EMPIRICAL CORRELATIONS FOR THERMAL FLOWMETERS COVERING A WIDE RANGE OF THERMAL-PHYSICAL PROPERTIES

J. E. Hardy
J. O. Hylton
T. E. McKnight
Oak Ridge National Laboratory*

Paper prepared for the
National Conference of Standards Labs (NCSL)
1999 Workshop and Symposium
Charlotte, NC
July 19-22, 1999

"The submitted manuscript has been authored by a contractor of the U.S.Government under contract no. DE-AC05-96OR22464. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes."

* Research sponsored by the U.S. Department of Energy and performed at Oak Ridge National Laboratory, managed by Lockheed Martin Energy Research Corporation for the U.S. Department of Energy under contract DE-AC05-96OR22464.

Empirical Correlations for Thermal Flowmeters Covering a Wide Range of Thermal-Physical Properties

Presenter: J.O. Hylton
Oak Ridge National Laboratory
Oak Ridge, TN 37831-6011
Phone (423) 574-0349 Fax (423) 574-1249

Paper Authors: J.E. Hardy, J.O. Hylton, and T.E. McKnight
Oak Ridge National Laboratory

ABSTRACT

Thermal flowmeters can provide direct mass flow measurement of gases and vapors over a wide range of process conditions without the need for density corrections based on pressure and temperature. They are widely used in industrial processes that contain toxic, corrosive, or highly reactive gases. It is often not possible to calibrate the flowmeter on the process gas in which it will be used. In this case a non-hazardous "surrogate" gas is used for calibration, and a theoretical model used to predict the meter's response in the process gas. This can lead to large measurement errors because there are no accurate and straightforward methods for predicting the performance on one kind of gas based on the calibration on another gas because of the complexity of the thermal processes within the flow sensor. This paper describes some of the commonly used models and conversion methods and presents work done at ORNL to develop and experimentally verify better thermal models for predicting flowmeter performance.

1.0 INTRODUCTION

Many industrial applications require accurate flow measurements of various process gases and vapors. Usually the quantity of interest is the mass flow rate or the equivalent volume flow at some reference condition (i.e. standard temperature and pressure). Most gas flowmeters are influenced by fluid density and other properties such that they cannot directly measure the mass flow rate. The process pressure and temperature must also be measured so that an appropriate "density" correction can be made on the flowmeter signal. Thermal flowmeters however, which are based on convective heat transfer effects, have become very popular because they can provide accurate and economical mass flow measurements over a wide range of flow rates that are essentially independent of process conditions. This gives thermal flow meters a significant advantage in that they can be calibrated at ambient conditions on a specific gas and then be used at process conditions to make mass flow measurements without the necessity for monitoring pressure and temperature in order to make density corrections.

If the process gas is highly reactive or toxic, it may be very difficult or impossible to perform a calibration, even at ambient conditions. In these cases it is a common practice to calibrate the meter on a substitute (surrogate) gas that is safer to handle and which matches the thermal characteristics of the process gas as closely as possible. Unfortunately, because of the complexity of the thermal processes within the flow sensor, there is no accurate and straightforward method for predicting the performance on the real process gas. Although most thermal flowmeter manufacturers provide conversion factors from one gas to another, they also state that large errors (300% or more) can occur for process gases that have not been tested by calibration. Because many process gases are so hazardous and difficult to handle, very little work has been done to obtain experimental flow calibration data on them.

Because of the widespread use of thermal meters, especially in the semiconductor industry, there is strong motivation both for mass flow calibrators that will handle hazardous gases, and for better theoretical models in predicting meter performance with different gases. Work has been done at Oak Ridge National Laboratory to develop better predictive thermal models and to develop hazardous gas flow calibrators. This paper presents some results from the thermal model development work. The gas flow calibrator work has been presented elsewhere. [1]

2.0 THERMAL FLOWMETER OPERATING PRINCIPLES

Thermal flow sensors measure fluid velocity by the cooling effect of the fluid motion on a heated sensor in the flow stream. There are two basic configurations of commercially available thermal flowmeters in widespread use today. One type is based on laminar flow inside a heated capillary tube, usually arranged in parallel with a larger laminar flow path. The second type is based on thermal sensors called “hot-wire anemometers” which have been an important fluid dynamics research tool for several decades.

