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FOR THE DEPARTMENT OF ENERGY

ORNL/TM-2006/157

Neutron Detection Algorithm Development

November 2006

Prepared by
C. S. Blessinger
R. L. York

UT-BATTELLE
ORNL-27 (4-00)

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ORNL/TM-2006/157

Nuclear Science and Technology Division

NEUTRON DETECTION ALGORITHM DEVELOPMENT

C. S. Blessinger
R. L. York

Date Published: November 2006

Prepared by
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831-6283
managed by
UT-BATTELLE, LLC
for the
U.S. DEPARTMENT OF ENERGY
under contract DE-AC05-00OR22725

CONTENTS

Page

LIST OF FIGURES v
ABSTRACT 1
1. GROSS COUNTING WITH A POISSON DISTRIBUTION 1
   1.1 OVERVIEW OF THE PROBLEM 1
   1.2 NATURE OF THE NEUTRON BACKGROUND 1
   1.3 EXPLANATION OF THE FALSE ALARM RATE 4
   1.4 DETECTION OF A NEUTRON SOURCE 6
   1.5 HOW TO CALCULATE THE THRESHOLD 7
   1.6 ROUND-OFF ERROR 7
2. MAXIMIZING THE FIGURE OF MERIT AS A FUNCTION OF DECISION TIME 11
   2.1 SAMPLING TIME AND THE RUNNING AVERAGE 15
3. DETERMINATION OF THE MINIMUM DETECTABLE LIMIT (MDL) AS A FUNCTION OF RPM PARAMETERS 17
4. CONCLUSIONS 21
5. FUTURE WORK 23
Appendix A. CONTRASTED WITH SPRT A-1
   A.1 WHAT IS SPRT? A-1
   A.2 SPRT APPLIED TO RADIATION MONITORING A-2
   A.3 ADVANTAGES OF SPRT A-4
   A.4 DISADVANTAGES OF SPRT A-5
Appendix B. GAMMA DETECTION B-1
   B.1 USING POISSON STATISTICS IN GAMMA DETECTION B-1
   B.2 CALCULATING THE MDL USING GAUSSIAN GROSS COUNTING ALGORITHM B-1

LIST OF FIGURES

Figure		Page
1	A neutron background measured over 100 1-s intervals	2
2	A histogram (shown on linear and log scales) of the neutron background for the same detector measured over 2.5 * 104 s (about 3 d)	3
3	A sample neutron background histogram with an arbitrary threshold	.5
4	The background histogram and a source with 50% detection probability	7
5	Flowchart for calculating the alarm threshold	8
6	A plot of the FAP vs threshold for five backgrounds	9
7	Three background histograms (on two scales) with means of 1, 1.5, and 2	10
8	Left, the motion of a point source; right, the expected count rate	11
9	Figure of merit of an arbitrary point source traveling as a function of count time R = 2 m; υ = 2 m/s	13
10	Five point running sum	15
11	A simulated Poisson background (blue) and a moving source (red, 5 m/s with a pillar spacing of 2.3 m).	16
12	The MDL of a sample detector as a function of false alarm probability and background	18
A.1	Sample Poisson distributions	A-3

ABSTRACT

The detection of neutron sources in commerce is similar to gamma detection in many ways. Geometry issues regarding detector spacing and source speed are identical. The neutron signal from a vehicle must be compared to background to determine if a source is present. The background must be updated periodically to account for changes in the environment.

But there are also some very important differences. There are no natural neutron sources transported in commercial traffic. This means that there are no NORM (Naturally Occurring Radioactive Material) alarms associated with neutrons. All neutron sources found in a vehicle should be labeled and licensed, or they are contraband. Such events are very rare. Neutron nuisance alarms are therefore false alarms, not innocent alarms. False alarms are caused by statistical fluctuations of the background that coincide with the passage of vehicles.* For most of the neutron alarms generated by a RPM (Radiation Portal Monitor), there is no neutron source present.

Because there are no NORM neutron sources and the threat represented by a real neutron source can be severe, the response to each neutron alarm can be intense. Combine this with the fact that most neutron alarms are false (i.e., no source is present), and it can create a situation that is very frustrating for the response personnel. Controlling this false alarm rate is a critical issue.

The scope of this project is limited to development of the alarm decision process. The goal is to develop an algorithm with the best neutron sensitivity possible while maintaining a controlled false alarm rate.

1. GROSS COUNTING WITH A POISSON DISTRIBUTION

1.1 OVERVIEW OF THE PROBLEM

Gross counting has long been used as a method of gamma detection where the goal is optimize sensitivity vs false alarm rate. The problem with directly applying gamma algorithms to neutron detection is that most gamma algorithms assume that the background fluctuations are Gaussian in nature. In general, Poisson distributions apply to all radiation sources, gamma or neutron. At high count rates, a Poisson distribution can be very accurately described by a Gaussian distribution. For large-area gamma detectors used in radiation portal monitors (RPMs), the background count rates are generally very large, generally hundreds or thousands of counts per second. For large-area neutron detectors used in RPMs, the background count rates can still be quite small, often less than a count per second.

In this section, we will rework the gross counting algorithm without the standard assumption of a Gaussian distribution. Instead we will carry the Poisson distribution all the way through.

1.2 NATURE OF THE NEUTRON BACKGROUND

The number of neutrons counted during a specified unit of time is the *count rate*. The count rate can be a fractional quantity if the measurement time and the unit of time that it is expressed are not the same. If this count rate is generated by the background radiation, it is called the *background count rate*. The neutron background count rate is due to electronic noise or cosmic ray interactions.

As shown in Figs. 1 and 2 the background count rate varies with time. There are two causes of this phenomenon. The first reason is the statistical fluctuation of the count rate.

As mentioned above, each count is due to a singular event, a neutron striking the detector and being detected. The exact direction and exact time that a neutron source emits a neutron is essentially

*Neutron nuisance alarms are also caused by detector malfunctions or poor choice of operating parameters. But this report is assuming everything is working as intended.

random. The counts produced by any neutron source (background or not) in a detector can be described as a Poisson distribution whose mean is a dependent on the activity of the source. The mean count rate is μ. The count rate of a specific sample is x.

$$P(\mu, x) = \frac{\mu^x}{x!} e^{-\mu} \tag{1}$$

The standard deviation of the Poisson distribution is equal to the square root of the background.

$$\sigma = \sqrt{\mu} \tag{2}$$

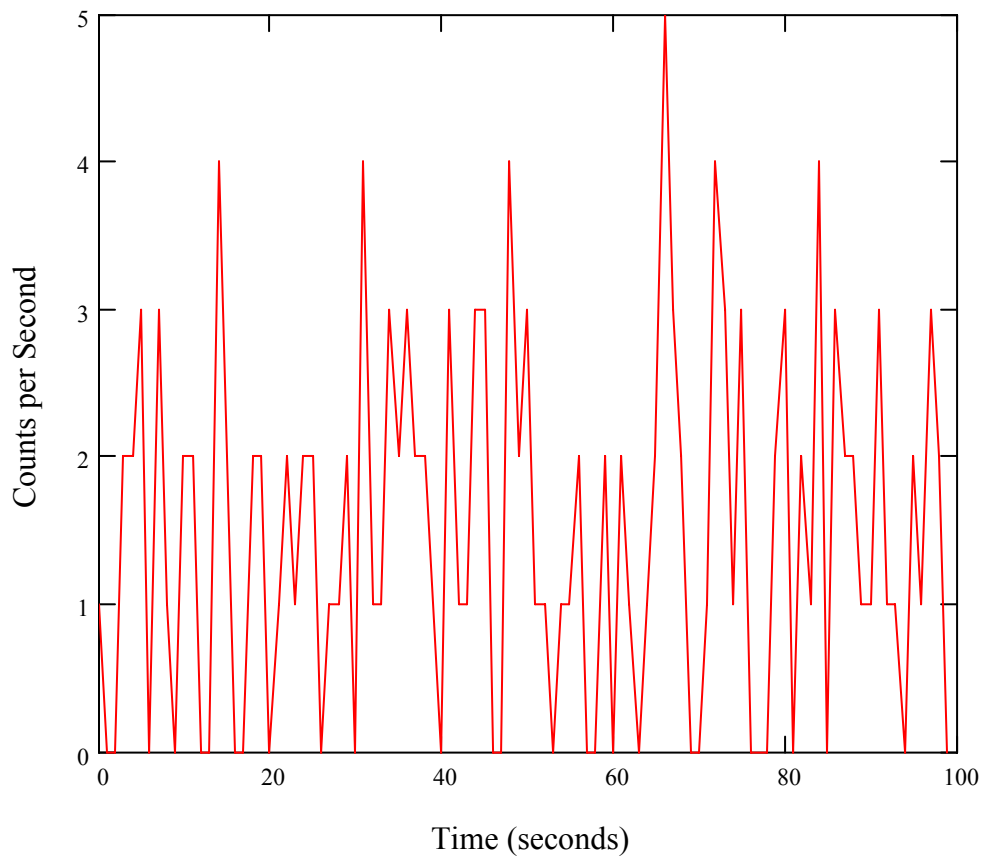

Fig. 1. A neutron background measured over 100 one second intervals.

