Statistical Analysis of Fissile Mass

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Objective

Statistical analysis of the confidence level measurement currently used in the Blend Down Monitoring System (BDMS)

Outline

- Introduction to BDMS
- Software Model
- Description of “Quality”
- Measures Taken to Strengthen “Quality”
- Conclusions

Blend Down Monitoring System

1993

- Russia and the United States agreed on the exchange of 500 MTU to be extracted from dismantled Russian nuclear warheads, to be concluded in 2013

2006

- 10% of total American power drawn from agreement
- More than 250 MTU (equivalent to 10,000 warheads) dismantled

Blend Down Monitoring System (cont.)

- **BDMS is a transparency measure taken to ensure that the nonproliferation objectives are being met**
- **System measures fissile material characteristics to ensure process is taking place**
- **Enrichment and fissile mass flow rate are parameters obtained**

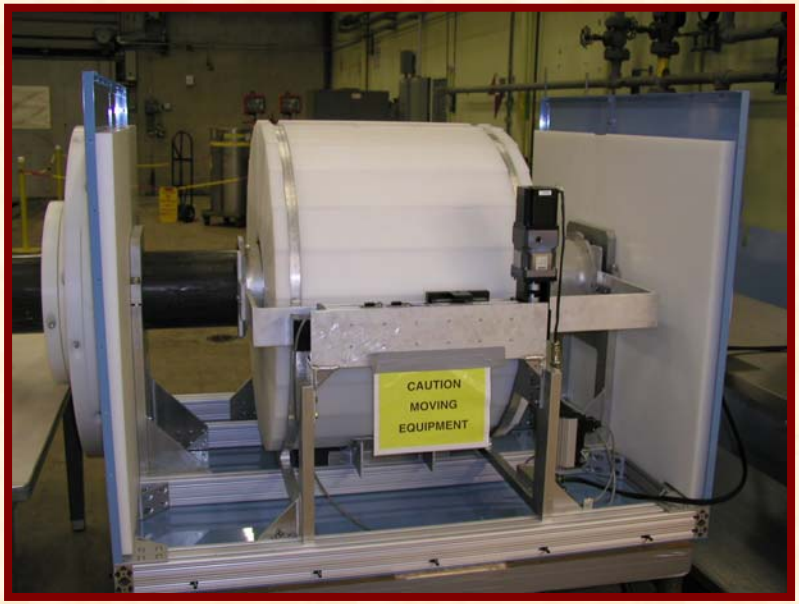
Blend Down Monitoring System (cont.)

- **Fissile Mass Flow Monitor (FMFM) measures fissile mass flow rate**
- **Enrichment Monitor (EM) measures enrichment**

FMFM EM

Blend Down Monitoring System (cont.)