2.1 Laminar Flow Thermal Meters

The laminar flow tube configuration is well suited for low flow rates and is the basis for the popular mass flow elements and mass flow controllers used widely in industry and research. The basic configuration for the flow element is illustrated in figure-1.

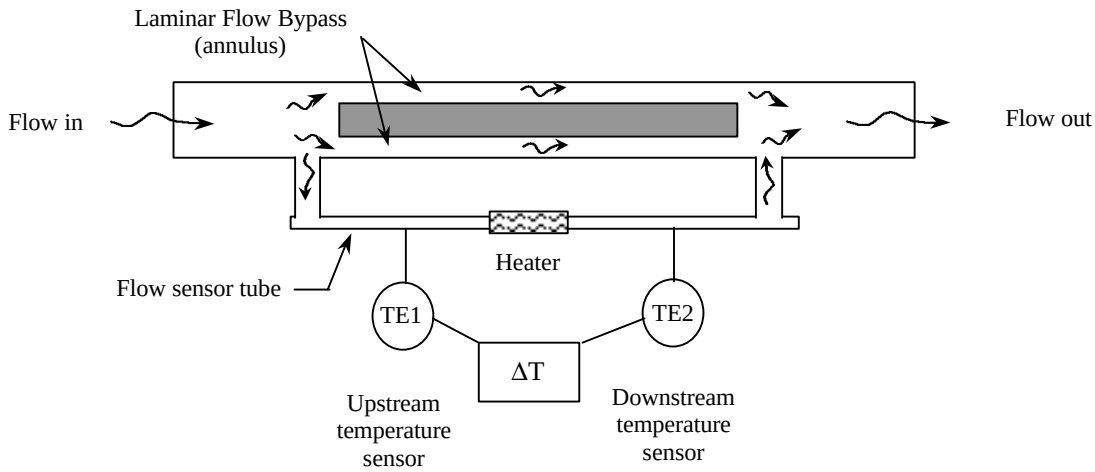

Figure-1: Laminar Flow Tube Thermal Sensor Configuration

The smaller portion of the flow that goes through the sensor tube is proportional to the total mass flow and is measured by the thermal elements as follows;

An energy balance on the flow sensor tube yields the following relationship

(eq 1) $$J_{el} = \dot{m} C_p (\Delta T)$$

J_{el} = Electrical power to the heater that is transferred as heat to the fluid
DT = Upstream / downstream temperature difference (TE1 - TE2)
$\dot{m}$ = Mass flow rate of gas in the sensor tube
C_p = Specific heat (constant pressure) of gas

If the electrical power and the thermal losses are constant, then equation 1 provides a relationship between the mass flow rate and the temperature difference. This relationship is valid until the gas travels so fast that it cannot absorb all of the applied heat as it travels past the heater. Below this limit, however, equation-1 is quite accurate and the mass flow is inversely proportional to the temperature difference. It is common to express the mass flow rate in terms of volume flow at "standard temperature and pressure" (STP) [1].

(eq 2) $$\dot{m} = r_{std} Q_{std}$$

Thus

[1] Standard pressure is generally accepted as 1 atmosphere (14.7 psia) ; Standard temperature is defined differently depending on the user or industry. It is usually 70 degF, 20 degC, or 0 degC. Since there is no universally accepted definition the user must remember to check the value used by the manufacturer.

(eq 3)

Where

rstd = Density of gas at standard temperature and pressure (STP)
Qstd = Volume flow rate of gas at standard temperature and pressure (STP)

The factor *N* , used by some manufacturers, is called a “molecular structure” correction factor. It

because they are small and constant. The ratio of heater power to probe “over-temperature” is a measure of *h*.

(eq-5) $$h = \left(\frac{1}{A_h}\right)\frac{J_{el}}{\Delta T}$$

If the heater power (J_{el}) is held constant, the convective heat transfer coefficient (*h*) varies with fluid velocity and the temperature difference (ΔT) can be used as a measure of the mass flow rate. The procedure is illustrated below.

The *Reynolds* Number of the flow over the sensor is given by

(eq-6) $$\mathrm{Re} = \frac{rvd}{m} = \frac{\dot{m}d}{Am}$$ (*Reynolds* Number)

where

$\dot{m}$ = the mass flow rate of the gas
d = a characteristic dimension of the system (sensor diameter)
A = the cross sectional area of the pipe.
m = viscosity of the gas
v = average free stream velocity of the gas
r = Density of gas at process conditions

The relationship between the convective heat transfer coefficient and the velocity can be very complex, especially in a turbulent flow boundary layer. Empirical relationships, often expressed in terms of dimensionless property groups, can be found in the literature and can be used to obtain a mathematical model of the instrument response.