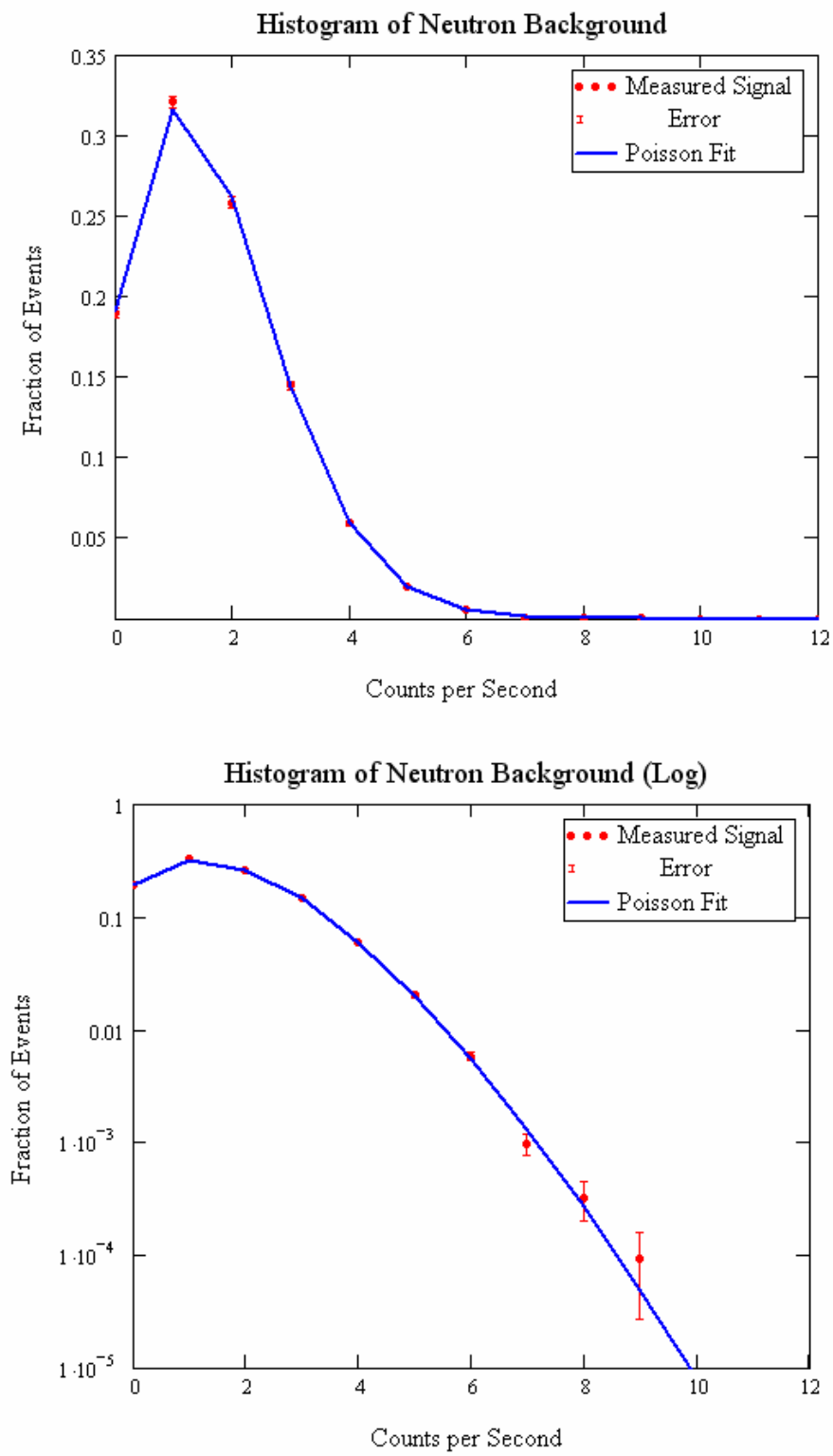

Fig. 2. A histogram (shown on linear and log scales) of the neutron background for the same detector measured over 2.5 * 10^4 seconds (about 3 days). The blue line is a fit of the Poisson distribution made by measuring only the mean of the count rate.

As the background is measured for longer and longer times, the statistical fluctuations cancel out, and the mean value of the count rate is known more exactly. The uncertainty of the background count rate is the *reduced standard deviation* (or the *standard deviation of the mean*). The number of samples is *n*.

$$\sigma_{\mu} = \sqrt{\frac{\mu}{n}} = \frac{\sigma}{\sqrt{n}} \tag{3}$$

The standard deviation of the mean goes to zero as the number of samples goes to infinity. We would like to count for as long as possible to accurately characterize the background, which brings us to the second reason that background changes with time. That is, the background changes with time because the environment changes with time. The neutron background is part of the weather that people not using radiation detectors do not see. Changes in air pressure change the amount of shielding that the atmosphere provides and therefore change the rate of cosmic ray interactions in the detector. In short, the mean of the underlying Poisson distribution changes on a time scale equivalent to that of the weather. This means in turn, that in order to have an accurate value for mean background, the background measurement must be updated at a faster rate. This results in a compromise between statistical accuracy and systematic accuracy. Most RPMs update the background at a rate between once per 30 seconds and once per 15 minutes.

When a previous background measurement is used to predict the background fluctuations in a new measurement, the uncertainty of the mean contributes to the total uncertainty. But in practice, the uncertainty of the mean of the background measurement is small compared to the standard deviation and is generally dropped.

$$\sigma_{total} = \sqrt{\sigma_{\mu}^{\ 2} + \sigma^2} \tag{4}$$

$$\sigma_{total} = \sqrt{\left(\sqrt{\frac{\mu}{n}}\right)^2 + \left(\sqrt{\mu}\right)^2} \tag{5}$$

$$\sigma_{total} = \sqrt{\left(\frac{\sigma}{\sqrt{n}}\right)^2 + \sigma^2} \tag{6}$$

$$\sigma_{total} = \sigma\sqrt{\frac{1}{n} + 1} \approx \sigma \quad \text{when} \quad n >> 1 \tag{7}$$

As mentioned above, the background measurement is typically between 30 and 900 seconds, which results in the uncertainty of the mean contributing between 1.7% and 0.056% to the uncertainty of a one second sample measurement. Throughout the rest of this document we will assume that the uncertainty of the background measurement is negligible compared to that of a particular sample. In other words, the background count time is long compared to the drive through time.

1.3 EXPLANATION OF THE FALSE ALARM RATE

A false alarm is an alarm that is caused by a statistical fluctuation in the normal background. This is distinct from an innocent alarm, which is caused by the passage of a legal source.

Figure 3 shows the probability distribution of the background count rate with an arbitrary threshold set above the mean. From the form of the Poisson distribution, one can see that there is a finite probability of the background producing any count rate. Higher count rates are exponentially less likely to occur. The probability that background alone produces a count rate that is larger than the threshold is the false alarm probability (FAP). The FAP is shown in Fig. 3 as the area under the probability distribution that is above the threshold.

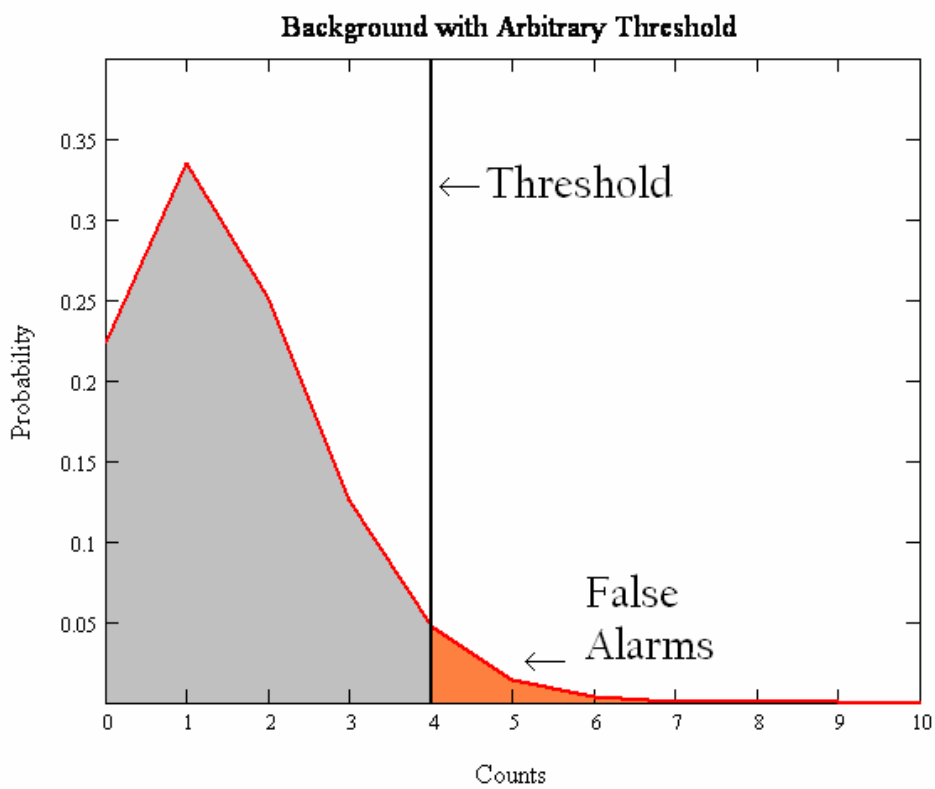

Fig. 3. A sample neutron background histogram with an arbitrary threshold. For any given background there is a finite probability that a specific number of counts will be measured in a given interval. The area under the curve and above the threshold is the false alarm rate.

We can therefore predict the false alarm rate of a given threshold setting from the mean of the background measurement.

