INCORPORATION OF NEUTRON AND GAMMA MULTIPLICITY MEASUREMENTS IN THE ORNL NUCLEAR MATERIALS IDENTIFICATION SYSTEM (NMIS): PASSIVE AND ACTIVE MEASUREMENTS

R. B. Perez, J. L. Munoz-Cobo*, T. E. Valentine, J. T. Mihalczo, and J. A. Mullens

Oak Ridge National Laboratory
P.O. Box 2008, Oak Ridge, Tennessee 37831 U.S.A.
(423) 574-5571

*Universidad Politécnica de Valencia
Chemical and Nuclear Engineering Department

ABSTRACT

A rigorous stochastic theory of the neutron and photon induced multiplicities has been developed.

1. INTRODUCTION

Nuclear material safeguards efforts necessitate the use of non-destructive methods to determine the attributes of fissile materials and to perform non-destructive assay of spent nuclear fuels. Extensive work has been done in this field of applications using multiplicity measurements (1-7). In this technique a detector event initiates an inspection time interval of length τ_o. During the inspection time, τ_o, the number of counts including the trigger following the trigger is recorded. After background correction for accidentals, which are obtained from random triggering of the inspection interval, the neutron multiplicities are used to obtain the moments which are related to the ^{240}Pu-effective mass.

This paper deals with a proposal for the incorporation of neutron and photon multiplicity active and passive measurements in Nuclear Materials Identification System s (NMIS). We consider a sample of, V_o (cm^3), containing an effective ^{240}Pu-effective mass was with an average fission rate, F_s $\left(\frac{\text{fissions}}{\text{gr.sec}}\right)$. The sample is interrogated by the neutrons and photons from an external Cf-252 source of, strength S_o, $\left(\frac{\text{fissions}}{\text{sec}}\right)$. The theoretical basis for this work is provided by the results of a general stochastic transport theory of the neutron and photon coupled fields. (8)

2. THEORETICAL BACKGROUND

Let, $P_N(\tau_o)$, be the probability that upon insertion of a neutron and photon source at time 0, there will be N-correlated counts registered during the inspection time interval, τ_o. This multiplet probability is obtained by registering, the number of times that N-counts (including the triggering

event) were measured during a block of duration, τ_o, and then normalizing to the number of blocks. In terms of the P_N-probabilities, one defines the source probability generating function [S P G F], G_s (Z, τ_o):

$$G_s = \sum_{N=0}^{\infty} Z^N P_N \quad , \tag{2.1}$$

that in turn is the basis for the calculation of the qth factorial moment.

$$M_q = \overline{N\,(N-1)\ldots(N-(q-1))} = \left(\frac{\partial^q G_s}{\partial Z^q}\right)1 \tag{2.2}$$

In view of (2.1) and (2.2), one easily obtains the relation below between the factorial moments and the P_N-multiplet probabilities, that is the basis for our theoretical interpretation of multiplicity measurements

$$M_q = q! \sum_{n=q}^{\infty} \binom{n}{q} P_n \quad . \tag{2.3}$$

The SPG function is given by (8)

$$G_s = (Z, t_o) = \exp\{E_s + E_c\} \tag{2.4}$$

with

$$E_S = \sum_j \sum_q e_{sjq} Q_s \left[(T_{s,k} G_k)^j (T_{s,Q} G_Q)^q - 1\right] \tag{2.5}$$

$$E_c = \sum_j \sum_q e_{cjq} Q_c \left[(T_{c,k} G_k)^j (T_{c,Q} G_Q)^q - 1\right] \tag{2.6}$$

where we introduced the operators

$$T_{s,ck} = \int d\mathrm{v} \frac{\mathrm{d}\Omega}{4p} \; c_{s,c}(\mathrm{v}) \; ; \; T_{scQ} = \int d\mathrm{E} \frac{\mathrm{d}\Omega}{4p} \; \Theta_{s,c}(\mathrm{E}) \tag{2.7}$$

$$Q_S = m_{os} \frac{F_s}{V_o} \int_{-\infty}^{\infty} dt \int d\underline{r} \; Q_c = S_o \int_{-\infty}^{\infty} dt \int d\underline{r} \; d(r - \bar{r}_s) \; . \tag{2.8}$$