Exact solutions for the boundary layer equations can be obtained for only a few simple geometrical arrangements. In most cases, however, empirical correlations must be used and are usually expressed in the form;

(eq 7) $$(\quad)(\quad)$$

Where *Nu, Re,* and *Pr* are dimensionless groupings of fluid properties and geometrical parameters, and *f(Re)* and *g(Pr)* are functions of the dimensional groups.

The *Nusselt* Number (*Nu*) is a dimensionless group which characterizes temperature gradients in the boundary layer. The *Nusselt* Number is used to obtain the convective heat transfer coefficient *(h).*

(eq 8) $$Nu = \frac{hd}{k}$$ (*Nusselt* Number)

*T*he *Prandtl* Number is a dimensionless group that characterizes the relative thickness of the velocity profile and the temperature profile in the boundary. It is the ratio of the kinematic

viscosity, which affects the velocity profile, to the thermal diffusivity, which affects the temperature profile.

(eq 9) $$\Pr = \frac{mC_p}{k}$$ (*Prandtl* Number)

where

k = the thermal conductivity of the gas
C_p = Specific heat (constant pressure) of gas

The correlation functions (in equation 7) for various geometrical configurations can be found in the literature. If the correlation functions are known, the relationship between mass flow and sensor temperature can be established.

From equations 5, 7, and 8;

(eq 10) $$h = \left(\frac{k}{d}\right) Nu = \left(\frac{J_{el}}{A_h}\right)\left(\frac{1}{\Delta T}\right)$$

(eq 11) $$Nu = f(\mathrm{Re})g(\mathrm{Pr}) = \left(\frac{d}{k}\right)\left(\frac{J_{el}}{A_h}\right)\left(\frac{1}{\Delta T}\right)$$

(eq 12) $$f(\mathrm{Re}) = \left(\frac{d}{k}\right)\left(\frac{J_{el}}{A_h}\right)\left(\frac{1}{g(\mathrm{Pr})}\right)\left(\frac{1}{\Delta T}\right) = (Jel)\left(\frac{b}{\Delta T}\right)$$

(eq 13) $$b = \left(\frac{d}{k}\right)\left(\frac{1}{A_h}\right)\left(\frac{1}{g(\mathrm{Pr})}\right)$$

The parameter *b* contains only fluid property and sensor geometry terms. Equation 12 can be solved to provide a mathematical relationship between mass flow and *DT*.

3.0 CONVERSION FROM ONE GAS TO ANOTHER

If a thermal flowmeter is calibrated on a “surrogate” gas, then the math models in section 2 can be used to predict the instrument response on the actual process gas if the appropriate fluid properties are known. The accuracy of the prediction depends, of course, on how well the model describes the heat transfer processes taking place in the sensor. This section will illustrate this conversion process for the two different types of thermal meters and discuss the results of efforts to develop and experimentally verify more accurate thermal models.

3.1 Predictive Models for Laminar Flow Thermal Meters

Figure-1 shows the basic configuration of these flowmeters and equation 3 gives the mathematical model used by most manufacturers for converting from one gas to another. Suppose that a Mass Flow Controller (MFC) is calibrated to read Standard Liters per Minute (SLM) of nitrogen, but it is desired to use the instrument in an argon system. Equation 3 can be used to calculate a "conversion factor" for argon.

The output signal (or reading) of the meter is derived from the *DT* produced by a certain flow rate of nitrogen. The "conversion factor" is the ratio of argon flow to nitrogen flow required to produce the same *DT*. This can be calculated from equation 3 as follows:

(eq 14) $$\frac{Q_{ARGON}}{Q_{N2}} = \left(\frac{N_{AR}}{N_{N2}}\right)\left(\frac{r_{N2}}{r_{AR}}\right)\left(\frac{Cp_{N2}}{Cp_{AR}}\right)$$

The *Q's* and the *r's* are the volume flow and density at standard temperature and pressure. MFC manufacturers provide tables of gas properties or conversion factors for many different process gases. They also provide a disclaimer that the inaccuracy of the conversion factor can be 10% or greater.