The false alarm probability, which is the probability that the background count rate will exceed the threshold K, is given by Eq. (8):

$$FAP(\mu, K) = \int_{K}^{\infty} \frac{\mu^x}{x!} e^{-\mu} \partial x \tag{8}$$

This is actually an infinite sum due to the discrete nature of the distribution:

$$FAP(\mu, K) = \sum_{x=K}^{\infty} \frac{\mu^x}{x!} e^{-\mu} \tag{9}$$

Calculation of the false alarm probability is difficult due to the infinite sum. It can be simplified by calculating the true negative probability and subtracting from one. Equation (10) shows the sum of Poisson distribution calculated for all values of *k* for zero to infinity. The first term is the true negative probability; the second term is the FAP. Equation (12) then gives the FAP in terms that do not include an infinite sum.

$$\sum_{x=0}^{K-1} \frac{\mu^x}{x!} e^{-\mu} + \sum_{x=K}^{\infty} \frac{\mu^x}{x!} e^{-\mu} = 1 \tag{10}$$

$$\sum_{x=K}^{\infty} \frac{\mu^x}{x!} e^{-\mu} = 1 - \sum_{x=0}^{K-1} \frac{\mu^x}{x!} e^{-\mu} \tag{11}$$

$$FAP(\mu, K) = 1 - \sum_{x=0}^{K-1} \frac{\mu^x}{x!} e^{-\mu} \tag{12}$$

Note that the false alarm rate cannot be determined solely by the number of standard deviations from the background as it can be when using Gaussian statistics.* When the false alarm rate is estimated in the field, it is important to remember that the FAP calculated here is per RPM decision not per vehicle.

1.4 DETECTION OF A NEUTRON SOURCE

Radioactive sources generate neutrons that can interact with the detector as well. This is also a statistical process. For any given source, there will be a probability distribution which describes the number of counts we expect to see. Equations (13) and (14) describe the situation, where μ_B is the mean number of counts due to the background and μ_A is the mean number of counts in the detector when the source is present (due to the background and source). Figure 4 graphically shows the relationship of the background count rate and count rate due to a source whose probability of detection is 50%.

$$P(\mu_B, x) = \frac{\mu_B^{\ x}}{x!} e^{-\mu_B} \tag{13}$$

$$P(\mu_A, x) = \frac{\mu_A^{\ x}}{x!} e^{-\mu_A} \tag{14}$$

*Gaussian statistics are commonly applied to gamma detectors.

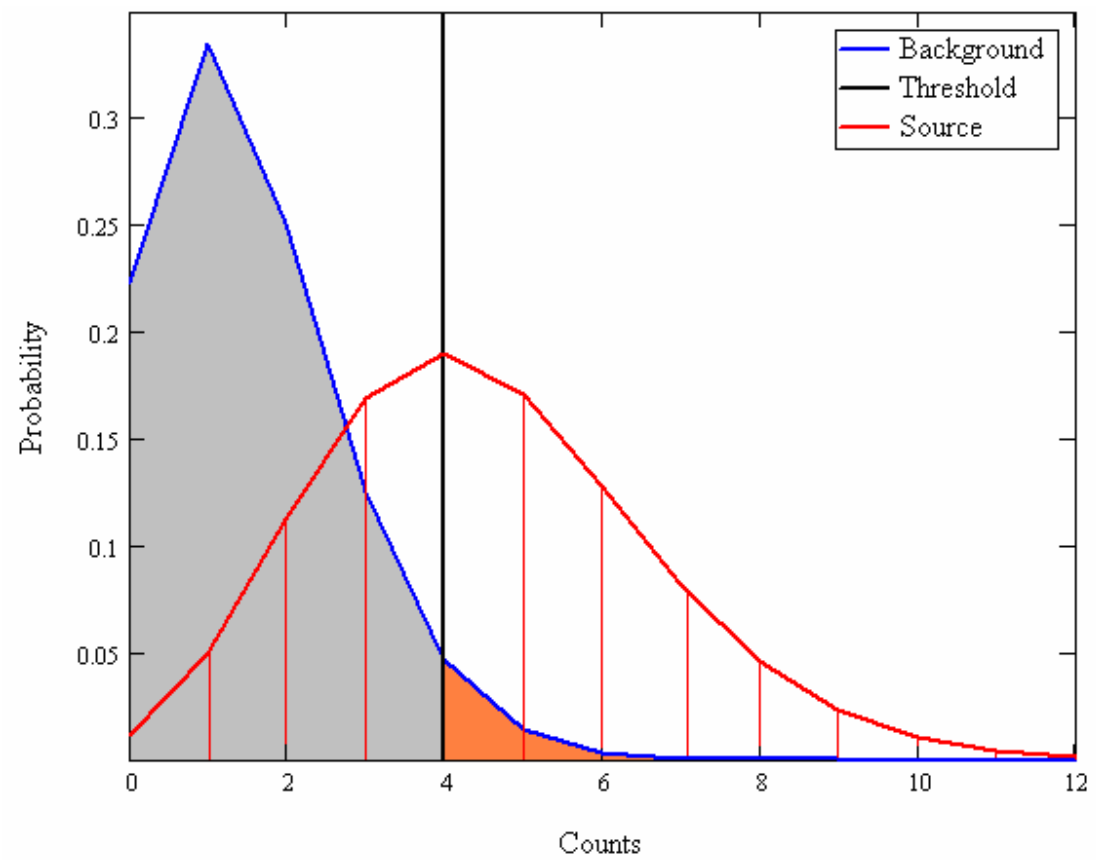

Fig. 4. The background histogram and a source with 50% detection probability. The area of the red plot to the right of the black line represents a 50% detection probability, and the orange shaded area represents the false area rate associated with this detection probability.

1.5 HOW TO CALCULATE THE THRESHOLD

Because the Poisson distribution is discrete, it is not possible to choose the FAP exactly. It is possible to set an upper bound on the FAP and then find the minimum count rate, K, that yields a FAP that is less than the bound. The method is illustrated in the flowchart in Fig. 5. Figure 6 shows the variation of FAP as a function of threshold for different background values.

1.6 ROUND-OFF ERROR

When counting neutrons, only integer values have meaning. A detector never counts half of a neutron. This creates a temptation to use integer arithmetic in all of the calculations. The problem is in the assumption that the error in the neutron background count rate is negligible. The background count rate is an integer number of counts divided by a finite time interval; that is, fractions are important. If the count rate is high (e.g., 400 cps), the error introduced by rounding off to the nearest count per second is small compared to the statistical error of the measurement. Alternatively, if the count rate is low (e.g., 1 or 2 cps), a plus or minus half count per second shift in the background measurement changes the Poisson distribution significantly. Therefore, the round-off error needs to be controlled to limit its effect. Using normal floating point arithmetic reduces round-off error to well below counting statistical error. Figure 7 demonstrates the problem of round-off error. In this case, a half of a count per second round-off changes the false alarm rate by an order of magnitude.

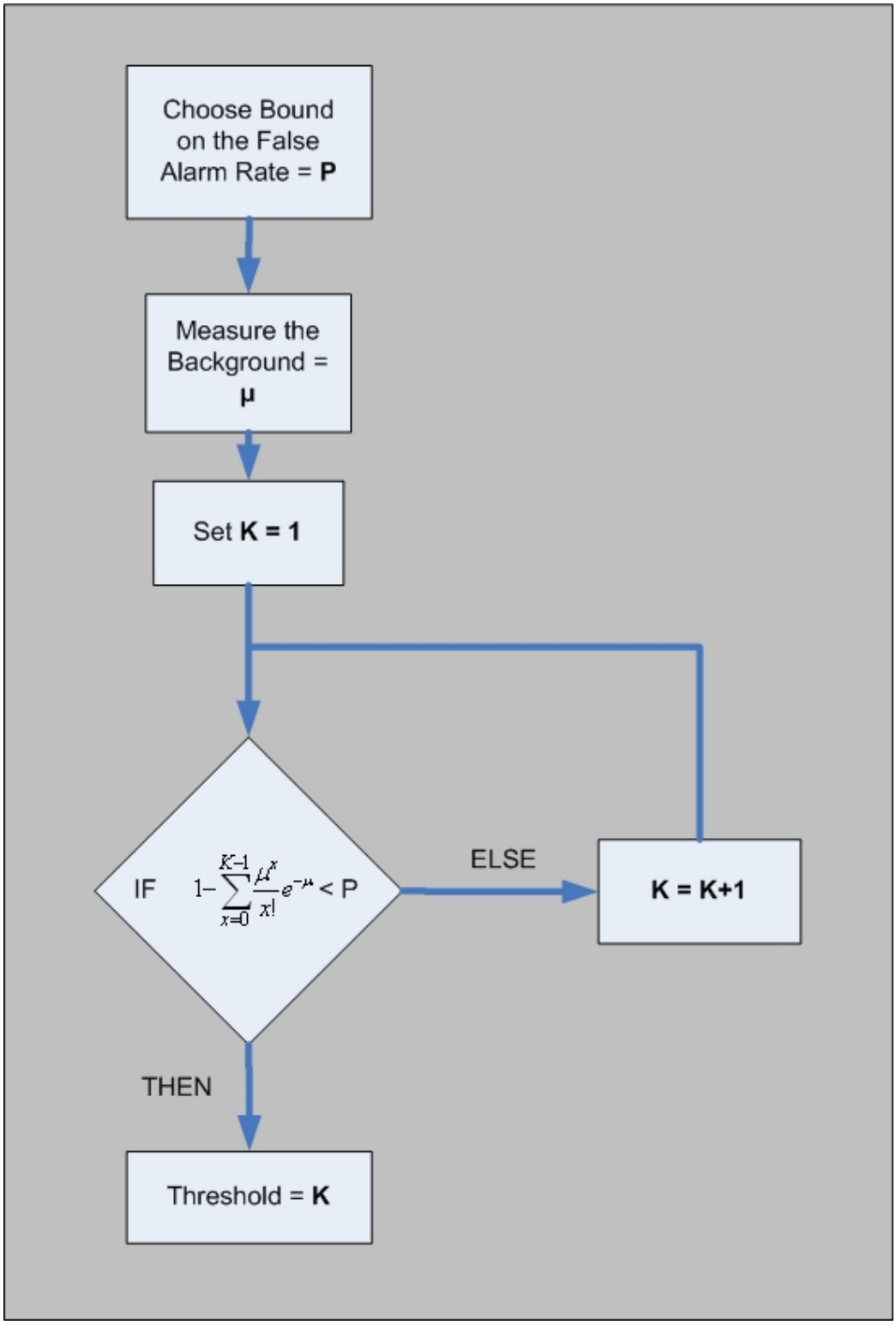

Fig. 5. Flowchart for calculating the alarm threshold.

