the probability that statement x is true" or , simply, "the probability of x," is always assumed to be conditioned on some set of circumstances such as an experimental arrangement. This conditioning or contextuality may be explicitly given by the notation x|y which is read "x given that the statement y is true." The mapping p obeys the product and sum rules [Cox, 1946], which state that p(xy) = p(x) p(y|x) and p(x+y) = p(x) + p(y) - p(xy), where x,y ∈ X, "x+y" is the disjunction of the two statements and "xy" or "x,y" is their conjunction. Note that both conjunction and disjunction are associative and commutative, and that conjunction distributes over disjunction as in ordinary arithmetic. A probabil-

ity distribution $\{p_j\colon p_j \in [0,1]\}$ is a particular assignment to, or a mapping from, the elements $x_j$ of a denumerable set of statements, to the interval [0,1] such that $p_j = p(x_j)$.

In quantum mechanics, the physical state of a quantum system is represented by a unique direction in Hilbert space; all proportional vectors represent the same state. The concept of a state comes from Newtonian physics and is taken to be a primitive notion. The postulate tells us that the vector $\alpha|\psi\rangle$, where $\alpha \neq 0$, and $|\psi\rangle$ represent the same state. Any physical observable of a quantum system is represented by a linear, self-adjoint operator on the corresponding Hilbert space, and the square of the absolute value of an amplitude of a normalized vector in a Hilbert space is the probability of obtaining the eigenvalue belonging to that eigenvector in a measurement of the corresponding operator. These postulates of quantum mechanics and the definition of a probability distribution allow one to associate a probability distribution on the discrete set of basis functions or eigenvalues. Suppose the probability of obtaining the eigenvalue $\lambda_m$ belonging to the eigenfunction $\phi_m$ in a measurement of the observable $\mathcal{A}$ on the state $|\psi\rangle$ is $p(\lambda_m)$, then this probability distribution is given by

$$p(\lambda_k) = |\langle\phi_m \mid \psi\rangle|^2 . \tag{1}$$

To take a specific example, consider a die with $k$ sides. The physical-state postulate lets us represent the state of this possibly actual object by a vector in a suitable Hilbert space. If the observable of interest is the number of spots appearing in a toss of the die, the space has $k$ dimensions and we assume that the number of spots that can show in a given toss is an element of the set $Z_k=\{1,\ldots, k\}$. Of course, spots as distinguishing features are arbitrary, so one could have colors or engraved marks or any other symbol set to distinguish between the sides; the assumption is that we can distinguish between each of the $k$ sides in any toss.

*Tensor products of Hilbert spaces*

The single observable on a single Hilbert space adequately represents a game situation where one die is tossed: the states of a single die belong to a simple Hilbert space as shown in the previous section. To describe a game with more than one die, a larger space is needed. The states of a set of dice in a single toss can also be considered as states in a new Hilbert space. The new space is a direct product of the individual, simpler spaces, one associated with each die. For n dice of $k$ sides each, the product space has $k\,n$ dimensions.

Define the direct or Cartesian product of two vector spaces, $\mathcal{H}_1$ and $\mathcal{H}_2$ as the set of all ordered pairs $|a\rangle|b\rangle$ where the first factor belongs to the first space and the second to the second. Such pairs are to represent vectors in the Hilbert space denoted by $\mathcal{H}_1 \otimes \mathcal{H}_2$. A state consisting of two dice is then written as

$$|\,\psi\rangle = \sum c_{i,j}\,\big|\,i\big\rangle\,\big|\,j\big\rangle, \tag{2}$$

and we can now ask questions of the form, "What is the probability, in a single toss of a pair of dice, of observing eigenvalue m for one die and eigenvalue n for the other?" The answers to such questions are joint probabilities for the joint occurrences of the two states in the same experiment or toss. The joint probability for obtaining eigenvalue m for the state on $\mathcal{H}_1$ and the eigenvalue n for the state on $\mathcal{H}_1$ is given by

$$p(m,\,n) = \,|\,\langle m,\,n\,|\,\psi\rangle\,|^2 = \,\Big|\,\sum c_{i,j}\,\langle m\,|\,i\rangle\,\langle n\,|\,j\rangle\,\Big|^2 = \,|\,c_{m,n}\,|^2 \tag{3}$$