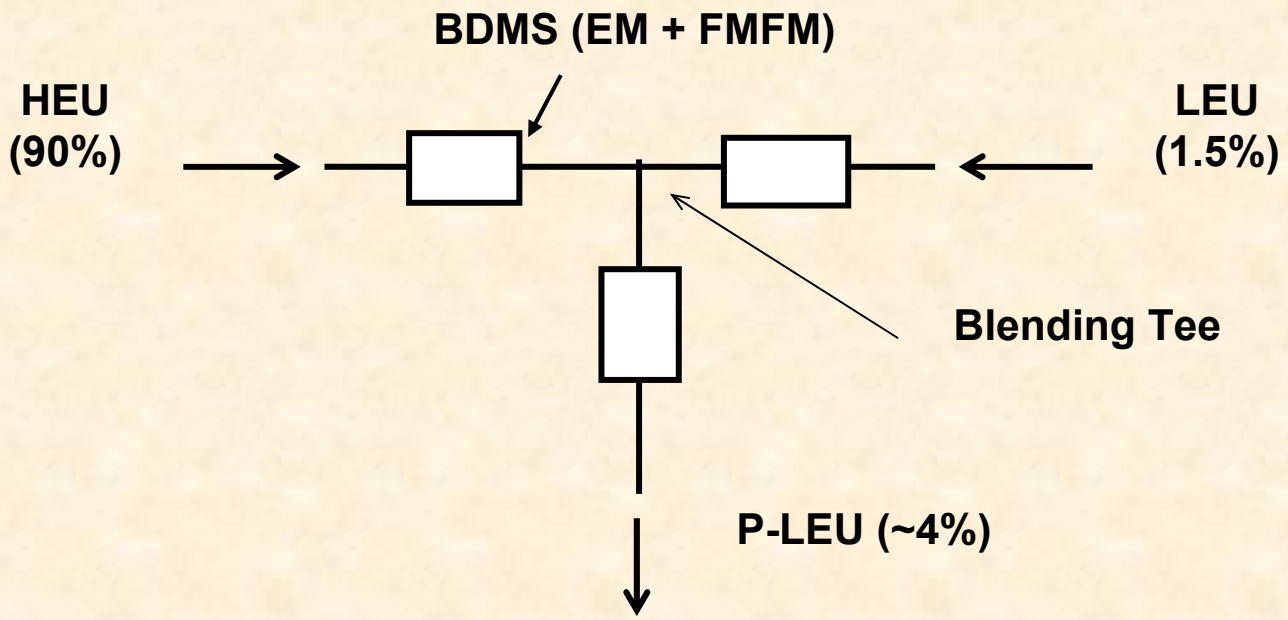
FMFM



EM

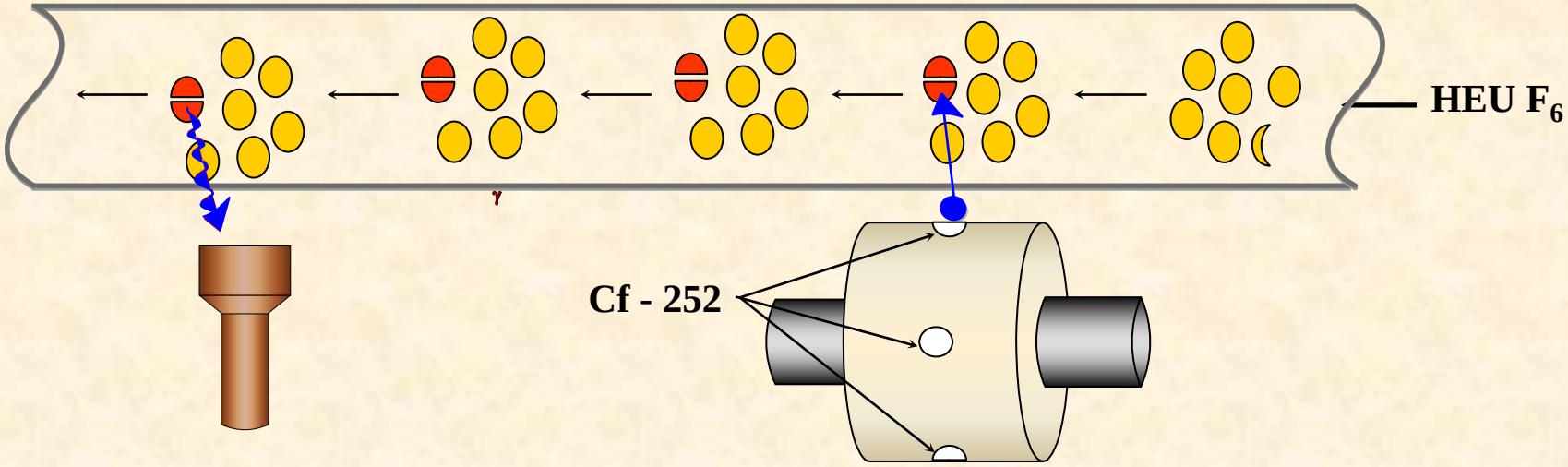
Blend Down Monitoring System (cont.)

