CALCULATING NEUTRINO-NUCLEUS INTERACTIONS

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I briefly review the role of weak interactions on nuclei. I then review the status of calculations necessary to provide high-quality results for a description of relevant experimental data that may come from the proposed ORLaND facility.

1 Introduction

A variety of experiments may be performed using the neutrinos originating from the decay at rest pions that will be produced at the Oak Ridge Spallation Neutron Source. For example, one fundamentally important quantity is the strangeness content of the nucleon which could be measured provided that a useful experiment could be identified.[1] Neutrino interactions with the deuteron (e.g. $\nu_e + d \to \nu_e + n + p$) are very important reactions used by the Sudbury Neutino Observatory (SNO) to study the solar neutrino flux. This reaction has recently been extensively studied in effective field theory,[2] and was shown to depend on one coefficient in the next-to-leading order expansion. SNO is unable to accurately measure this cross section, and a calibration experiment would be extremely valuable.

Weak interactions on nuclei occur naturally during the various phases of supernova evolution. During the final stages of the stellar evolution of a ten to twenty solar mass star, an iron core develops as the nuclear fuel is exhausted. Outward pressure from liberated electrons keeps this core from collapsing until it reaches the Chandrasekhar mass. At this point, electrons are captured by nuclei through the Gamow-Teller resonance, thus depleting the lepton number within the core region. The interaction $e^- + A(N,Z) \to A^*(N+1, Z-1) + \nu_e$ becomes a source of electron neutrinos within the supernovae.

As the collapse proceeds, a bounce develops when the central density in the neutron rich core exceeds nuclear matter density; however, numerical studies indicate that the energy obtained from this nuclear bounce is not enough to overcome the gravitational attraction of the matter. Neutrinos again play an important role as they may deposit their energy to the matter in the stalling shock front, imparting sufficient energy to overcome gravitation. Once a supernovae explodes, neutrinos again become important in r-process nucleosynthesis, particularly of heavy elements through a mechanism of neutrino spallation.

2 Neutrino-nucleus interaction

The formalism necessary to calculate total and differential neutrino cross sections on nuclei has been known for many years.[3] The charged-current cross section at a given incident neutrino energy E_ν is

$$\sigma_{CC}(E_\nu) = \frac{G}{2\pi}\int_{-1}^{+1} d\cos\theta\delta(E_i - E_f + E_\nu - E_l)p_l E_l \mid M \mid^2 , \qquad (1)$$

where E_i and E_f are initial and final energies of the nuclear states involved, E_l and p_l are the energy and momentum of the emitted lepton, p_ν is the neutrino momentum, and $\cos\theta = \frac{p_l \cdot p_\nu}{|p_l||p_\nu|}$. The reduced matrix elements of the transition matrix element M, which connects states i and f

$$\begin{aligned}&\left\langle J_f, T_f, Tz_f \mid\mid \Omega^{J,T} \mid\mid J_i, T_i, Tz_i \right\rangle = \\ &\sum_{j_a j_b} \left\langle J_f T_f Tz_f \mid\mid \left[a^\dagger_{j_a} \times a_{j_b}\right]^{J,T} \mid\mid J_i T_i Tz_i \right\rangle \langle j_a t_a \mid\mid \Omega^{JT}(q) \mid\mid j_b t_b \rangle \qquad (2)\end{aligned}$$

where $J_{i,f}$, $T_{i,f}$, and $Tz_{i,f}$ are the initial and final spin, isospin, and isospin projection of the initial and final nuclei, and j_α and t_α are single-particle spin and isospin labels of the individual nucleons in the nucleus. We represent the expansion of the weak interaction in terms of multipoles Ω^{JT} of a given spin and isospin (JT) multipolarity. We note that Ω carries a momentum transfer dependence q. The one-body matrix elements $\langle f \mid\mid \Omega^{JT} \mid\mid i \rangle$ were tabulated,[4] and are readily calculable. The more difficult computation involves the one-body density matrix elements which are calculated between the final and initial nuclear *many-body* states.

3 Methods of solution to the nuclear many-body problem

Various approaches to solutions of the nuclear many-body problem exist. For light nuclei ($A \leq 8$) Greens Function Monte Carlo methods have been employed to solve for ground-state properties using realistic two- and three-nucleon interactions.[5,6] These methods are, however, limited to light systems and have not been developed for calculating scattering cross sections (although their antecedent Variational Monte Carlo methods have been used to study proton-nucleus reactions.[7])

The shell model provides a tractable solution to the nuclear many-body problem in a truncated model space. Thus one divides the complete Hilbert space into an active and inactive space. The active space is usually referred to as the valence space. One natural form of truncation is to work in a major

oscillator shell. For example, if we choose ^{40}Ca as a core of inactive nucleons, we can work in the region of $21 \leq N, Z \leq 39$. Thus, by solving the resulting eigenvalue problem for a given nucleus, we obtain detailed information concerning its spectroscopy and one-body density matrix.