Although equation 3 is theoretically valid and can be very accurate under certain conditions, there are some practical factors which limit it's accuracy. Equation 3 only measures the flow in the sensor tube. The total flow measurement depends on maintaining a constant ratio between the flows in the sensor tube and the bypass tube. This is affected by such factors as entrance length boundary layer development, inertial losses, thermal losses, and flow path design. McKnight [2] has performed an extensive theoretical and experimental analysis of laminar flow MFM's to determine the validity of predictions based on equation-3, and to see if better analytical models can improve the predictions. Figure-3 shows a comparison of model predictions and experimental data on argon for three different MFM's. The line labeled "Mfg Published" is the correction factor based on equation 14. The curve labeled "model data" is from a finite element (CFD) model of the sensor and bypass flow paths. As can be seen, the equation-14 conversion factor is in error by nearly 10% at the higher flows, but the CFD model tracks the experimental data within about 3% for most of the points.

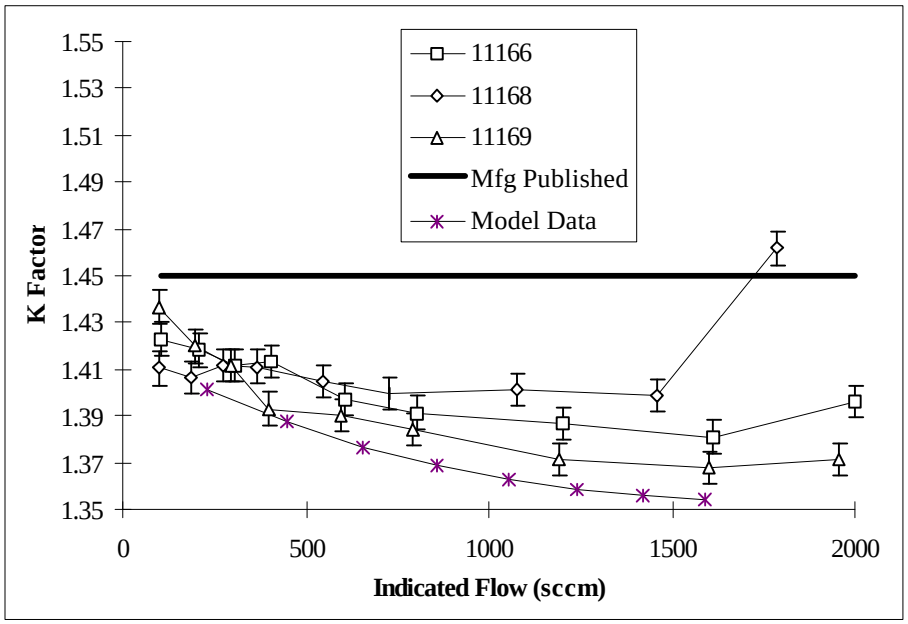

Figure-3: Comparison of Model Predictions and Experimental Data for Laminar Flow Thermal Mass Flow Meters on Argon.

3.2 Predictive Models for Hot-wire Thermal Flow Meters

Figure-2 shows the basic configuration of these flowmeters and equation 12 gives the general form of a mathematical relationship between mass flow, fluid properties, and sensor temperature. Unlike the laminar thermal meters discussed in the previous section, the hot-wire flowmeter operates in turbulent as well as laminar flow. The heat transfer phenomena is somewhat more complicated and does not lend itself well even to CFD computer analysis. It is possible though to calculate conversion factors from one gas to another using equation 12 if an appropriate heat transfer correlation of the form in equation 7 is known. Such correlations for different geometric configurations are available in the literature but they are very sensitive to differences in geometry and can result in errors of over 100% in some cases. Manufacturers will provide conversion factors in some cases but will state that large errors may result unless the meter is calibrated on the new gas.

This section will present the results of work done to obtain conversion factors for a hot-wire thermal meter that was calibrated on air but was intended for use on uranium hexafluoride (UF_6). The geometrical configuration of the sensor is shown in figure 2. The manufacturer provided a conversion factor from air to UF_6 but had no way to experimentally verify it. The flowmeter was

installed in the user's facility where there was also no way to verify the calibration, but there were some indirect means to infer the flow measurement. It was suspected that the manufacturer's conversion factor was in error by a large amount. The purpose of the work presented here was to experimentally verify the air to UF_6 conversion factor for this meter and, if necessary, to obtain a better model.