- **Process occurs at a point referred to as the Blending Tee**



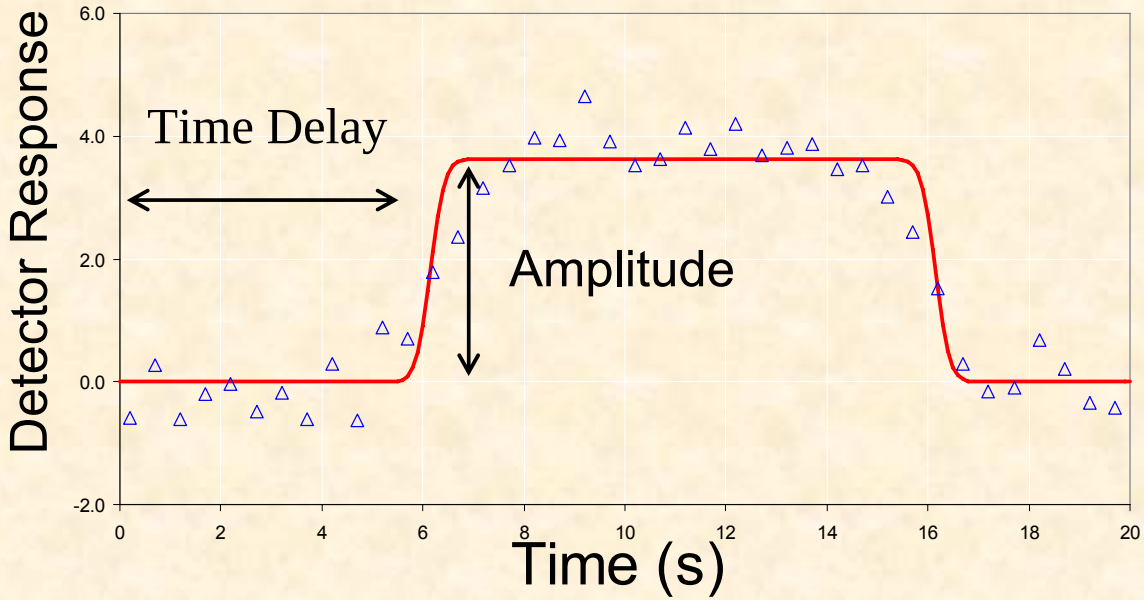
Blend Down Monitoring System (cont.)

- **Californium source is placed about pipe and modulated by linear positioner**
- **When source is opened, neutrons emitted induce fission in process stream**
- **Detector System obtains counts from delayed gammas released by fission products remaining in pipe**



Blend Down Monitoring System (cont.)

- **Modulating linear positioner provides time signature for neutron emission**
- **Time signature at detector provides time delay**
- **Detector response profile contains data to determine fissile mass flow**



Software Model

- **Response is a function of three variables…**
  - **Correlated background (from positioner motion)**
  - **Uncorrelated background**
  - **Delayed gamma counts**
- **Resulting gamma counts follow a certain profile**
- **Profile matched with best fitting model predicted profile**