The neutron and photon kernel probability functions, G_k and G_Q, respectively are determined by non-linear differential integral equations (8). The neutron and photon transport moments, n_q^+ and p_q^+, are given in terms of the kernel probability functions above by

$$n_q^+ = \left(\frac{\partial^q G_k}{\partial Z^q} \right)_1 ; \; p_q^+ = \left(\frac{\partial^q G_Q}{\partial Z^q} \right)_1 \tag{2.9}$$

and satisfy linear transport equations. For instance the first and second moments (q = 1, 2) satisfy the coupled neutron-photon transport equations

$$\underline{M}^+ \overline{V}_1^+ = \overline{S}_1^+ ; \; \underline{M}^+ \overline{V}_2^+ = \overline{S}_2^+ \tag{2.10}$$

where we define the following matrices and vectors

$$\underline{M}^+ = \begin{bmatrix} L_n^+ & -O_g^+ \\ 0 & L_g^+ \end{bmatrix} ; \; \left(V_1^+\right)^T = \left[n_1^+ , \, p_1^+ \right] ; \; \left(V_2^+\right)^T = \left[n_2^+ , \, p_2^+ \right] \tag{2.11}$$

$$\left(\overline{S}_1^+\right)^T = \left[S_{in}^+ , \, S_{ig}^+ \right] ; \; \left(\overline{S}_2^+\right)^T = \left[S_{2n}^+ , \, S_{2g}^+ \right] \tag{2.12}$$

and introduced the operators

$$L^+_n = -1/\mathrm{v} \frac{\partial}{\partial \mathrm{t}} - \Omega . \overline{\nabla} + \sum_T - \left(O^+_S + O^+_{IN} + O^+_F \right) \tag{2.13}$$

$$L^+_g = -1/\mathrm{c} \frac{\partial}{\partial \mathrm{t}} - \Omega . \overline{\nabla} + m_t - O^+_{Sg} \tag{2.14}$$

$$O^+_g = \int dE' \, d\Omega' / 4p \, \{ \overline{m}_c \sum_c (\vec{r}, \mathrm{v}) \, \Theta_c (E') + \overline{m}_{IN} \sum_{IN} (r, \mathrm{v}) \, \Theta_{IN} (E') + \tag{2.15}$$

$$\overline{m}_F \sum_F (\vec{r}, \mathrm{v}) \, \Theta_F (E') \}$$

Where the elastic scattering, inelastic scattering, and fission adjoint operators O^+_s, O^+_{IN} and O^+_F respectively are given by

$$O^{+}_{s} = \int dv' \, d\Omega' \, [\sum_{s} (\vec{r}, v) \, f_{s}(\vec{r}, v, \vec{\Omega} | v', \vec{\Omega}') + \sum_{SD} (\vec{r}, v) \, . \tag{2.16}$$

$$O^{+}_{IN} = \int dv' \, d\Omega' \, [\sum_{IN} (\vec{r}, v) \, f_{IN}(\vec{r}, v, \vec{\Omega} | v' \vec{\Omega}')] \tag{2.17}$$

$$O^{+}_{F} = \int dv' \, d\Omega' / 4p \; \bar{v} \sum_{F} (\vec{r}, v) \; c(v') \tag{2.18}$$

The gamma scattering adjoint operator O^{+}_{Sg}, is given by:

$$O^{+}_{Sg} = \int dE' \, d\Omega \, [m_{s}(E\vec{\Omega} | E'\vec{\Omega}') + m_{SD} \, g_{D}(E\vec{\Omega} | E'\vec{\Omega}')] \tag{2.19}$$

and for plastic scintillation detectors the adjoint sources in (2.12) are

$$S^{+}_{IN} = e_{n} U_{o}(t | t_{o}) \sum_{SD} \; ; \; S^{+}_{1g} = e_{g} U(t, t_{o}) (m_{aD} + m_{SD}) \tag{2.20}$$

$$S^{+}_{2n} = Q_{T} + 2 e_{n} U_{o}(t | t_{o}) \sum_{SD} H_{nD} \; ; \; S^{+}_{2g} = 2 e_{g} U(t, t_{o}) m_{SD} H_{gD} \tag{2.21}$$

with

$$Q_{T} = Q_{ng} + Q_{2} \; ; \; Q_{ng} = 2(\bar{m}_{IN} \sum_{IN} H_{IN} I_{IN} + \sum_{F} \overline{vm}_{F} X_{F} I_{F}) \tag{2.22}$$

$$Q_{2} = \overline{m_{c} (m_{c} - 1)} \sum_{c_{F}} I_{c}^{2} + \overline{m_{IN} (m_{IN} - 1)} \sum_{IN} I_{IN}^{2} + \overline{m_{F} (m_{F} - 1)} \sum_{F} I_{F}^{2} + \\ + \overline{v(v-1)} \sum_{F} X_{F}^{2} \tag{2.23}$$

where, H_{no}, H_{go}, H_{IN}, I_{IN}, I_{F}, I_{c}, and X_{F} are spectral weighted factors (8) and the following moments of the neutron and pn6 Tc -002. Tc (v'0 TD -0.252 199/F3ors (() Tj ET Q q4 Tc (4F20 0 0 F23390.4 -36 Tj

v0122ÊD -354 -0.1599.122 Ò72-0.06.48 ±asTD 0.053.6j 0 Tc (m16 TD2.01T 27dj 051D -3 Tw (9.04 34 0 TD (8.4 6 TD T

The transport equation (2.11) for the first moments, n_1^+ and p_1^+, are driven by the neutron-photon detector. These moments are in fact the adjoint neutron and photon importances and can be interpreted as the detector field-of-view, whereas the governing equations for the higher transport moments, n_2^+ and p_2^+ are driven by the weighted capture inelastic and fission reaction rates.

