Local One-Dimensional ICRF Full-Wave Solutions Valid to All Orders in $k_\perp\rho$

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Abstract. High harmonic ion cyclotron resonances are important for understanding future fast wave heating experiments on NSTX[1] as well as recent ICRF flow drive experiments on PBX-M[2] and TFTR[3]. Unfortunately, many of our ICRF wave analysis codes are based on an expansion to second order in $k_\perp\rho$ where $k_\perp$ is the perpendicular wave number, and ρ is the Larmor radius. Such codes are limited to cyclotron harmonics less than or equal to 2. Integral codes [4, 5] on the other hand, are valid to all orders in both $k_\perp\rho$ and ρ/L where L is the equilibrium scale length. But velocity space integrals in these codes require long running times. Here we take a simpler approach which assumes a local plasma conductivity ($\rho/L << 1$), while still retaining all orders in $k_\perp\rho$. This allows high harmonic fast wave and flow drive applications, while requiring less computing time than conventional integral codes.

INTRODUCTION

Recently there has been interest in developing full-wave ICRF models which are applicable at high harmonics of the ion cyclotron frequency. This interest stems in part from the National Spherical Tokamak Experiment (NSTX) where the primary ICRF heating system is designed to operate at high harmonics of the majority deuterium frequency[1]. Other examples include recent ion Bernstein wave (IBW) flow drive experiments on PBX-M[2] and TFTR[3] where directly launched high harmonic IBW is used to drive poloidal flow near minority ion cyclotron resonance layers. A realistic theoretical treatment of these cases requires a full-wave model which goes beyond the usual second order finite Larmor radius expansion used in standard finite difference and finite element codes. Integral codes[4, 5] have been developed in one-dimension (1-D) which are valid to all orders in $k_\perp\rho$ as well as ρ/L where ρ is the ion Larmor radius, $k_\perp$ is the perpendicular wave number, and L is the gradient scale length for the equilibrium magnetic field and plasma profiles. But the usefulness of these codes is limited by long computing times associated with calculating velocity space integrals for the plasma current. Here we try a simpler approach which assumes a local plasma conductivity ($\rho/L << 1$) while still retaining all orders in $k_\perp\rho$. The advantage is a much shorter running time than required by conventional integral codes.

SPECTRAL SOLUTION TO THE WAVE EQUATION

To avoid the usual second order gyroradius expansion used in finite difference and finite element codes, we apply a spectral method to solve the vector wave equation,

$$-\nabla\times\nabla\times\mathbf{E}+\frac{\omega^2}{c^2}\left(\mathbf{E}+\frac{i}{\omega\epsilon_0}\mathbf{J}_p\right)=-i\omega\mu_0\mathbf{J}_{ext}\,. \tag{1}$$

The RF electric field **E** and plasma current $\mathbf{J}_p$ can be expanded in Fourier harmonics as

$$\mathbf{E}(x)=\sum_{k_n}\mathbf{E}_{k_n}e^{ik_nx}\,,\qquad \mathbf{J}_p(x)=\sum_{k_n}\widetilde{\sigma}(x,k_n)\cdot\mathbf{E}_{k_n}e^{ik_nx} \tag{2}$$

where $k_n = 2\pi n/d$, and x is the radial dimension in a 1-D perpendicularly stratified plasma model ($0 \le x \le d$). The dependence on x in $\sigma(x, k_n)$ is through plasma equilibrium quantities n_e, T, and B. We typically keep $n \le 2048$ Fourier harmonics in Eqs. (1–2). With this representation, the vector wave equation can be solved for individual Fourier harmonics of the electric field. For $\sigma(x, k_n)$, we apply the well-known local expression for the plasma conductivity with an isotropic Maxwellian velocity distribution. To lowest order in ρ/L, this gives [6]

$$\widetilde{\sigma}(x,k_n)=\frac{\epsilon_0\omega}{i}\sum_{l=-\infty}^{\infty}\frac{\omega_p^2}{\omega^2}\,\zeta_0\;\;e^{-\Gamma_n}\,\tilde{T}_l \tag{3}$$