Least Squares Fit

- **Data is fitted to these three parameters:**

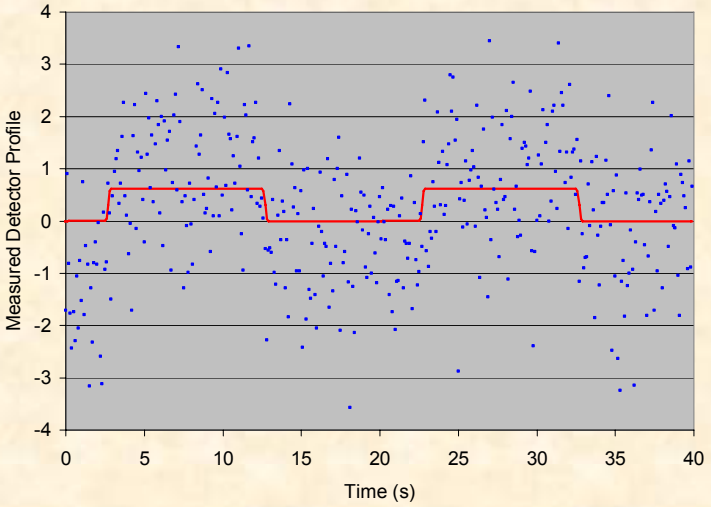
$$X_{i,fit}(u) = Uncorr + \beta Corr_i + \kappa(u) P_i(u)$$

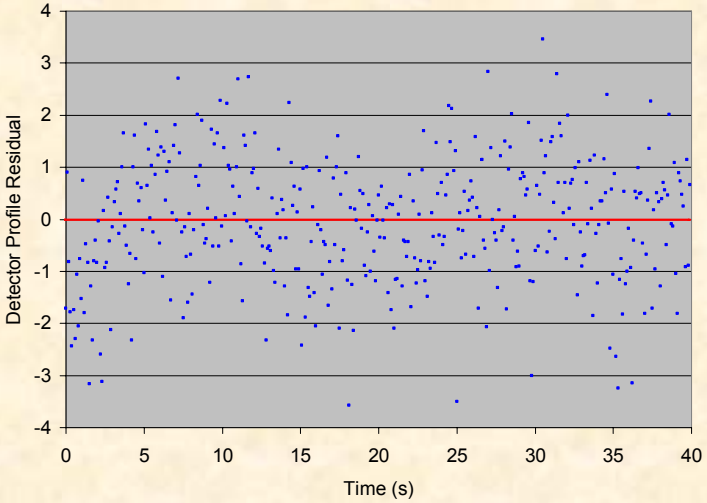
- **P_i is the ith velocity-dependent least-squares profile variable**
- **κ is the velocity-dependent profile term**
- ***Uncorr* is the constant uncorrelated background**
- ***$Corr_i$* is the ith data points correlated background**

The velocity resulting in the least error is selected, and information is obtained from the resulting profile, $P_i(u)$

Statistical Test

- **The fitted data result in a particular variance with respect to the raw data**
- **A test is performed in which the raw data are fitted to only the background, assumes NO FLOW**
- **If flow is present, the variance assuming flow will be lower**





F Test

- **An F test determines the probability that two sets of numbers come from the same distribution by observing the variance of each**
- **An F test is performed on these variances providing probability that they are the same**
- **Number provides probability of equality; probability of inequality needed**

Quality

- **In addition, we must avoid center point; if variances are equal, what do we do?**