It may appear that multiple, $k$-sided dice are quite boring if they are fair (fairness implies uniform marginal probabilities). Note that the implication chain 'uniform conditionals'→'uniform joint'→'uniform marginals'→'dice are fair' is valid as is the converse starting with 'unfair dice.' However, it is easy to show that the assumption of fair dice does not imply uniform conditionals with the single counterexample of the joint probability for two quasi coins given by

$$\mathrm{P}_{e_1,e_2} = \frac{1}{2}\begin{pmatrix} \sin^2\theta & \cos^2\theta \\ \cos^2\theta & \sin^2\theta \end{pmatrix} \tag{4}$$

$P_{e1,e2}$ is uniform with all entries equal to 1/4 only on a countable subset of the continuous parameter $\theta$, namely $\{\theta : \theta = (2n+1)\pi/4, n \in N\}$; for almost all values of $\theta$, $P_{e1,e2}$ is non-uniform with non-uniform conditionals given by $\sin 2\theta$ and $\cos 2\theta$. Yet the marginals are all 1/2 so both coins are fair. This particular joint matrix appearing in underlies the quantum-mechanical "violation" or counterexample to the famous Bell inequalities [Bell, 1987] where $\theta$ is half the angle between the two inhomogeneous magnetic field directions, one magnet for measuring the polarization of electron 1 and the other for electron 2.

### *Entangled Dice*

We are now in a position to explain the subtle yet bizarre behavior of entangled particles. To motivate the definition of entanglement, note that the state of a pair of dice as given above can produce a non-uniform joint probability array with non-uniform conditional probabilities if the $k^n$ amplitudes are not equal in absolute value. Such a situation would seem to imply that the dice would be unfair and hence easily detectable in a game. However, properly entangled dice may be fair yet allow a magician to win consistently in a game of dice. To explain this situation, we need the following definition.

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A function on the product space of $n$ $k$-dimensional Hilbert spaces, $\otimes_j \mathcal{H}_j$ where $j$ ranges from 1 to $n$, is unentangled if it can be written as a single term with $n$ factors, one belonging to each of the $n$ spaces $\mathcal{H}_j$. If the function requires more than one but less than $k$ terms, it is partially entangled; if it requires exactly $k$ terms, it is maximally entangled.

The expansion for a fair coin as in Eq. 2, with all coefficients equal to $\frac{1}{2}$, may be written as the product $(|1\rangle+|2\rangle)/\sqrt{2}$ $(|1\rangle+|2\rangle)/\sqrt{2}$ where the first factor belongs to $\mathcal{H}_1$ and the second to $\mathcal{H}_2$. On the other hand, the vector with $c_{1,1} = c_{2,2} = 0$ given by $|\text{coins}\rangle = c_{1,2}\,|1\rangle\,|2\rangle + c_{2,1}\,|2\rangle\,|1\rangle$ is maximally entangled for any choice of the two remaining amplitudes, neither being zero. Entanglement depends on the particular choice of the amplitudes, which are experimental parameters.

Imagine a game wherein two coins are tossed and the state of the pair is given by the normalized vector $|\text{coins}\rangle$ above with $|c_{1,2}| = |c_{2,1}|$. Suppose that the first coin is observed to land with 'heads' showing while the second one is still in the air spinning. Entanglement implies that the second coin will necessarily show 'tails' when it comes to rest. This is a deterministic event in the

sense that it has probability 1 of happening when conditioned on the knowledge of ‘heads’ for the first coin. The coins never show the same face no matter how many tosses are examined. The magician always wins by betting on ‘opposite’ even though the toss is fair (of course, a particular result for any given coin cannot be predicted with certainty). Entangled coins and dice are indeed “magic” whereas gloves, even with the same joint and conditional probabilities, are not. From a gaming standpoint, even the most naïve “mark” allowed to bet on the occurrence of ‘same’ would recognize immediately that the gloves or shoes are appearing in pairs, the first either left handed or right handed, the second just the opposite.