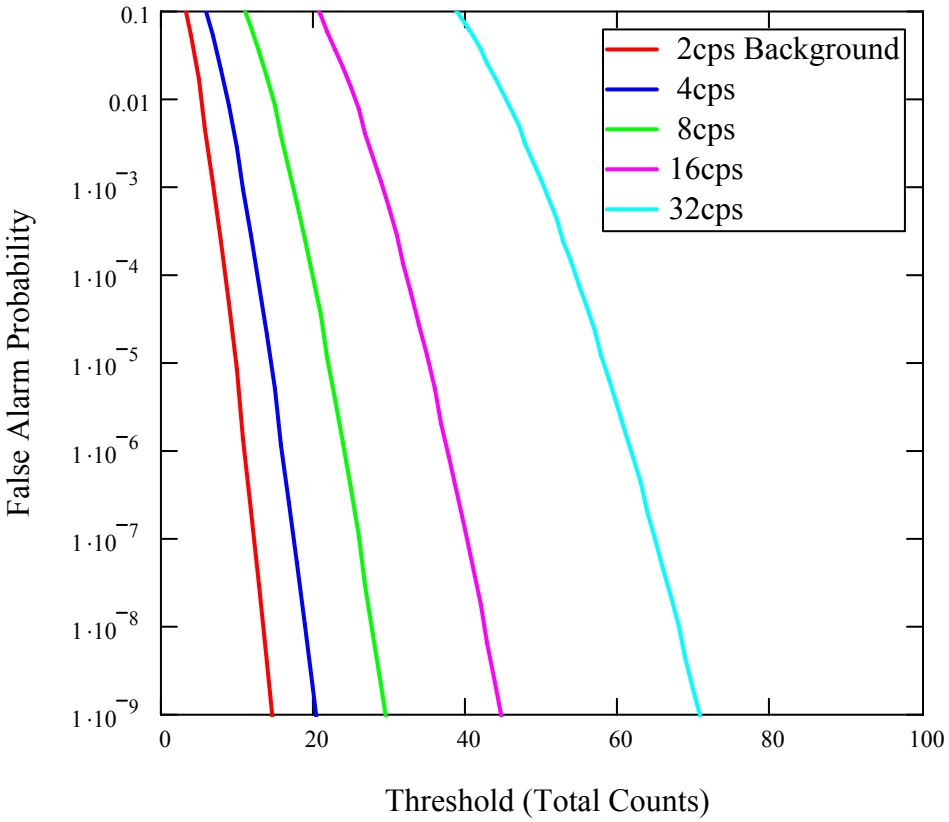

Fig. 6. A plot of the FAP vs threshold for five backgrounds.

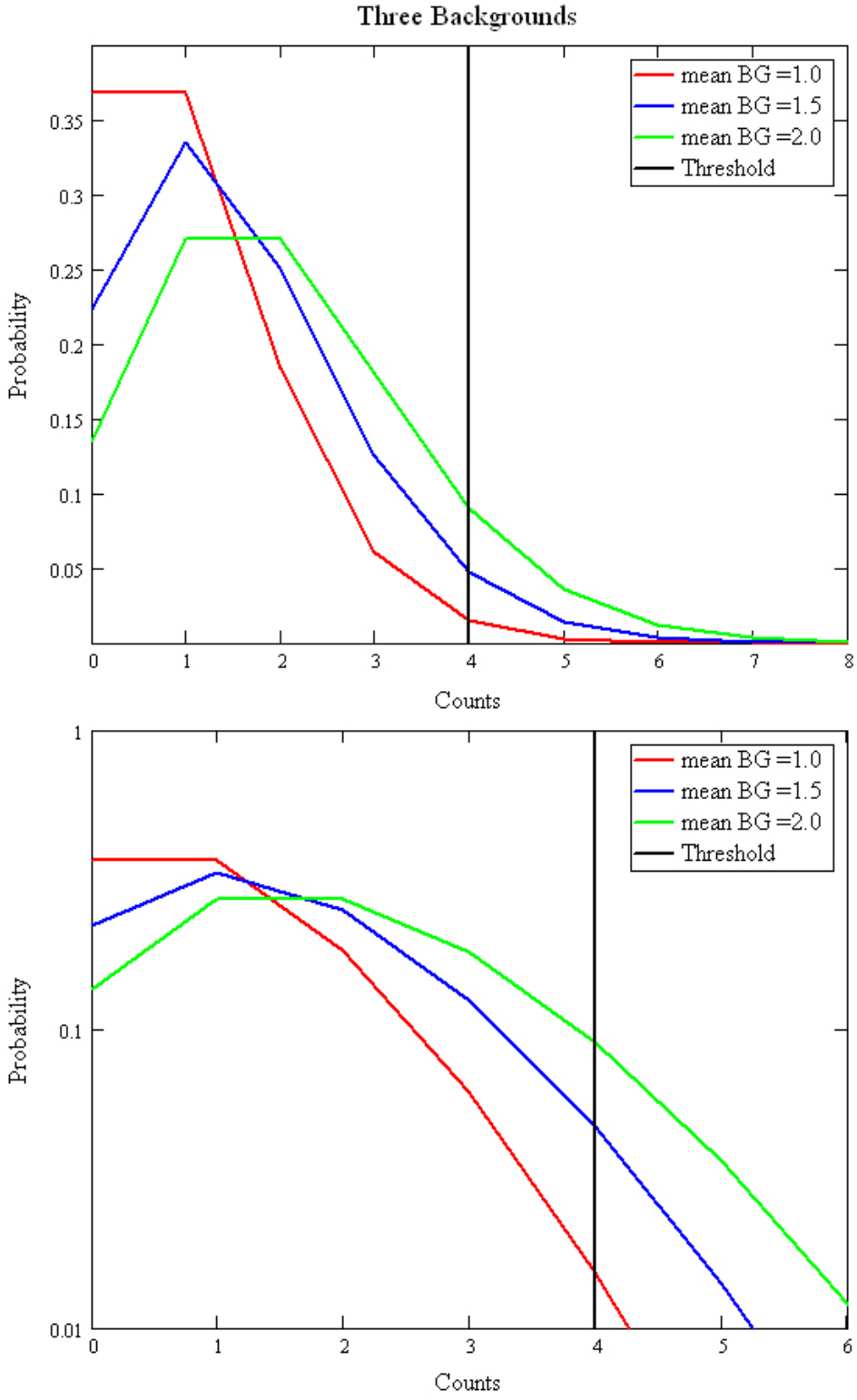

Fig. 7. Three background histograms (on two scales) with means of 1, 1.5, and 2. These plots show the importance of not rounding the background off to the nearest integer when the count rates are low.

2. MAXIMIZING THE FIGURE OF MERIT AS A FUNCTION OF DECISION TIME

Gross counting is an excellent method to measure the smallest possible change in a quantity with a controlled false alarm rate. But it is important to measure the change in the quantity of greatest concern. The detector's solid angle with respect to the source changes with time as the vehicle moves past the detector. To minimize the statistical error, one should count for as long as possible. But counting when the source is not present averages the source strength with the background. This section shows how to maximize the signal strength against the standard deviation of the background. The figure-of-merit (sometimes called the signal-to-noise ratio) is shown in Eq. (15).

$$F = \frac{N}{\sqrt{B}} \tag{15}$$

F is the figure-of merit. N is the total net counts generated in the detector by the source during the decision time. B is the total count rate generated by the background during the decision time.

In Fig. 8 the left plot shows a schematic of a vehicle passing through an RPM, and the right plot shows the expected signal and the background count rate as a function of time. To calculate the figure of merit as a function of the decision time, we need to integrate the expected net counts and the expected background as a function of the decision time. We will begin with the net counts, N, from a point source as a function of position. R is the distance of closest approach; x is the source position on the centerline of the portal. Velocity of the source is υ, and the time is t. The quantity, (N_0/R^2), is the count rate generated by a stationary source at the center of the RPM. N_0 is a function of source activity, source spectrum, detector area, and detector efficiency, etc. If measured for a particular source and RPM, N_0, will be a constant. For a particular RPM spacing, N_0/R^2 will also be constant.

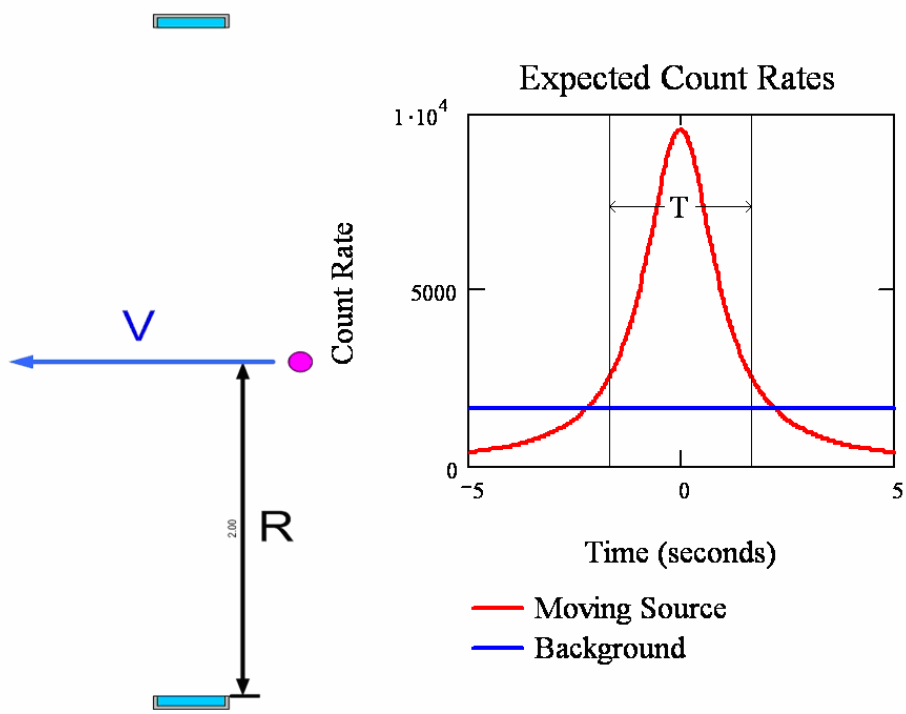

Fig. 8. Left, the motion of a point source; right, the expected count rate.