The use of some literature correlations to calculate conversion factors will be illustrated and compared to actual calibration results on a variety process gases. A new predictive model developed for this particular sensor geometry will also be presented and compared to the experimental data.

3.2.1 Calculation of Conversion Factors for Hot-Wire Sensors

Suppose that the flowmeter has been calibrated on air but it is desired to calculate a conversion factor for use with argon. Equation 12 gives the relationship between Reynolds Number (*Re*) and sensor temperature rise (*DT*). Since the Reynolds Number is directly proportional to mass flow (equation 6), this relates the mass flow ($\dot{m}$) to *DT*. The conversion factor is the ratio of argon flow to air flow that produces the same *DT*. This can be calculated as follows:

a) Calculate the air Reynolds Number for the desired air mass flow using equation 6 and the Prandtl Number for air using equation 9.

b) Calculate the factor *b* for air using equation 13.

c) Calculate *f(Re)* for air using the heat transfer model (equation 7)

d) Calculate *f(Re)* for argon ;

(eq 15) $$f(\mathrm{Re})_{ARGON} = \left(\frac{b_{ARGON}}{b_{AIR}}\right) f(\mathrm{Re})_{AIR}$$

e) Solve equation 15 for the argon Reynolds Number, then solve for the argon mass flow.

f) Convert air and argon mass flows to standard volume flow using equation 2, then calculate the ratio of argon flow to air flow.

3.2.2 Correlations for Calculation of Conversion Factors

The flowmeter was calibrated on several different gases for comparison with calculated conversion factors. The gases and their properties are listed in table –1.

TEST GAS	Thermal Conductivity (W/cm-C)	Specific Heat (Cp) Joules/g-C	Viscosity (μ) Micro-poise	Density 20 C & 1 atm mg/cc
Air	2.62E-4	1.007	186	1.209
Argon	1.79E-4	0.520	229	1.667
SF_6	1.41E-4	0.6663	156	6.012
CO_2	1.68E-4	0.775	149	1.837
Helium	15.67E-4	5.19	201	0.167
Ammonia	2.4E-4	2.13	101	0.7354
UF_6	0.783E-4	0.377	193	14.57

Table-1: Test Gases and Properties

Two different literature correlations were used to calculate conversion factors.

The first was a correlation by Whitaker [3] for single cylinders in cross flowing gases or liquids for Reynolds Numbers from 1.0 to 100,000 and Prandtl Numbers from 0.67 to 300.

(eq 16) *Whitaker*

The second was developed by Churchill & Bernstein [4] which also was for cylinders in

3.2.3 Experimental Results

Figures 4 and 5 give comparisons of conversion factors calculated from the three models (eq 16, 17, & 18) to the experimental data for argon and UF_6. The UF_6 data was inferred by comparison to a research device installed in the operating process. The argon data was obtained by calibrating an identical flowmeter on argon in an ORNL calibration loop.

For Argon the ORNL correlation is seen to agree well with the experimental data, and the manufacturer's correlation is within a few percent. Both of the literature correlations are low by 30 to 40%.