In some sense, there is a mechanism for paring, for ensuring that paired objects such as gloves are produced from the “glove box.” A mechanism is something that has a function (in the mechanical sense of “the function of X is to perform Y”). The presence of a mechanism renders superfluous the need for actions-at-a-distance; mechanisms can be tenuous such as gravitational fields or involve material connections such as rods and levers. The function of a mechanism is described by means of variables or parameters. One might say that the mechanism is a function (now in the mathematical sense of the word) of the variables. A putative mechanism connecting entangled quantum particles is termed a “hidden-variable” theory. The assumption of hidden variables, which implies a yet-undiscovered mechanism, has testable consequences in the form of Bell’s inequalities [Bell, 1987], which restrict the range of certain expectation values of observables. All experimental tests of Bell’s inequalities made over the past 25 years are inconsistent with the assumption of hidden variables. No hidden variables, no hidden mechanisms; there are certainly no ostensible mechanisms governing entangled particles, thus there are no mechanisms.

For describing entangled dice (or electrons), a mechanism is not required. Although the process for producing entangled states is given by the general quantum-mechanical prescription of “bring two particles together for a certain time, let them interact and then allow them to separate,” quantum-mechanical dice are not manufactured in definite immutable states corresponding to ‘left’ and ‘right’ as are gloves; neither are actual coins or dice, for that matter (exceptions such as two-headed coins being forbidden by the rules). Thus, any two electrons, any two photons, or any two atoms can be entangled by a quantum procedure wherein the two objects form a single pure state

on a suitable Hilbert space; it is the procedure that produces entanglement and the result of the procedure necessitates a description on a product space.

We are now in a position to address the central issue of this discussion: what can a (measured) joint probability table tell us about entanglement? It might appear that a non-uniform joint probability array is a sure sign of entanglement; however, this turns out to be too general as there are non-uniform joint arrays that are derivable from non-entangled particles. To be specific, consider two $k$-sided dice. Leaving the issue of fairness aside for now, the concept connecting non-entangled dice with the joint probability array is one of row or column dependence from ordinary linear algebra.

It is fairly straight forward to prove the following two lemmas.

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A normalized, unentangled state on a product space generates a joint probability array with all rows or columns proportional.

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A $k$ by $k$ joint probability array with all rows or columns proportional represents a normalized, unentangled state on a product Hilbert space.

Combining the negation of the converses of these lemmas proves the following theorem relating joint probabilities to entangled states.

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On a product space, a normalized state is entangled if and only if the corresponding joint probability array has two or more non-proportional rows or columns.

Note that “all rows proportional” is a stronger requirement than merely demanding that the determinant of the joint probability array vanish. A vanishing determinant means merely that at least one row or column is proportional to some other row or column (the vectors of the corresponding rows or columns are parallel), indicating partial entanglement.

Physically, a set of quantum entities is entangled if a measurement of a property of one member of the set allows one to infer information about the others, information that could only otherwise be obtained by performing actual measurements on the remaining entities. To take a specific example, two electrons are entangled in the spin variable if observation of the spin of one electron allows one to infer the probability distribution for the spin of the other beyond that which

is known before a measurement on the first electron. Consider Eq. 1, which is the joint probability array for two spin-entangled electrons, where the array is assumed to have rows labeled with ‘up’ and ‘down’ for the first electron and columns labeled with ‘up’ and ‘down’ for the second. Suppose the spin of the first electron is measured and found to be ‘up.’ It can then be inferred, from the way in which the entangled state was originally prepared, that the probability distribution for the second electron is precisely ($\sin 2\theta$, $\cos 2\theta$) for the two eigenvalues ‘up’ and ‘down.’ Note that this distribution is the conditional distribution of the second electron’s spin conditioned on specific knowledge of that of the first. If the spins were uncorrelated, the joint probability array would give no additional information about the second electron. Presented in this prosaic manner, entanglement seems not at all strange. Indeed, as Bell remarked [Bell, p. 139], “The philosopher in the street, who has not suffered a course in quantum mechanics, is quite unimpressed by Einstein-Podolsky-Rosen correlations [entanglement].” Had Bell’s philosopher met with entangled dice, however, he might have been more curious.