The net counts, N, as a function of position, x, is expressed in Eq. (16).*

$$N(x) = \frac{N_0}{R^2 + x^2} \tag{16}$$

The net counts as a function of time are given by Eq. (17).

$$N(t) = \frac{N_0}{R^2 + (\upsilon \cdot t)^2} \tag{17}$$

The net counts as a function of decision time, T , centered on the expected peak count rate, can be determined from the following integral. In the next section, we will discuss how to center the decision time on the peak. But for now we will concentrate on how long the decision time should be.

$$N(\mathrm{T}) = \int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} \frac{N_0}{R^2 + (\upsilon \cdot t)^2} \partial t \tag{18}$$

which reduces to

$$N(\mathrm{T}) = \frac{2N_0}{\upsilon^2} \int_0^{\frac{\mathrm{T}}{2}} \frac{1}{t^2 + \left(\frac{R}{\upsilon}\right)^2} \partial t \tag{19}$$

In general, it is true that

$$\int \frac{\partial x}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} \tag{20}$$

Substituting Eq. (20) into Eq. (19) results in

$$N(\mathrm{T}) = \frac{2N_0}{\upsilon^2} \left[\left(\frac{\upsilon}{R} \right) \left(\arctan \left[\frac{\upsilon}{R} \cdot \frac{\mathrm{T}}{2} \right] - \arctan[0] \right) \right] \tag{21}$$

$$N(\mathrm{T}) = \frac{2N_0}{\upsilon R} \arctan \left[\frac{\upsilon \cdot \mathrm{T}}{2R} \right] \tag{22}$$

If μ_B is the background count rate (taken before the arrival of the vehicle), the total background counts as a function of decision time can be approximated as:

*We recognize that this is a simplification of the problem; in reality, this is a three-dimensional problem. The value of R is function of the position within the detector. The change in R is not necessarily trivial in large detectors. But the efficiency of the detector is also a function of the position, and it is not generally known well. If we do not make this approximation, the complexity of the calculation escalates with diminishing returns.

$$B(\mathrm{T}) = \int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} \mu_B \partial t \tag{23}$$

$$B(\mathrm{T}) = \mu_B \left(\frac{T}{2} - \left(-\frac{T}{2} \right) \right) \tag{24}$$

$$B(\mathrm{T}) = \mu_B \cdot T \tag{25}$$

Combining the equations for the net counts and the background counts into the figure of merit yields:

$$F(\mathrm{T}) = \frac{N(\mathrm{T})}{\sqrt{B(\mathrm{T})}} \tag{26}$$

$$F(\mathrm{T}) = \frac{\frac{2N_0}{\upsilon R} \arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R} \right]}{\sqrt{\mu_B \cdot \mathrm{T}}} \tag{27}$$

$$F(\mathrm{T}) = \frac{2N_0}{\upsilon R \sqrt{\mu_B}} \cdot \frac{1}{\sqrt{\mathrm{T}}} \cdot \arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R} \right] \tag{28}$$

For example, Fig. 9 is a plot of the figure of merit as a function of T given, $R = 2$ m and $\upsilon =$ 2 m/s.

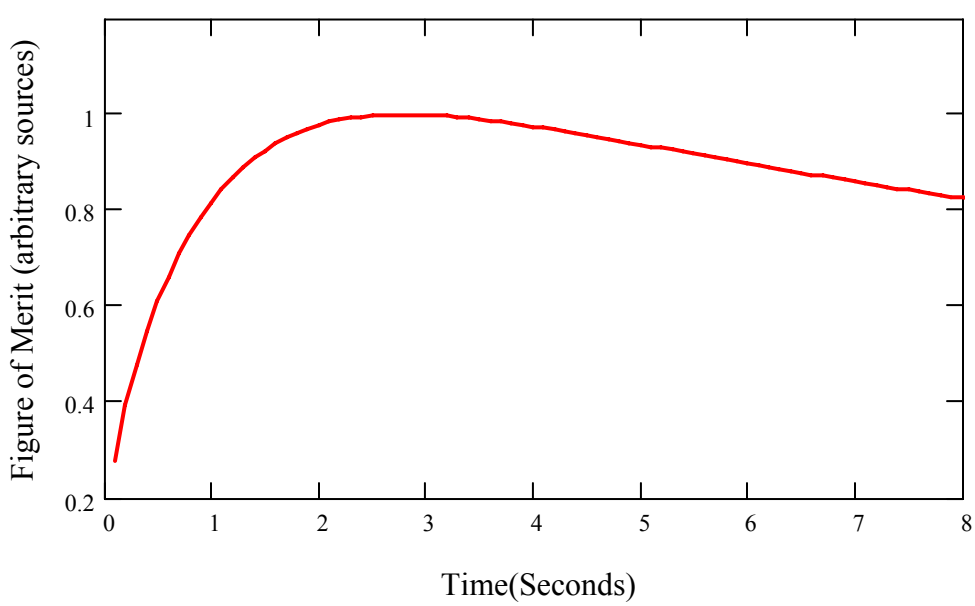

Fig. 9. Figure of merit of an arbitrary point source traveling as a function of count time R = 2 m; υ = 2 m/s.

To find the decision time that optimizes the figure of merit, we can find the root of the derivative of figure of merit equation.

$$\frac{\partial F}{\partial \mathrm{T}} = 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial \mathrm{T}^2} < 0 \tag{29}$$

$$\frac{\partial F}{\partial \mathrm{T}} = \frac{2N_0}{\upsilon R\sqrt{\mu_B}}\left[-\frac{1}{2}\mathrm{T}^{-\frac{3}{2}}\arctan\left[\frac{\upsilon\cdot\mathrm{T}}{2R}\right] + \mathrm{T}^{-\frac{1}{2}}\cdot\frac{\upsilon}{2R}\cdot\frac{1}{1+\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)^2}\right] \tag{30}$$

$$-\frac{1}{2}\mathrm{T}^{-\frac{3}{2}}\arctan\left[\frac{\upsilon\cdot\mathrm{T}}{2R}\right] + \mathrm{T}^{-\frac{1}{2}}\cdot\frac{\upsilon}{2R}\cdot\frac{1}{1+\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)^2} = 0 \tag{31}$$

$$-\frac{1}{2}\mathrm{T}^{-\frac{3}{2}}\arctan\left[\frac{\upsilon\cdot\mathrm{T}}{2R}\right] + \mathrm{T}^{-\frac{3}{2}}\cdot\frac{\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)}{1+\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)^2} = 0 \tag{32}$$

$$\arctan\left[\frac{\upsilon\cdot\mathrm{T}}{2R}\right] - \frac{2\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)}{1+\left(\frac{\upsilon\cdot\mathrm{T}}{2R}\right)^2} = 0 \tag{33}$$

$$\arctan\left[X\right] - \frac{2\left(X\right)}{1+\left(X\right)^2} = 0 \quad \text{where} \quad X = \frac{\upsilon\cdot\mathrm{T}}{2R} \tag{34}$$

Solved numerically yields one solution:

$$X = 1.392 \tag{35}$$

$$\frac{\upsilon\cdot\mathrm{T}}{R} = 2.784 \tag{36}$$

The important feature to note is that the decision time that maximizes the figure of merit depends on the distance of closest approach and vehicle speed, but not on source strength, detector efficiency, or the background count rate. It is also important to note that this calculation assumes a clear field of

view from the detector to the source. These assumptions are generally good for neutron sources, but do not necessarily apply to gamma sources.*

This relationship between detector and distance may also apply for detectors besides RPMs.†

2.1 SAMPLING TIME AND THE RUNNING AVERAGE

The calculation of the figure-of-merit as a function of the decision time assumed that the decision time was exactly centered on the peak of the expected net counts. To ensure that this happens, the count rate is sampled at a much faster rate and than the decision time used for the alarm decision and is then assembled by a running sum. The formula for computing the sum is shown in Eq. (37), where T is the decision time, t is the time of a single sample, and s_i is the number of counts in a single sample. Figure 10 shows graphically what is happening.

$$S_i = \sum_{k=0}^{T/t-1} s_{i+k} \quad . \tag{37}$$

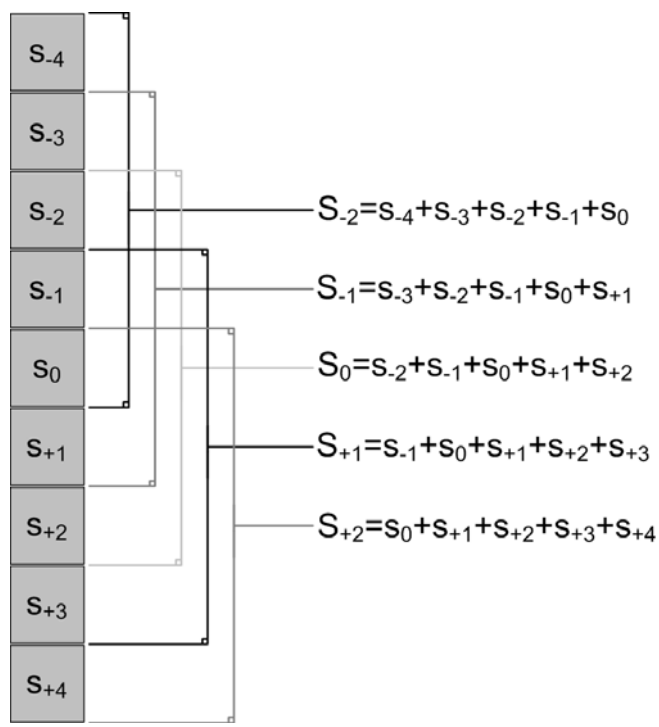

Fig. 10. Five point running sum.