One must necessarily use many-body perturbation theory to project the realistic nucleon-nucleon interaction into the model space.[8] This approach, while theoretically correct, encounters problems for nuclei towards the mid-shell for a variety of reasons. Perhaps the least well investigated at the moment involves inclusion of a realistic three-nucleon interaction into the many-body perturbation theory. GFMC results seem to indicate that one cannot properly describe the mass dependence of nuclei as a function of nucleon number without the three-body interactions, and thus it is not surprising that the resultant shell-model interaction derived from many-body perturbation theory still requires some modifications by slightly tuning to experimental data. In light nuclear systems such as ^{12}C, it is possible to obtain directly from experimental data enough information to fit nuclear matrix elements.[9] This is also possible in the *sd*-shell (nuclei from ^{16}O to ^{40}Ca).[10] Over the years, good effective interactions have also been developed for the *fp*-shell (iron group nuclei).[11]

Even in cases where reasonable effective interactions are available, one always faces the explosion in the dimension of the many-body basis in which one must diagonalize the effective Hamiltonian. Increasing computer power allows one to incrementally solve for larger problems by diagonalization. Alternate methods of solution based on quantum Monte Carlo techniques have been developed and used in regions where standard diagonalization could not at the time be successfully applied.[12] Another complication comes when one extends calculations beyond one major oscillator shell. In such cases, contamination from spurious center-of-mass effects must be corrected. Furthermore, overlapping methods to account for the giant-resonance regions are probably necessary.

Thus, calculations of the neutrino-nucleus cross sections within a shell model approach will always have a systematic uncertainty arising from a) the effective interaction used; b) the size of the active model space; and c) center-of-mass contaminations.

RPA-based approaches[13] typically begin with Skyrme Hartree-Fock generated basis states, thus determining the mean-field and single-particle occupied levels of the nucleus. The unoccupied levels are obtained by diagonalizing the Hartree-Fock mean field using a harmonic oscillator basis. Thus, the continuum part of the single-particle spectrum is discretized, and discrete particle-hole configurations coupled to the appropriate spin and parity are used as a

basis in order to cast the RPA equations into a matrix form. Hartree-Fock is sometimes augmented by BCS pairing correlations. In this case, uncertainties arise from the effective interactions used and the allowed correlations available for the ground state. The lack of correlations beyond RPA typically causes an increased total cross section and a narrower distribution of strength within the various interaction channels.

Recent applications of continuum RPA and the shell model to ^{12}C scattering give $13.4 - 14.5 \times 10^{-40}$ cm^2 (shell model[15]) and $(18 - 20) \times 10^{-40}$ cm^2 (CRPA[14]) for the total cross sections. The experimental value is 12.3×10^{-40} cm^2.[16] At present theoretical calculations have an associated 25-35% systematic uncertainty.

4 A guide from electron capture

Based on the above assessment, one could argue that we should simply avoid complicated calcuations with which to calibrate and interpret experimental data. This conclusion would be mistaken, as can be shown from investigations of the Gamow-Teller transitions in nuclei. A number of studies have shown that inclusion of the full space is necessary if one wishes to describe the Gamow-Teller strength distributions. Such $0\hbar\omega$ calculations have been carried out in the p-, sd-, fp-, and gds-shells. For example, total Gamow-Teller strengths are well reproduced for a variety of fp-shell nuclei using a modified version of the Kuo-Brown interaction for this region.[17] The strength distributions for several nuclei are shown in Fig. 1. These calculations were carried out using shell-model Monte Carlo techniques.[12] We renormalize the total strength by $1/1.26^2$, which is a standard factor for spin operators in $0\hbar\omega$ model spaces. These shell-model results should be compared with the more naive non-interacting shell-model results of Fuller, Fowler, and Newmann.[18] The total strengths of FFN are usually a factor of 2-10 larger than the full calculation. FFN places that strength into one transition peak (solid verticle line in Fig. 1). Clearly, the strength is fragmented across many states, and the centroid is usually displaced from the FFN value. These results may have serious consequences for the evolution of the collapse phase of type II supernovae.[19,20] The inverse reaction, neutrino scattering or capture by iron, should also be studied both experimentally and theoretically.

5 Conclusions

Several important reactions that have a significant scientific impact may be studied at the proposed ORLaND facility. Each reaction requires sufficient

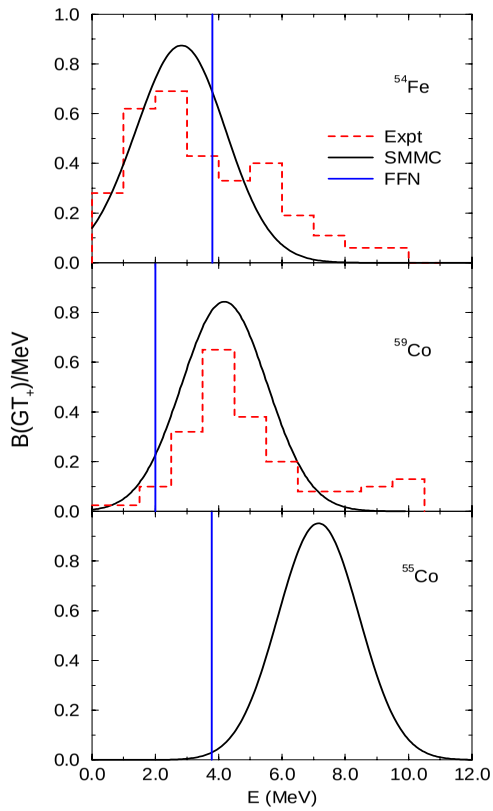

Figure 1. Gamow-Teller strength distributions for selected nuclei. The dashed line represents experimental data, while the solid curve are the shell-model Monte Carlo calculations. Solid vertical lines represent the FFN parameterization.

nuclear structure computation in order to understand the resulting experimental signal. In the Introduction, I mentioned two such experiments: 1) $\nu_e + d \to \nu_e + p + n$, and 2) a well-designed neutral current experiment to detect strangeness content of the nucleon. Additionally, several interesting experiments could be done with iron in order to study neutrino heating. For light systems such as the deuteron, effective field theory may be a useful theoretical framework in which to understand experimental findings. For heavier nuclei (carbon, oxygen, iron) the shell model and RPA will be useful tools.

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