A slightly more dramatic example, one that illustrates the physical meaning of partial entanglement as well, is found in the state representing a pair of $k$-sided dice where $k = 3$. Preparing the state by selecting certain amplitudes rather than others produces the joint probability array shown in Table 1, where $a^2 + b^2 = 1$.

| | ‘1’ on die 2 | ‘2’ on die 2 | ‘3’ on die 2 |
|---|---|---|---|
| ‘1’ on die 1 | $\frac{1}{3}a^2$ | 0 | $\frac{1}{3}b^2$ |
| ‘2’ on die 1 | 0 | $\frac{1}{3}$ | 0 |
| ‘3’ on die 1 | $\frac{1}{3}b^2$ | 0 | $\frac{1}{3}a^2$ |

Table 1

A Joint Probability Array for a Pair of 3-Sided Dice. The rows are labeled with the number of spots for the first die; the columns with the number of spots for the second.

Suppose the first die is observed with one spot ‘up’, corresponding to the eigenvalue ‘1.’ The first row, divided by the sum over the first row, gives the conditional probability distribution ($a^2$, 0, $b^2$) for observing the faces ‘1’ or ‘2’ or ‘3’ for the second die. That this knowledge of which of the three possible probability distributions for the second die was not available until the first die was observed. Incidentally, this distribution is unique among the three possibilities. If the observa-

tion of the first die gave a ‘2,’ then the probability distribution for the second die would be (0, 1, 0), indicating that the second die would necessarily land with ‘2’ showing. This pair of dice is maximally entangled since no row is proportional to any other.

The ability given to infer probability distributions for outcomes on other members of a set of particles by making possibly remote observations on one of them is the physical meaning of entanglement. The mathematical behavior, presented in the theorem its two corollaries, parallels this physical behavior and justifies the above definition of entanglement.

Entangled coins and dice do exist and their behavior is precisely that suggested by the theorem. Electrons and photons are two commonly found quantum “coins”; each has two states of the spin or polarization observable. Entangled photons are routinely produced in the process of parametric down conversion wherein a single photon from a “pump” laser is converted into two fourth-order correlated photons in a non-linear optical crystal. Necessarily, the sum of the energies of the two down-converted photons must equal that of the original pump photon; likewise, their vector momenta must sum to that of the pump photon. The production process thus conserves both energy and momentum and the corresponding state in Hilbert space must reflect this fact.

Can one hope to manufacture macroscopic coins or dice that are entangled? This is still an open question; whatever method is used, it must take strict account of the conservation of both energy and momentum for the particular entities involved. There must be no “leakage” of energy or momentum to or from the environment until the dice are tossed and have come to rest. Conservation laws imply the selection rules that choose certain amplitudes rather than others, producing entangled states. The particular selection of $c_{1,1} = c_{2,2} = 0$ illustrating the definition is a case in point; here the conservation law is that of energy supposing the two basis vectors are eigenfunctions of the energy operator.

## General product spaces

Hilbert space was chosen for the above illustration of quantum entanglement simply because it has been shown experimentally that such entangled entities, both atoms and photons, exist and play a role in quantum computing and information. The formalism of product spaces is not limited to Hilbert space and any space admitting a tensor product is capable of supporting entangled behaviors.

The richness of this product-space formulation was recognized by Robert Rosen [1991] to distinguish descriptions of organisms from those of machines and simulations. In Rosen's formulation, the entanglement of quantum entities is replaced by a more general recursive relation within the Aristotelian causal structure: an organism is a system that is closed to efficient cause. Rosen further goes on to show that a mechanism or simulation of a natural system is described on a direct-sum space of the component parts and that an organism must be described on the direct-product space.