On average, the counting window will be misaligned against the peak by half of an individual sample time. Therefore, it is important that the sampling intervals be small compared to the decision time.

*In particular, sensitivity to gamma sources is limited by the presence of NORM and not the sensitivity of the detector. Secondly, it is much more likely that a gamma source will be shielded for part of its approach.

†For example, the same geometry would apply if a hand-held search instrument was moved past a source at a constant speed and distance of closest approach.

Use of the running sum is a form of digital filtering. The optimization of the figure-of-merit as a function of decision time is analogous to setting the bandwidth of the digital filter.* Figure 11 shows the effect of using the running sum on a set of sample data.

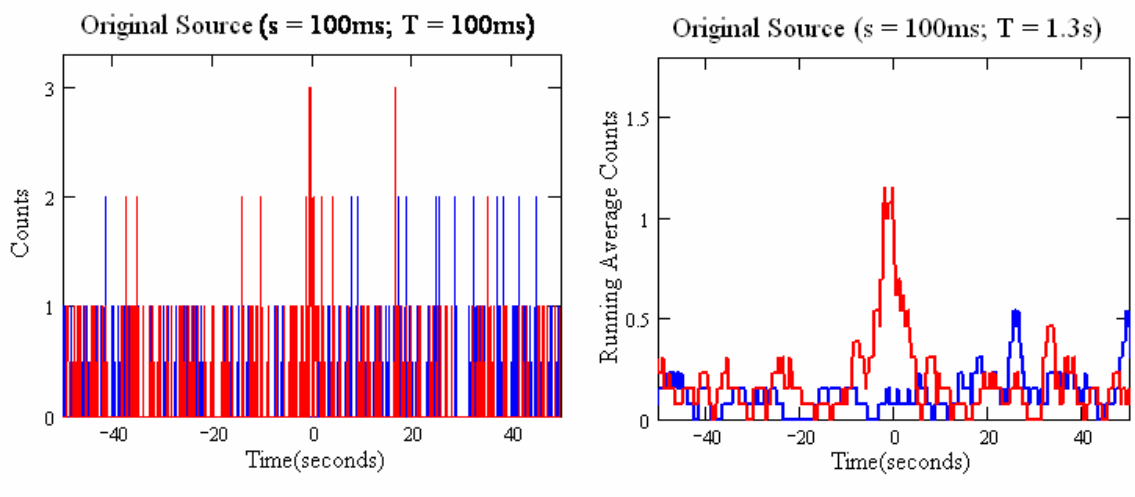

Fig. 11. A simulated Poisson background (blue) and a moving source (red, 5 m/s with a pillar spacing of 2.3 m). The plot on the left is 100-ms data before filtering. The plot on the right is the identical data set with an optimized block filter.

*Electrical engineers familiar with digital signal processing (DSP) will be used to a very different method for determining the length of the running sum. Because this is not a repeating waveform, the Fourier analysis is more complex than the integral over time and yields essentially the same result. The derivation will therefore be left for of those who are so inclined.

As a word of caution we would mention that most DSP techniques assume that the only significant uncertainty is the quantization error of the sampling time. RPM data will have significant uncertainty due to counting statistics as well. And the uncertainty due to counting statistics will increase as the sample time is reduced. This is why sampling frequencies over 10 Hz are seldom employed in RPMs.

3. DETERMINATION OF THE MINIMUM DETECTABLE LIMIT (MDL) AS A FUNCTION OF RPM PARAMETERS

Operating an RPM at a constant false alarm means the sensitivity of the RPM is a function of background. Here we present a calculation for the MDL.

If the source plus background exceeds the threshold, then an alarm is sounded.

$$N + \mu_B > K \tag{38}$$

N is the expected net counts, calculated from Eq. (22) [reprinted here as Eq. (39)].

$$N = \frac{2N_0}{\upsilon \cdot R} \arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right] \tag{39}$$

K is the threshold calculated by the flowchart in Fig. 5 and Eq. (12) [reprinted here as Eq. (40)].

$$FAP(\mu, K) = 1 - \sum_{x=0}^{K-1} \frac{\mu^x}{x!} e^{-\mu} \tag{40}$$

Combining Eq. (38) and Eq. (39) yields:

$$\frac{2N_0}{\upsilon \cdot R} \arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right] + \mu_B > K \tag{41}$$

(N_0/R^2) is the count rate due to the presence of the source placed at its center of the RPM. N_0 is a function of the source activity (A) in neutrons per second (not decays per second), detector area (D), and the detector's intrinsic efficiency for that source (η_s). The source activity is surface activity; that is, *source activity does not include either shielding or multiplications effects*. Equation (42) describes N_0 using a far field approximation. This may not be entirely accurate depending on the RPM dimensions, but N_0 will always be proportional to the source activity. We call the proportionality constant, H_s, the detector efficiency. Because it is difficult to calculate, we prefer to measure it. H_s is a constant given a specific combination of detector and isotope. For example, H_{Cf} and H_{AmBe} will not be equal. This is because the neutron energy distributions of the sources are different and η_s will be a function of energy. As a generally rule, it is a good approximation to use H_{Cf} for any fission source. Note that H_s is in units of m^{-2}.

$$N_0 = A \cdot \left(\frac{\eta_s \cdot D}{4\pi}\right) \equiv \frac{A}{\mathrm{H}_s} \tag{42}$$

$$\frac{2A}{\upsilon \cdot R \cdot \mathrm{H}_s} \arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right] + \mu_B > K \tag{43}$$

Solving Eq. (43) for the source activity yields an expression for the minimum activity required to generate an alarm, which is the MDL, denoted just as M in Eq. (45). It is important to remember that the MDL is defined as the source that has a 50% probability of alarming in a drive through.

$$A > \mathrm{H}_s \cdot \frac{\upsilon \cdot R}{2} \cdot \frac{K - \mu_B}{\arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right]} \tag{44}$$

$$M = \mathrm{H}_s \cdot \frac{\upsilon \cdot R}{2} \cdot \frac{K - \mu_B}{\arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right]} \tag{45}$$

The final value for the MDL is not in an elegant form. This is primarily because the value of K must be calculated via a numeric integral with each background and FAP (but this is done anyway as part of the alarm algorithm). Although this is not simple, most algorithms in use do not present a method for calculating the MDL at all.

The sample set of performance curves are shown in Fig. 12.

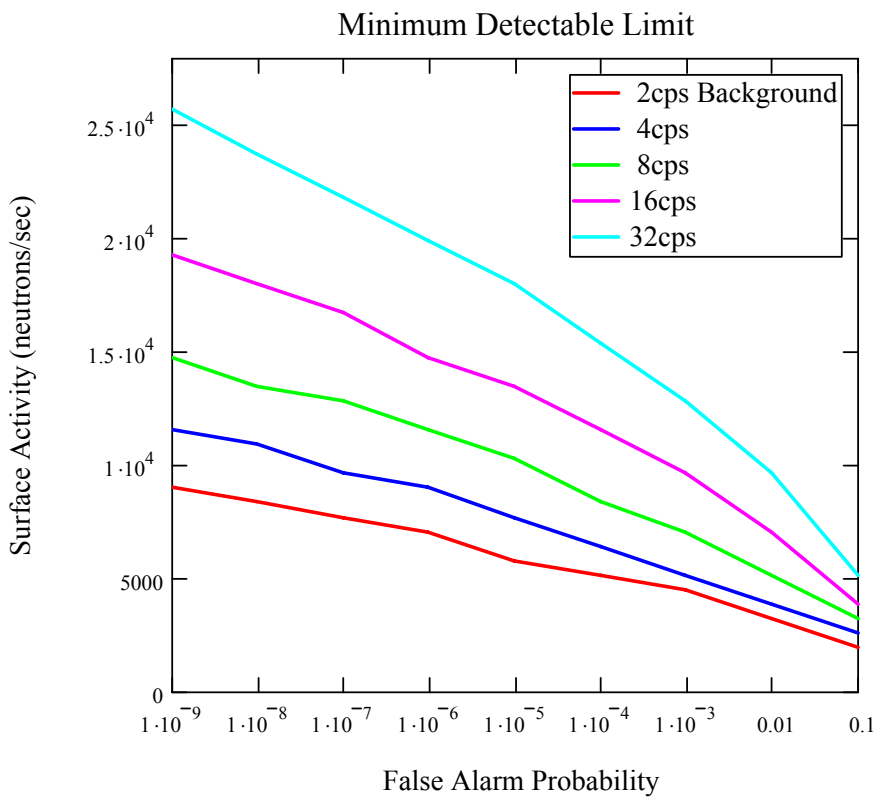

Fig. 12. The MDL of a sample detector as a function of false alarm probability and background.