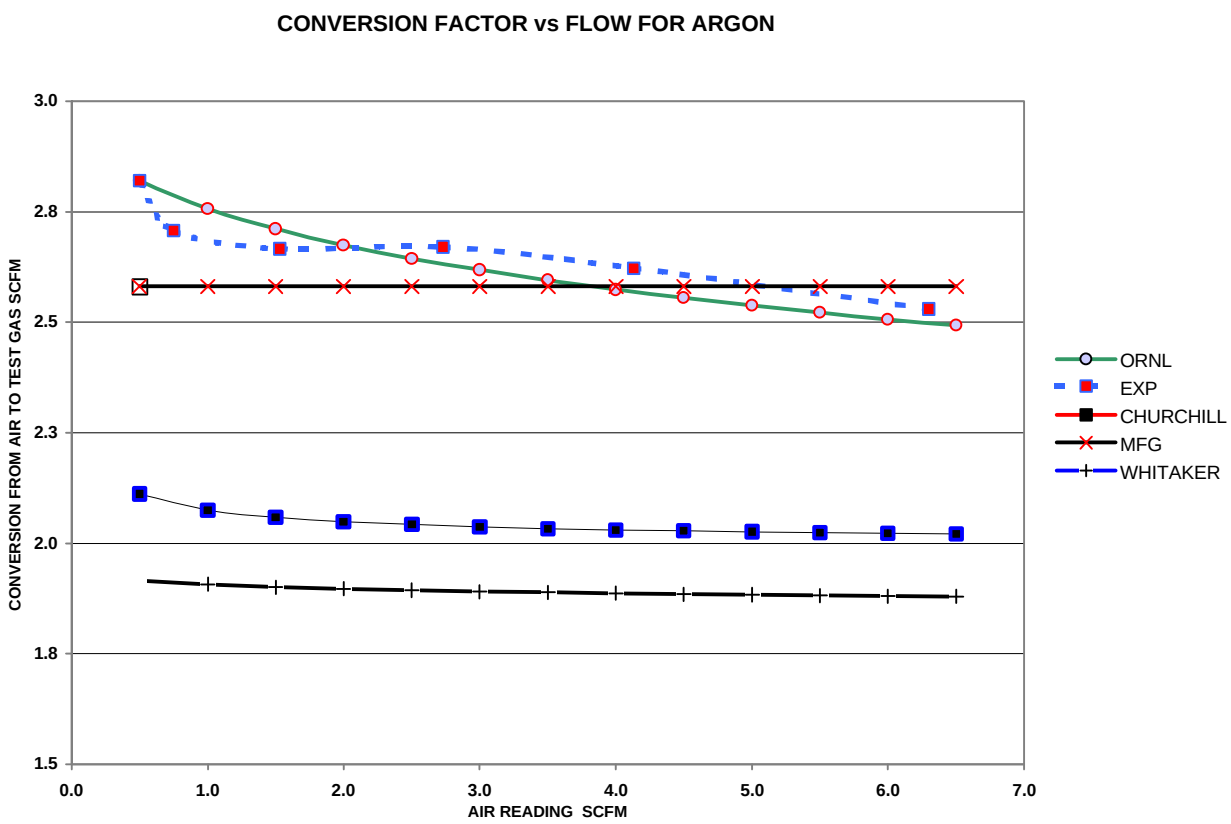

Figure-4: Experimental vs Calculated Conversion Factors for Argon

For UF6 the ORNL and the Churchill correlations appear to track the inferred process flow fairly well. The Whitaker correlation is quite low and the manufacturer's correlation is over 100% too high.