3. ACTIVE AND PASSIVE MULTIPLICITY MEASUREMENTS

This section is devoted to the development of analytical expressions for the first and second moments, M_1 and M_2, of the multiplet distributions. In a first step equations (2.3) and (2.4) are used for q=1 and 2. This step yields the M_1 and M_2 moments as functions of the system physical properties and the adjoint moments n_1^+, $p_1^+ \cdot n_2^+$, and p_2^+. These moments are then expressed in the single-mode approximations as a function of the fundamental eigen mode (Φ_o^+, g_o^+), i.e.,

$$n_1^+(1,t) = T_1(t)\,\Phi_o^+(1)\,;\; n_2^+ = T_2(t)\,\Phi_o^+(1)\,;\; p_1^+ = Z_1(t)\,g_o^+(1)\,;$$
$$p_2^+ = Z_2(t)\,g_o^+(1) \tag{3.1}$$

that satisfies the coupled transport equations

$$\underline{M}\,V_o^+ = \alpha_o\,\underline{C}\,V_o^+\;;\;(V_o^+)^T = \left(\Phi_o^+, g_o^+\right);\; \underline{C} = \begin{pmatrix} \frac{1}{v} & 0 \\ 0 & \frac{1}{c} \end{pmatrix} \tag{3.2}$$

where, α_o, is the fundamental eigenvalue (sec^{-1}).

The model amplitudes are determined by inserting equation (3.1) into equations (2.11) and forming the inner product between this result and the vector $(V_o)^T$. Upon insertion of this result into the expressions for the moments and integration over the system volume and over the inspection time, τ_o, one obtains the results

$$M_1/t_o = M_{1c} + M_{1s}\,m_o\,\left(\sec^{-1}\right) \tag{3.3}$$

$$\left(\frac{M_2}{t_o}\right)/t_o = M_{ca} + M_{cb}\,M_{oF} + \left(M_{sa} + M_{sb}\,M_{oF}\right)m_{os} + M_1^2/t_o \tag{3.4}$$

$$M_{1s} = e_n\,H_{gn} + e_g\,H_{sg}\,\left(\sec.gr\right)^{-1}\right) \tag{3.5}$$

$$M_{ca} = e_g^{\,2} H_{1cg} + e_n^{\,2} H_{1cn} + e_g e_g H_{cng} \quad (\sec^{-1}) \tag{3.6}$$

$$M_{cb} = e_g^{\,2} W_{cg} + e_n^{\,2} W_{cn} + e_n e_g W_{cng} \quad (\sec^{-1}) \tag{3.7}$$

$$M_{sa} = e_g^{\,2} H_{1sg} + e_n^{\,2} H_{1sn} + e_n e_g H_{sng} \quad (\sec^{-1})\,(gr)^{-1} \tag{3.8}$$

$$M_{sb} = e_g^{\,2} W_{sg} + e_n^{\,2} W_{sn} + e_n e_g W_{sng} \quad (\sec^{-1})\,(gr)^{-1} \tag{3.9}$$

where, H_{cn}, H_{cg}, H_{sn}, H_{sg}, H_{1cg}, H_{1sg}, H_{1cn}, H_{1sg}, H_{cng}, H_{sng}, W_{cg}, W_{sg}, W_{cn1}, W_{sn}, W_{cng}, W_{sng} are adjoint weigo1.6 Tj ETcb W

4. CONCLUSIONS

This paper sets up the basis for a rigorous stochastic theory of both neutrons and photons multiplicity measurements. As was the case in previous theoretical multiplicity formulations,[2] it was found convenient to express the theoretical results in terms of the factorial moments of the multiplet distribution and to use equation (2.3) for their measured values. Results for the first factorial moments for both neutron and photons are given in terms of average neutron and photon multiplicities and a set of weighting factors that depend on the physical parameters of the system, such as the ^{240}Pu-effective mass, the system multiplication, the neutron and photon spectral distributions, and the detector field-of-view for neutrons and photons. A polynomial dependence of the factorial moments on the Pu-240 effective mass has been found.

The incorporation of photon multiplicities both increases the order of the measured multiplicities and enhances the dependence of the measured moments on the mass of the spontaneous fissioning material (see equations 3.4, 3.8, and 3.9). Finally, the ability to separate the effects of the sample induced and spontaneous fission process will help in the forth-coming challenging problems involved in the evaluation of a diversity of nuclear material samples.