A second feature of Eq. (45) is that it can be used to estimate the surface activity of a given source seen by the portal. This can be done in the time in which it takes to generate the alarm. In order to do this, all we need to do is substitute the measured count rate, Z, for the threshold, K.

$$\text{Source Activity Estimate} = \mathrm{H}_s \cdot \frac{\upsilon \cdot R}{2} \cdot \frac{Z - \mu_B}{\arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right]} \tag{46}$$

This estimate of the source activity assumes that the source passes through the middle of the portal and that there are no shielding or moderator effects and therefore is not necessarily accurate. But the source activity estimate will give the inspector a strong indicator as to whether or not the alarm is noise-induced and whether or not a health hazard exists.

As an alternative, one could calculate the FAP associated with respect to a specific event. By substituting Z for K in Eq. (12) we get Eq. (47).

$$FAP(\mu, Z) = 1 - \sum_{x=0}^{Z-1} \frac{\mu^x}{x!} e^{-\mu} \tag{47}$$

When the FAP associated with a particular event is very small compared to the FAP associated with the alarm threshold, it is a strong indicator that the source is real.

4. CONCLUSIONS

Use of this algorithm will result in an optimized sensitivity while maintaining a controlled false alarm rate. Neither the gross counting algorithm nor the digital filtering method is a groundbreaking technique. But neither is employed consistently by commercial RPM vendors.

The new features of this algorithm are the ability to calculate the MDL of the monitor given its parameter settings and the ability to estimate the size of the passing source. We feel that these features are very important because they help inspection personnel evaluate the size of sources that they encounter.

5. FUTURE WORK

One possible method of improving this technique is to use a shaped filter instead of a box filter. The filter could be shaped to match the shape of the moving point source. This would require very good speed control/measurement but could result in a superior signal-to-noise ratio. We did not explore this technique because the filtered background would become non-Poisson, which would make the effect on the false alarm rate extremely difficult to calculate.

A second path could be to optimize the filter to maximize the signal-to-K [Eq. (12)] instead of against the signal to standard deviation of the background. This is in fact, a more proper filter, but once again, the mathematics [differentiating to find the maxima of the numerical integral of Eq. (12)] gets out of control very quickly.

A third possibility would be to replace the two-dimensional figure of merit optimization with a more detailed calculation.

Given more time, any of these options could have been explored.

Appendix A. CONTRASTED WITH SPRT

A common counterargument to this GCPS (Gross Counting with Poisson Statistics) will be the suggestion to use the SPRT (Sequential Probability Ratio Test). The principle weight behind this comes from the fact that, historically, most applications of SPRT have handled Poisson statistics correctly, whereas many applications of gross counting have not. SPRT has many disadvantages, not the least of which is the fact that it is conceptually more complex, which leads it to be misunderstood and misapplied. This appendix is included to help the reader understand the differences between the algorithms.

A.1 WHAT IS SPRT?

SPRT is a generic algorithm designed to determine the most likely distribution from which a sample was taken, given two possible alternatives in the least number of trials. It is often used in applications that have nothing to do with radiation monitoring, such as quality control in industry.

Given two known distributions, P_A and P_B, one can perform a SPRT on a single sample, s. This in fact is just the PRT (Probability Ratio Test). The decision tree looks like this.

$$
\begin{aligned}
&D = \frac{P_A(s)}{P_B(s)} \\
&\text{IF:} \quad D > 10^{\alpha} \quad \text{THEN: the sample is from } P_A \\
&\text{IF:} \quad 10^{-\beta} \le D \le 10^{\alpha} \quad \text{THEN: the origin is unknown} \\
&\text{IF:} \quad D < 10^{-\beta} \quad \text{THEN: the sample is from } P_B
\end{aligned}
\tag{A.1}
$$

The thresholds are defined as α and β. When it is 10^{α} times more likely that the sample came from distribution A, we accept A as the correct distribution. When it is 10^{β} times more likely that the sample came from distribution B, we accept B as the correct distribution. Otherwise, we make no decision.

SPRT is PRT performed on successive samples until a decision is made. The probability ratio is calculated for each sample as it is taken and then multiplied by the product of all previous probability ratios. Then, the inequality is tested. If a decision between P_A and P_B is made, stop. Otherwise, take another sample. With n as the individual sample number and N as the total number of samples in the decision, the decision tree looks like this.

$$
\begin{aligned}
&D_N = \prod_{n=1}^{N} D_n = \prod_{n=1}^{N} \left(\frac{P_A(s_n)}{P_B(s_n)} \right) \\
&\text{IF:} \quad D_N > 10^{\alpha} \quad \text{THEN: the sample is from } P_A \\
&\text{IF: } 10^{-\beta} \le D_N \le 10^{\alpha} \quad \text{THEN: the origin is unknown} \\
&\text{IF:} \quad D_N < 10^{-\beta} \quad \text{THEN: the sample is from } P_B
\end{aligned}
\tag{A.2}
$$

These inequalities are often expressed in terms of logarithms.

$$\log_{10}\left(D_N\right)=\sum_{n=1}^{N}\log_{10}\left(\frac{P_A(s_n)}{P_B(s_n)}\right)$$

$$\begin{aligned}&\text{IF:}\quad \log_{10}\left(D_N\right)>\alpha \quad \text{THEN: the sample is from } P_A\\ &\text{IF: } -\beta\le\log_{10}\left(D_N\right)\le\alpha \quad \text{THEN: the origin is unknown}\\ &\text{IF:}\quad \log_{10}\left(D_N\right)<-\beta \quad \text{THEN: the sample is from } P_B\end{aligned} \tag{A.3}$$

Although not obvious from Eq. (A.3), this step will often simplify the necessary calculation; as it will in our example of neutron detection.

A.2 SPRT APPLIED TO RADIATION MONITORING

The first step in applying SPRT to radiation monitoring is to select the two alternative distributions from which we would like it to make the selection, a background distribution and an alarm distribution. Selection of the background distribution is straightforward. First the mean background, μ_B, is measured over a time period which is long compared to the sample measurement time. This ensures that the uncertainty of the mean is a negligible concern. The background distribution is then described by the Poisson distribution.

$$P_B=\frac{\mu_B^{\,k}}{k!}e^{-\mu_B} \tag{A.4}$$

The selection of the alarm distribution is not necessarily clear. It should also be a Poisson distribution, and it should have a larger mean. But how large of a source should be required in order to alarm? How much overlap should there be between the background and alarm distributions? The selection of the alarm distribution is the first point in SPRT that is generally glossed over and is not clearly defined. This decision in the end is largely arbitrary. In that tradition we will define the alarm distribution as:

$$P_A=\frac{\mu_A^{\,k}}{k!}e^{-\mu_A} \tag{A.5}$$

Figure A.1 shows an example of a background and alarm distribution.
Calculating the probability ratio for the neutron counting statistics yields:

$$D_n=\frac{P_A(s_n)}{P_B(s_n)}=\frac{\dfrac{\mu_A^{\,k_n}}{k_n!}e^{-\mu_A}}{\dfrac{\mu_B^{\,k_n}}{k_n!}e^{-\mu_B}} \tag{A.6}$$

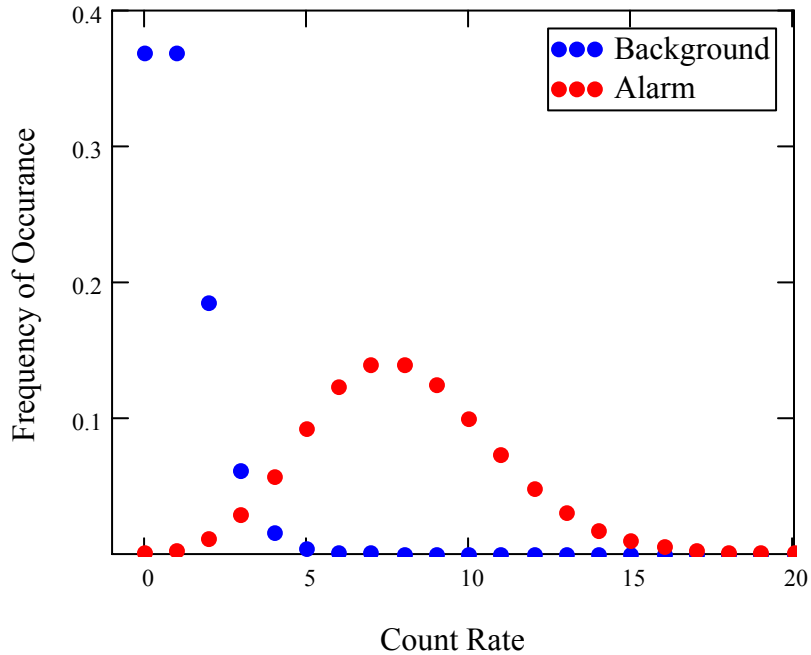

Fig. A.1. Sample Poisson distributions. In this example, the mean background is 1 count/second. The mean of the alarm distribution is 8 counts per second.