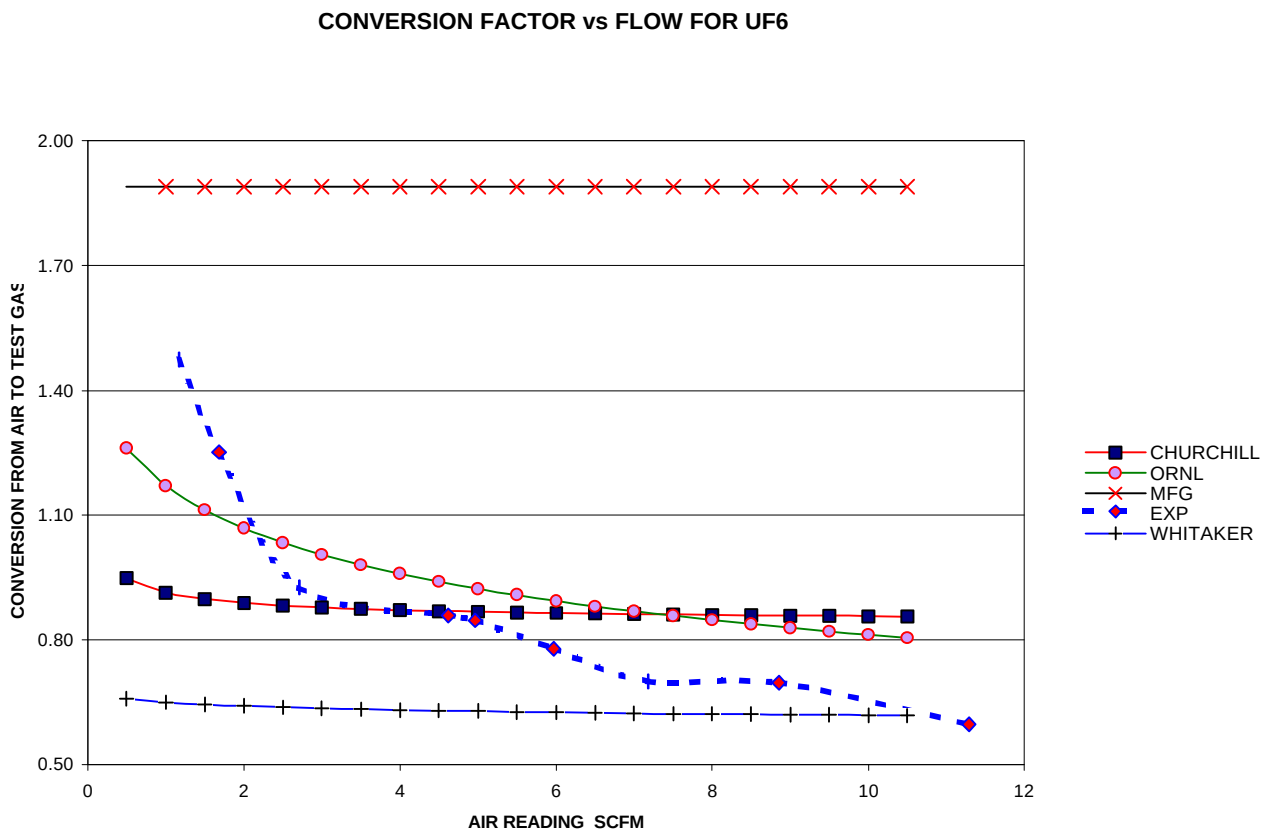

Figure-5: Experimental vs Calculated Conversion Factors for Argon

Results for all of the gases are shown in table-2.

GAS	Prandtl No	Experimental Conv. Factor	ORNL Correlation	Churchill & Bernstein	Whitaker Correlation
Air	0.7149	1.000	1.000	1.000	1.000
Argon	0.6653	2.769	2.616	2.043	1.8915
Helium	0.6657	0.062	0.0781	0.171	0.3020
CO2	0.6874	1.521	1.682	1.3466	1.2217
SF6	0.7372	0.728	0.752	0.5981	0.5129
Ammonia	0.8985	0.761	0.752	0.951	0.8874
UF6	0.9293	0.868 **	0.851	0.8637	0.6221

Table 2: Comparison of Experimental and Calculated Conversion Factors for Test Gases
(** UF6 experimental factor estimated from in-process observations)

4.0 CONCLUSIONS

Thermal flowmeters can provide accurate mass flow measurement of gases in a wide range of process conditions without having to make density corrections. They are capable of good accuracy (1% or better) if they can be calibrated on the actual process gas. Theoretical conversion from one gas to another is very difficult and can result in large errors, especially if using the manufacturer's conversion factors.

Theoretical conversion for laminar flow thermal mass flow controllers and mass flow meters can usually be expected to be within 10% or better when using manufacturer's factors based on equation 14. This accuracy can be improved significantly by using CFD models.

Theoretical conversions for hot-wire thermal flowmeters are quite complex and are not readily solvable even with CFD models. Using literature correlations for hot-wires can result in very large errors (>100%) for some gases. Manufacturer's conversion factors can result in similar errors. Empirical heat transfer correlations can be used to calculate conversion factors to within a few percent if the correlations match the geometrical configuration of the sensor. A Suitable correlation can be obtained by calibration on a few surrogate gases which have thermal properties (Prandtl Numbers) in the same range as the process gas.

REFERENCES

1 Hylton, James O and Remenyik, Carl J, "Instrument for Gravimetric Calibration of Flow Devices with Corrosive Gases", 4th International Symposium on Fluid Flow Measurement, June 1999.

2 McKnight, T.E. et al, "Flow Measurement Methods and Applications", John Wiley & Sons, New York, 1999, pp 137-203.

3 Whitaker, S. "Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, Single Spheres, and for Flow in Packed Beds and Tube Bundles", *A. I. Ch. E. Journal*, Vol. 18 (1972), pp. 361-371.

4 Churchill, S.W. & Bernstein, M., "A Correlating Equation for Forced Convection From Gases and Liquids to a Circular Cylinder in Crossflow", ASME Transactions, Vol 99, May 1977, PP 300-305.