Result is quality

$$Q = 1 - 2F\left\{\frac{\sigma^2_{no\ flow}}{\sigma^2_{flow}}\right\}$$

Inverted to provide inequality probability

Stretched to delete anything less than point of equality

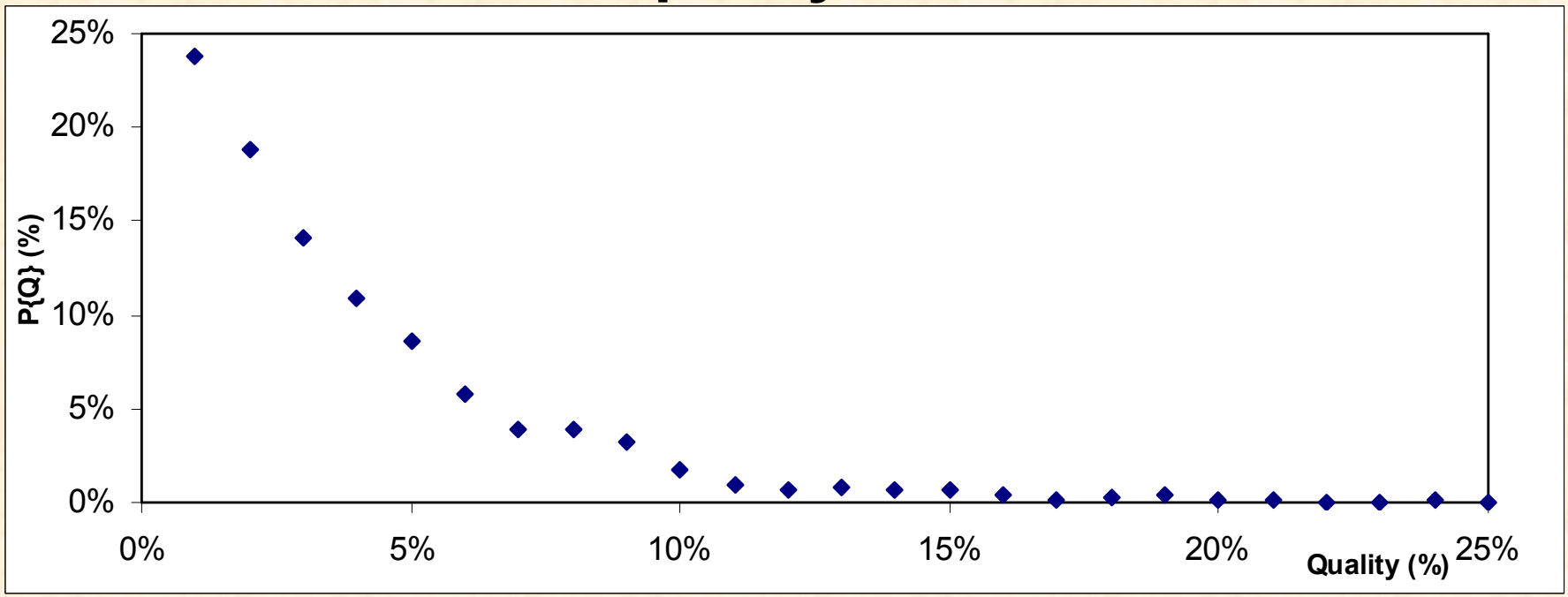
Quality (cont.)

- **In a noisy process, quality values can get low; how low can we go?**
- **Solution**
  - **Derive a threshold for low confidence**
- **Method**
  - **Simulate process**
  - **Enforce no flow**
  - **Observe behavior of quality**
  - **Obtain threshold from long-term behavior**