This greatly simplifies to

$$D_n = \left(\frac{\mu_A}{\mu_B}\right)^{k_n} e^{-(\mu_A - \mu_B)} \tag{A.7}$$

Expanding this to sequential decisions yields:

$$D_N = \prod_{n=1}^{N} \left(\frac{\mu_A}{\mu_B}\right)^{k_n} e^{-(\mu_A - \mu_B)} \tag{A.8}$$

Taking the log* of both sides:

$$\log_{10}(D_N) = \sum_{n=1}^{N} \log_{10}\left(\frac{P_A(s_n)}{P_B(s_n)}\right) \tag{A.9}$$

$$\log_{10}(D_N) = \sum_{n=1}^{N} \left[k_n \cdot \log_{10}\left(\frac{\mu_A}{\mu_B}\right) - (\mu_A - \mu_B) \cdot \log_{10}(e) \right] \tag{A.10}$$

*Many actual applications of SPRT will use the natural log here. A natural log makes the calculation of DN easier, but the calculation of α and β in the test condition will then be more difficult [see Eq. (A.3)]. Either method will result in a simplified calculation to perform during the sample measurement.

$$\log_{10}\left(D_N\right)=\left(\sum_{n=1}^{N}k_n\right)\cdot\left[\log_{10}\left(\frac{\mu_A}{\mu_B}\right)\right]-N\cdot\left[\left(\mu_A-\mu_B\right)\cdot\log_{10}\left(e\right)\right] \tag{A.11}$$

Everything that can be calculated before the arrival of the subject of the screening can be reduced to two constants.

$$\begin{aligned}\log_{10}\left(D_N\right)=C_1\cdot\left(\sum_n k_n\right)-C_2\cdot N \quad \text{where} \quad C_1=\log_{10}\left(\frac{\mu_A}{\mu_B}\right)\\ \text{and} \quad C_2=\left[\left(\mu_A-\mu_B\right)\cdot\log_{10}\left(e\right)\right]\end{aligned} \tag{A.12}$$

Placing this ratio into the logarithmic decision tree, Eq. (A.3), yields:

$$\begin{aligned}&\log_{10}\left(D_N\right)=C_1\cdot\left(\sum_n k_n\right)-C_2\cdot n\\ &\text{IF: }\left[C_1\cdot\left(\sum_n k_n\right)-C_2\cdot n\right]>\alpha \quad \text{THEN: the sample is from } P_A\\ &\text{IF: }-\beta\le\left[C_1\cdot\left(\sum_n k_n\right)-C_2\cdot n\right]\le\alpha \quad \text{THEN: the origin is unknown}\\ &\text{IF: }\left[C_1\cdot\left(\sum_n k_n\right)-C_2\cdot n\right]<-\beta \quad \text{THEN: the sample is from } P_B\end{aligned} \tag{A.13}$$

It is useful to divide both sides by C_1. This is because the factors, $\left(\frac{C_2}{C_1},\frac{\alpha}{C_1},\frac{-\beta}{C_1}\right)$, are all constants that can be calculated once the background is measured. k_n and n are the only terms that are measured during the passage of a vehicle.

$$\begin{aligned}&\text{IF: }\left[\left(\sum_{n=0}^{N}k_n\right)-\frac{C_2}{C_1}\cdot N\right]>\frac{\alpha}{C_1} \quad \text{THEN: the sample is from } P_A\\ &\text{IF: }\frac{-\beta}{C_1}\le\left[\left(\sum_{n=0}^{N}k_n\right)-\frac{C_2}{C_1}\cdot N\right]\le\frac{\alpha}{C_1} \quad \text{THEN: the origin is unknown}\\ &\text{IF: }\left[\left(\sum_{n=0}^{N}k_n\right)-\frac{C_2}{C_1}\cdot N\right]<\frac{-\beta}{C_1} \quad \text{THEN: the sample is from } P_B\end{aligned} \tag{A.14}$$

A.3 ADVANTAGES OF SPRT

SPRT has two major advantages over gross counting. The variable count time means that the decision to alarm on a large source is made in very little time, possibly one or two samples. At the same time, the SPRT maintains a high sensitivity to small sources by taking many samples in order to reach a decision. This is ideal for an application like a wait-in contamination monitor. In a wait-in

contamination monitor, the subject of measurement can be held as long as necessary. This is not true in portal monitoring. The subject of RPM comes and goes at its own speed. Because the position of the source is a function of time, the efficiency of the RPM is a function of time as well; SPRT algorithm chooses a count time assuming that the efficiency of the detector is constant. Therefore, the variable count time is *not* an advantage in the RPM application.

Secondly, the alarm condition requires few computer operations to calculate. Because so many terms in Eq. (A.14) are constants, there is very little calculation during the actual measurement. If the ratio term, D_N, is accumulated as a sum, one must perform an addition, a subtraction and then the comparison during each measurement interval. This is shown in Eqs. (A.15) to (A.18).

$$D_N = \left(\sum_{n=0}^{N} k_n \right) - \frac{C_2}{C_1} \cdot N \tag{A.15}$$

$$D_N = \left(\left(\sum_{n=0}^{N-1} k_n \right) - \frac{C_2}{C_1} \cdot (N-1) \right) + k_N - \frac{C_2}{C_1} \tag{A.16}$$

$$D_N = D_{N-1} + k_N - \frac{C_2}{C_1} \tag{A.17}$$

$$\begin{aligned} &\text{IF: } D_N > \frac{\alpha}{C_1} \quad \text{THEN: the sample is from } P_A \\ &\text{IF: } \frac{-\beta}{C_1} \le D_N \le \frac{\alpha}{C_1} \quad \text{THEN: the origin is unknown} \\ &\text{IF: } D_N < \frac{-\beta}{C_1} \quad \text{THEN: the sample is from } P_B \end{aligned} \tag{A.18}$$

The calculation has been extremely simplified. The time-consuming factorial and numerical integral of the gross counting method have been eliminated and replaced with three to five operations per sample. This is the point of using the logarithms and ratios that was alluded to in the beginning. In the 1970s, this was a fiendishly clever means of saving processing power. In a world where GHz CPUs are cheap, this advantage is archaic.

A.4 DISADVANTAGES OF SPRT

There are five principle problems with the use of SPRT, some of which have been mentioned above but are summarized again here.

First of all, the count time does not optimize the signal-to-noise ratio. The count time is a function of the random fluctuations of the measurement and not based on the motion of the source. Secondly, the count time is not necessarily centered on the peak of the source. The combination of these two factors means that the signal-to-noise ratio is seldom ideal. Another way to describe this is to say that the signal from the source is averaged with a variable quantity of the background.

Thirdly, the exact sensitivity is extremely difficult to calculate. Any algorithm which is designed to generate a constant false alarm rate will have a sensitivity that varies with time because the background varies as a function of time. But in the case of SPRT the entire history of the data used in the algorithm is necessary to calculate the sensitivity at any point in time. The starting point of any

one decision time is dependent on the number of intervals that all of the previous decisions took to reach conclusions.*

The fourth point is that SPRT is an algorithm that determines which distribution a given sample originated. The choice of the background distribution is straightforward, but the choice of the alarm distribution is largely arbitrary. The FAP given by SPRT is the probability that the background fluctuates in such a way as to mimic a specific arbitrary source. This is subtly different than the definition of FAP that is generally accepted.

The final point is SPRT is not well understood, even by people who are experts in the field of radiation monitoring. This often causes SPRT to be misapplied. Even when correctly applied, most operators do not have an understanding of how a change in the parameters will affect the performance of the RPM.

*In comparison, the gross counting algorithm proposed in the main body of this paper only requires knowledge of the previous background and the FAP in order to determine the sensitivity.

Appendix B. GAMMA DETECTION

B.1 USING POISSON STATISTICS IN GAMMA DETECTION

It is quite possible to use this Poisson gross counting algorithm with a gamma detector. Most RPMs use a Gaussian gross counting algorithm with their gamma detectors. As mentioned in the main body of the report, the Gaussian distribution is a very good approximation to the Poisson distribution is situations in which the count rate is large, which is true in most all RPM gamma detectors. But in other applications in which the expected count rate is very low, using the Poisson gross counting algorithm may be more appropriate. For example, a one cubic centimeter CZT detector in a handheld search instrument. When applying the Poisson gross counting algorithm to gamma detectors there are a couple of points to keep in mind.

Gamma-rays are much more susceptible to opportunistic shielding. Neutron sources can be shielded intentionally with special materials (e.g. borated polyethylene), but are generally not shielded by the presence of the vehicle conveying it. If someone makes an effort to shield neutron sources, it is reasonable to assume that it will be shielded from all directions. But gamma-rays are shielded by any dense material. A gamma source placed behind the engine block of a truck will be strongly shielded during the vehicles approach to the portal, but weakly shielded after the truck passes through. The practical effect of this phenomenon is that the profile will be narrower than depicted in Fig. 8, which in turn will make the ideal decision time shorter. One can construct any number of scenarios which possible. But without knowing the frequency of occurrence of these scenarios, it is difficult to set the count time exactly. In the field, ideal decision times tend to be less than or equal to those predicted in the laboratory. In the end, it is a judgment call on the part of those who are installing the RPM.

Using Eq. (46) to predict the size of a gamma source will be less accurate in general for several reasons. First of all, the opportunistic shielding mentioned above will attenuate the peak. Secondly, the intrinsic efficiency, η, of the detector will vary far more between gamma sources than it will between neutron sources. And finally, the gamma channel of the RPM is significantly affected by baseline suppression caused by the vehicle shielding the detector from the background.

B.2 CALCULATING THE MDL USING GAUSSIAN GROSS COUNTING ALGORITHM

It is also possible to use the same method to calculate the MDL of a RPM which uses the Gaussian gross counting algorithm. The alarm threshold of the Gaussian gross counting algorithm looks like this.

$$N > k\sqrt{B} \quad . \tag{B.1}$$

N is the net counts during the decision time. B is the background rate for the decision time. The threshold setting, k, is the number of standard deviations above background necessary to alarm. If B_0 is the background count rate in counts per second and T is the decision time, then B is simply

$$B \equiv B_0 \cdot \mathrm{T} \quad . \tag{B.2}$$

The derivation of the Gaussian MDL follows the Poisson MDL exactly with the single exception of the substitution of alarm conditions. The end result is shown in Eq. (B.3).

$$\mathrm{M} = \mathrm{H}_s \cdot \frac{\upsilon \cdot R}{2} \cdot \frac{k \cdot \sqrt{B_0 \cdot \mathrm{T}}}{\arctan\left[\frac{\upsilon \cdot \mathrm{T}}{2R}\right]} \quad . \tag{B.3}$$

It is important to remember that this value for the MDL does not include any corrections for background suppression or opportunistic shielding.