## Contextuality & structural causality

Entanglement in quantum mechanics can serve as a guide on extending the causal categories of Aristotle and Shopenhauer's causality as understanding to include what might be termed structural causality. "Structural" because its concept and action depend on the mathematical structure of product spaces and "causality" because this action, or correlation to harken back to Hume, can be shown to induce change. Quite properly, entangled quantum entities exhibit correlations as described by the joint probability relationships discussed above. The rather bizarre correlations arising from entangled particles have been known for over 25 years. The idea that the manifestations of entanglement go beyond correlations is seldom openly discussed; such possibilities are most often denied by appeal to relativity or common sense. Yet Bell [1987] carefully considered the issues involved and seemed inclined to give up Einsteinian causality and local realism before denying that such influences were impossible.

Furthermore, it would seem that the Kochen-Specker Theorem [Kochen and Specker, 1967] and Fine's Equivalences [Fine, 1982] would preclude just the type of joint-probability correlations

discussed above. However, both of these important theorems make the assumption of non-contextuality to show that quantum mechanics is incompatible with joint probabilities and hidden variables. If one takes experimental arrangements to be contextual in nature, that is, the particular experiment being done determines what answers will be found just as much as the nature of the particles on which the experiment is performed, then both local realism needs to be abandoned and structural causality begins to make realistic sense.

The most direct way to incorporate structural causality into quantum physics is to take the "transactional" approach as developed by Cramer [1986] from Feynman-Wheeler absorber theory. Simply put, a transaction in quantum mechanics is three-part process wherein a system desiring to emit a particle first sends out "offer" waves. These waves are solutions of the relativistic Schrödinger equation and accordingly consist of a temporally forward-traveling part (retarded wave) and a temporally backward-traveling part (advanced wave). The offer wave is absorbed at a suitable absorber and re-emitted, again as retarded and advanced waves. This "confirmation" wave travels back in time to the emitter; if all boundary conditions are satisfied, the transaction is confirmed and an actual absorption takes place at the absorber. All relevant probabilities as found in the usual quantum computations are present and the theory seems fully relativistic.

From the joint-probability examples above, it is a straightforward inference, in view of assumed contextuality, that a change in a configuration from a high probability of interaction of a particular state of one of two entangled particles to a low probability of interaction will alter the probability of finding the second particle in one of its possible states. In this sense, the structure of the product space describing the entanglement of two (or more) particles guides, within context, the changes that a system can undergo. Structure is thus a form of causality.

**Reaction Dynamics**

Consider a catalytic reaction at the quantum level where, perhaps, individual molecules are being assembled from metabolites by cellular machinery. The delicate nature of entanglement, which can be quickly destroyed by interaction with the environment (the "decoherence" effect), is assumed to be preserved within the cell wall. Assume that the catalyst can take two forms, active and passive; these forms are quantum states of the underlying enzyme. The products of the reaction are disposed elsewhere in the cell, perhaps leaving the cell altogether through appropriate gates. Given the contextual nature of quantum entanglement discussed above, it is likely that the active state of the enzyme, which is entangled with its product, is maintained as long as the reaction product interacts with the cell gate in a particular manner (it escapes, perhaps). Once the gate is signaled to close, by presumably external causes, the context of the entangled distributions change, immediately projecting the enzyme onto its passive state. There would be no need for the product to build up to the point where diffusion can provide a signal to shut down the reaction.

Reaction dynamics based on entanglement would appear to be much more efficient than those based on the diffusion equation. Such efficiency would certainly exceed that based on an equilibrium-thermodynamic calculation and might be just what evolution needed to provide a cellular machinery for organisms and life. If entangled, contextually based reactions occur at the molecular level and within a protected environment to guard against decoherence, a new thermodynamics of quantum-entangled reactions will be required.

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