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NMIS Active and Passive Measurements

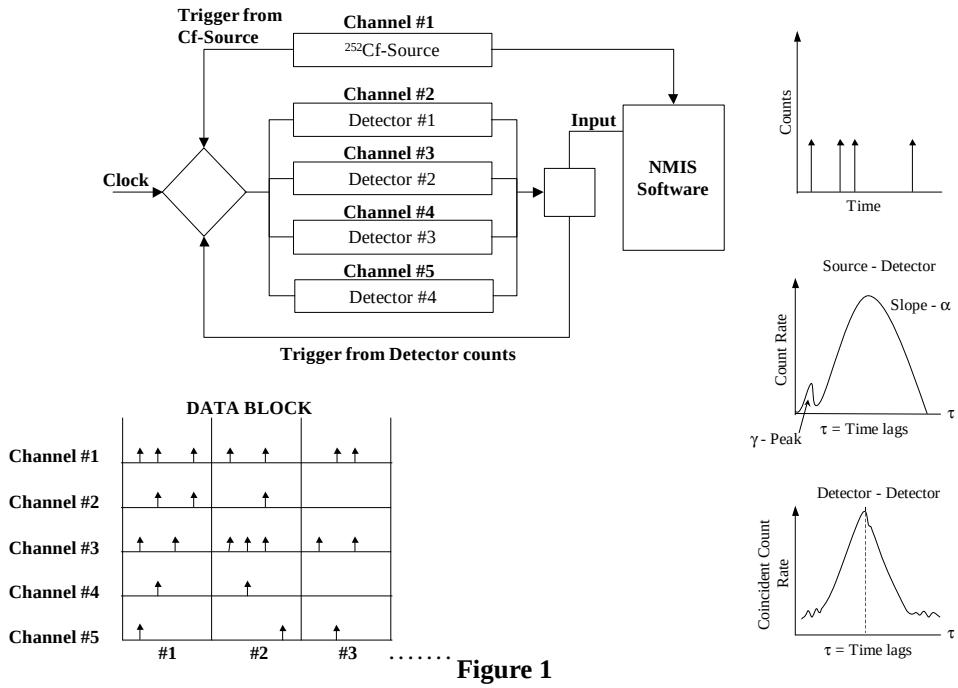

Figure 1

SYMBOLS

$\underline{dr}$ = space volume element
dv dΩ = speed and angular volume element
dE dΩ = energy and angular volume element
(1,t) = $\left(\vec{r}, \mathrm{v}, \vec{\Omega}, t\right)$ for neutrons
(1,t) = $\left(\vec{r}, \mathrm{E}, \vec{\Omega}, t\right)$ for photons
c = speed of light
ε_{jqp} = probability for the emission of j-neutron and q-photons per fission event in the fuel.
ε_{sjq}, ε_{cjq} = probability for the emission of j-neutrons and q-photons per fission event in the spontaneous Cf-252 sources.
$\overline{m_c}, \overline{m_{IN}}, \overline{m_F}$ = average number of photons, from capture, inelastic and fission events in the fuel.
g_{qc}, g_{qIN} = probability for the emission of q-photons per capture (inelastic) events.
$c_s(\mathrm{v}), c_c(\mathrm{v})$ = normalized fission spectrum (spontaneous Cf-252 sources)
$\Theta_s(E), \Theta_c(E)$ = normalized photon spectrum (spontaneous and CF-252 sources)
Σ_F, Σ_s, Σ_{ic}, Σ_F, Σ_{IN}, Σ_r = neutron cross sections
$\Sigma_{SD}(\vec{r}, \mathrm{v})$ = neutron scattering cross section in the detector material
$m_S(\vec{r}, E), m_a(\vec{r}, E), m_T(\vec{r}, E)$ = photon cross sections
$m_{aD}(\vec{r}, E), m_{SD}(\vec{r}, E)$ = photon absorption and scattering cross section in the detector
$f_s(\vec{r}, \mathrm{v}, \vec{\Omega}|\mathrm{v}', \vec{\Omega}')$; $f_{IN}(\vec{r}, \mathrm{v}, \vec{\Omega}|\mathrm{v}', \vec{\Omega}')$; $f_{SD}(\vec{r}, \mathrm{v}, \vec{\Omega}|\mathrm{v}', \vec{\Omega}')$ = neutron scattering probability function for elastic and inelastic processes in the sample, and scattering in the detector material.
$g(\vec{r}, E, \vec{\Omega}|E'\vec{\Omega}')$; $g_D(E, \vec{\Omega}|E'\vec{\Omega}')$ = photon scattering probability function for the sample and detector
ε_n, ε_g = neutron and photon detector efficiencies
$U_o(t \mid \tau_o)$ = square box window function of length τ_o.
M_{oF} = the ratio of the actual mass of fissile material in the sample to a reference mass.