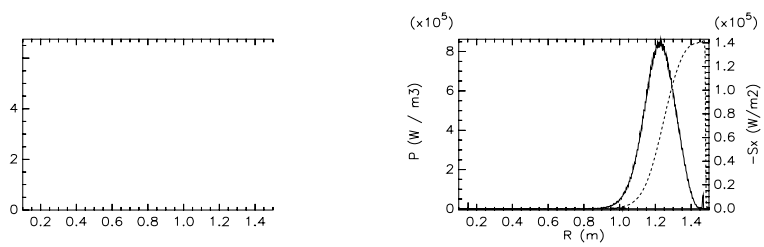
where

$$\widetilde{T}_l\;=\begin{pmatrix}\frac{l^2I_l}{\Gamma_n}Z_l, & -il(I_l-I_l')Z_l, & -\frac{lI_l}{\sqrt{2\Gamma_n}}Z_l'\\ il(I_l-I_l')Z_l, & \left[\frac{l^2}{\Gamma_n}I_l+2\Gamma_n(I_l-I_l')\right]Z_l, & -i\sqrt{\frac{\Gamma_n}{2}}(I_l-I_l')Z_l'\\ -\frac{lI_l}{\sqrt{2\Gamma_n}}Z_l', & i\sqrt{\frac{\Gamma_n}{2}}(I_l-I_l')Z_l', & -I_l\,\zeta_l\,Z_l'\end{pmatrix}. \tag{4}$$

A first order correction in ρ/L could be included by retaining terms in $\sigma(x, k_n)$ that are proportional to gradients in the equilibrium quantities. In Eqs. (3) and (4), I_l is the modified Bessel function of order l with argument $\Gamma_n = 1/2\,(k_n v_{th}/\Omega)^2$, and Z_l is the plasma dispersion function $Z(\zeta_l)$ with argument $\zeta_l = (\omega - l\,\Omega)/|k_z|v_{th}$. The derivative with respect to ζ_l is $Z'(\zeta_l) = -2[1 + \zeta_l Z(\zeta_l)]$. We typically keep $l_{max} \le 20$ ion harmonics in Eqs. (4) and (5). Metallic boundary conditions are simulated by a thin layer of high density plasma at each of the two boundaries, $x = 0$ and $x = d$. Comparison between the spectral code (AORSA) and a second order finite difference solution (ORION1D[7]) shows good agreement when $\omega/\Omega_{c,i} \le 2$ in the region of interest.

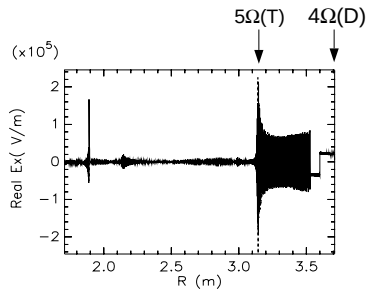
HIGH HARMONIC FAST WAVES IN NSTX

The ICRF heating system for NSTX is designed to operate at high harmonics of the ion cyclotron frequency. In this section, we compare the spectral solution described above with a second order, finite difference solution (ORION1D[7]) for NSTX parameters. In Fig. 1, high harmonic fast waves are launched at 30 MHz into a deuterium (D) plasma with k_z = 10 m^{-1}. The ion and electron temperatures are T_i = 1 keV and T_e = 2 keV, respectively, and the electron density on axis is 3×10^{19} m^{-3}. For these parameters, wave damping is due primarily to electron Landau damping and magnetic pumping near the outer edge of the plasma. High harmonic ion resonances occur throughout the region of interest with $4 \leq \omega/\Omega_{c,D} \leq 28$, but there is little ion absorption for $T_i \leq 1$ keV. Surprisingly, the second order expansion in Fig. 1(a) agrees quite well with the complete solution in Fig. 1(b) which includes up to 20 cyclotron harmonics. This agreement persists as the ion temperature is increased until ion absorption becomes significant. It is interesting that at least 20 ion harmonics are required in the spectral calculation to achieve the agreement in Fig. 1. Apparently, this large number of harmonics is required to accurately represent the ions in NSTX which are approximately unmagnetized[8]. The run time for the spectral case in Fig. 1(b) (with 256 Fourier modes and 20 ion harmonics) was less than 2 minutes on an IBM RS/6000 .



HIGH HARMONIC IBW LAUNCH IN TFTR

Recent experiments on PBX-M[2] and TFTR[3] have used high harmonic IBW to drive plasma flow near minority ion resonance layers. Figure 2 shows a calculation modeled after the TFTR experiment[3] in which 4*th* harmonic IBW is launched from the plasma edge and absorbed near the 5*th* harmonic of minority tritium (T). For $k_z \geq 9$ m^{-1}, there



is a cutoff which prevents the IBW from reaching the tritium resonance. But for the case shown ($k_z = 5\ \mathrm{m}^{-1}$), the IBW penetrates far enough to allow significant absorption at the $\omega/\Omega_{c,T} = 5$ resonance. Of course, this calculation is not possible with conventional second order codes. The run time for the case in Fig. 2 (with 2048 Fourier modes and 10 ion harmonics) was about 100 minutes on an IBM RS/6000.

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