Quality Distribution at Zero Flow

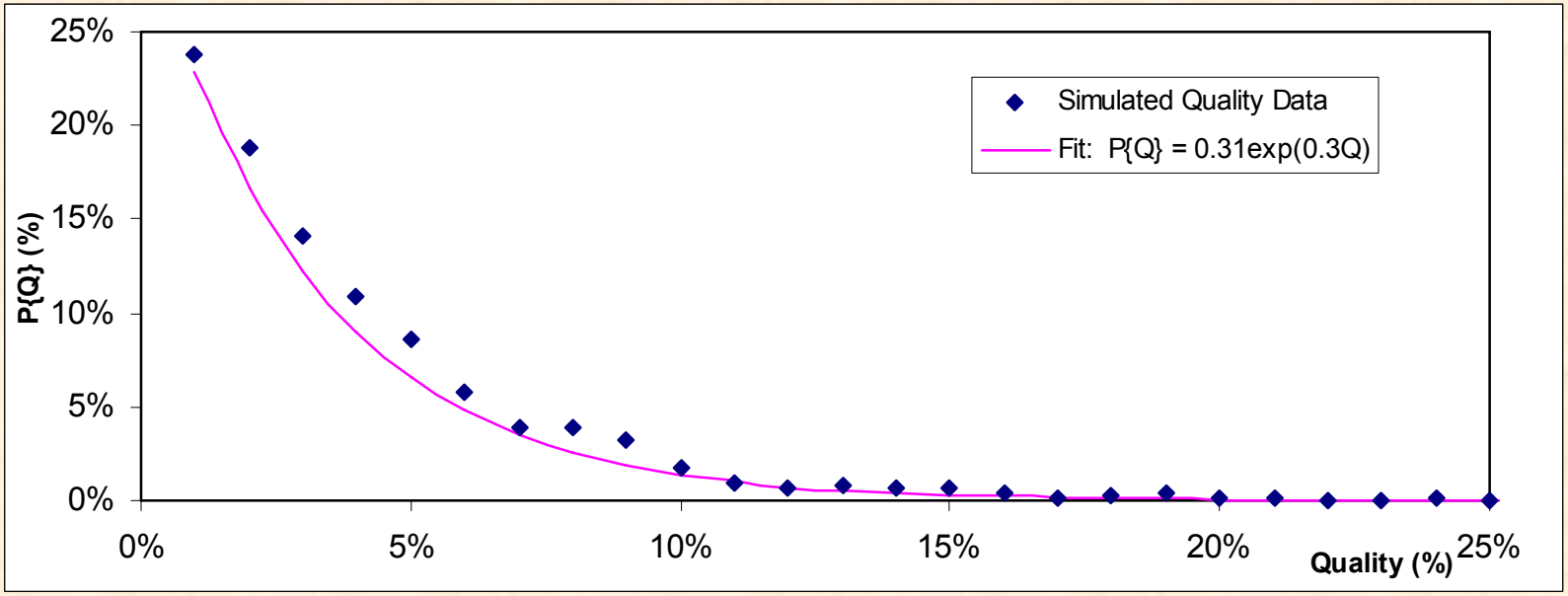
One test describes the matching of one profile over 60 seconds

Distribution of quality over 10,000 tests



Quality of "Quality"

- **Distribution of quality values at zero flow assumption decays exponentially**
- **Rate of decay is ~30%**



Importance of "Quality"

- **Deriving a threshold allows the capability of defining low confidence values**
- **According to fit, the probability of having a quality greater than 15 is less than 1%**
- **20 is a sufficient threshold; this data set states a probability less than 0.3% of obtaining quality greater than this value**

Derivation of Threshold

- **To be sure, an attempt at the significance of the argument against this threshold was performed**
- **This is calculated by finding the chance of having a cumulative probability less than some desired value, referred to as probable insignificance**
- **N normal tests are taken and a probability analysis is performed on each one**

Derivation of Threshold (cont.)

- **If a probable insignificance of *t* is chosen**
- **If N tests are taken, M of them will fall below *t***
- **Resulting in *M/N%* confidence that the chosen threshold is acceptable**
- **The significance of the argument against this threshold is *1 - M/N%***

Results of Applying Threshold

- **The data conform nicely to the exponential distribution** $P\{Q\} = 0.3e^{-0.3Q}$

- **A threshold of quality = 20 results in a confidence level of 95% at a probable insignificance of 1%**

Translation:

- **"In the field, we are 95% confident that any value above 20% presents high confidence in the presence of flow".**

Conclusions

- **Threshold for our purpose is 20%**
- **If a value less than this is obtained at full flow, it is 99% probable that no flow exists in the pipe**
- **If a value greater than this is obtained when flow is expected, there is a high probability that fissile flow is present in the pipe**

REFERENCES

- **Dunlop, Dorothy D., and Ajit C. Tamhane. Statistics and Data Analysis: From Elementary to Intermediate, 2000.**
- **Knoll, Glenn F. Radiation Detection and Measurement. Ann Arbor, Michigan: The University of Michigan, 1979.**
- **March-Leuba, J., *et al.*;"*Detector system for monitoring fissile mass flow in liquids and gases*", IEEE Transactions, 9-15 Nov. 1997.**