ANALYSIS OF NEUTRON SLOWING DOWN IN A CAPTURE-GATED DETECTOR

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ABSTRACT

Capture-gated neutron detectors are the object of recent investigations in the fields of nuclear safeguards, non-proliferation, and homeland security. These detectors are based on the use of organic scintillators doped with an absorbing medium, such as boron or lithium. The neutron detection occurs in two steps: first the neutron generates a scintillation pulse by multiple scatterings in the scintillator material (hydrogen and carbon); then, the neutron is captured in the absorbing medium. This capture occurs predominantly when the neutron has lost most of its energy in the previous scatterings. In the instrumentation, the scintillation pulse is gated by the subsequent neutron capture, which produces a pulse having fixed amplitude.

In previous studies, we have shown that the use of this type of detector facilitates the unfolding of the pulse height spectrum from the scintillation pulses to obtain the incident neutron energy spectrum. This feature is the result of the fact that the captured neutrons will have lost most of their energy in the scattering interactions, and therefore the scintillator pulse height can be related to the incident energy in a simple way.

In this paper, we present an analytical model to describe the neutron slowing down process in an infinite moderating medium that contains an absorber. The model can be used to compute the neutron spectra in the course of the slowing down process. We show how the resulting spectra are affected by the presence of the absorber. This information, together with the relevant cross-sections, can be used to calculate the number of scatterings that a neutron suffers before being captured. The first moment of this distribution is discussed. The results obtained with the analytical model are in excellent agreement with corresponding Monte Carlo simulations performed with the MCNP-PoliMi code.

1. INTRODUCTION

The process of neutron slowing down via elastic collisions on nuclei is a random process with peculiar and intriguing properties. For example, a well-known and counter-intuitive fact [1-4] is that the average number of collisions undergone by a neutron to slow down to, or below, a certain energy E from a starting energy E_0 cannot be calculated from the mean energy loss ratio per collision in the form of a geometric series, nor from the average logarithmic energy loss as an arithmetic series, although the latter yields a good approximation for large energy losses. Moreover, higher moments of the number of collisions do not seem to have been discussed even for the classical case of elastic scattering without absorption; we will address this question in this paper.

The number of collisions undergone by a neutron during slowing down in a scattering and absorbing medium was the object of recent Monte Carlo investigations [5]. The motivation arises from the need for enhanced information extraction from neutron detection methods for nuclear material management and accounting purposes, in connection with nuclear safeguards, non-proliferation, and homeland security. One application considers detectors that use organic scintillators doped with an absorbing medium, such as boron or lithium. The neutron detection occurs in two steps: first the neutron generates a scintillation pulse by multiple scatterings in the scintillating material (hydrogen and carbon); then the neutron is captured in the absorbing medium. This capture occurs predominantly when the neutron has lost most of its energy in the previous scatterings. In the instrumentation, the scintillation pulse is gated by the subsequent neutron capture, which produces a pulse having fixed amplitude.

Because of the non-linear relationship between energy transfer and light output, the latter is determined not only by the total energy transferred to the medium by the neutron, but also by the entire energy transfer statistics. Moreover, the energy transfer and the collision number are related quantities, and therefore the statistics of the number of collisions is of prime interest to understand the process of unfolding the detector light output into the incoming neutron energy spectrum. This heretofore disregarded fact constitutes much of the motivation behind the present work. In particular, we shall discuss the mean and the variance of the collision number leading to absorption, but higher moments can be computed as well. Regarding terminology, we have to distinguish between the number of scatterings (excluding the capture event) and the total number of collisions (including the capture event) that occur in the life history of the neutron. The scintillation light generation is related to the number of scatterings; however, when making a comparison with the classical case of slowing down without absorption, it is more natural to consider the total number of collisions. The first two moments of these random numbers are very simply related; the average total number of collisions exceeds by one the average number of scatterings, whereas their variances are equal. It follows that there will be some difference in the relative variances.

The interest in the statistics of the collision number comes also from another direction. Namely, the quantitative treatment of particle slowing down and spatial transport is simulated very accurately by Monte Carlo methods. Originally, Monte Carlo codes were developed to calculate first-order moments of particle transport based on probabilistic modelling of the stochastic transport, such as in the well-known MCNP package [6] for calculating neutron fluxes, leakage currents, and reaction rates. However, recently it has become clear that higher-order moments of the particle distribution are needed in a number of problems of interest. Examples are the variances of various quantities, as well as two-point (in time) distributions of the neutron detection process [7]. Hence, a new generation of particle transport Monte Carlo codes was developed that makes it possible to calculate entire probability distributions of various transport quantities, as well as temporal correlations among detections of various particles (neutrons, gammas, etc.), by implementing the correct statistics of individual reactions on an event-resolved basis. To carry out such simulations, the original MCNP code was modified to a full-statistics code for neutron and gamma ray transport, MCNP-PoliMi [8].

Using MCNP-PoliMi, various higher-order moments of the distribution of the collision number, or indeed any statistical quantity related to the neutron and photon transport, can be easily calculated. However, as is often the case with inverse tasks such as unfolding a measured light output into an incoming energy spectrum of source neutrons, a basic understanding of the process in simple terms is highly desirable. The objective of this paper is to provide a simple and accurate analytical model for the statistics of the neutron collisions upon slowing down in a homogeneous and absorbing medium.

2. ANALYTICAL MODEL

The calculation of the number of collisions required to slow down a neutron in hydrogen from energy E_0 to energy $E<E_0$ has been treated extensively [1-4]. The neutron energy distribution following the *n*th collision is

$$f_n(E) = \frac{1}{E_0(n-1)!}\ln^{n-1}\left(\frac{E_0}{E}\right) \qquad n \geq 1. \tag{1}$$

For n = 0, i.e., for the spectrum of the uncollided neutrons, one can write

$$f_0(E) = \delta(E - E_0)\ . \tag{2}$$

It is interesting to note that among the earliest derivations of Eq. (1), there is a reference to work in Russia; according to Mukhin [9], this formula was derived by Kurchatov, Artsimovich, and others in 1935, although he only gives a reference to a publication in the *Russian Journal of Theoretical and Experimental Physics* by the same authors in 1965 [10]. This derivation is of course not restricted to hydrogen, and it can be extended to other moderating materials.

Due to the form of Eq. (1), it is practical to turn to the lethargy variable

$$u = \ln\left(\frac{E_0}{E}\right). \tag{3}$$

Then, a simple substitution of Eq (3) into Eq. (1) yields the energy distributions as functions of lethargy per unit energy interval as

$$f_n(u) = \frac{u^{n-1}}{(n-1)!E_0}; \qquad f_0(u) = \frac{\delta(u)}{E_0}; \tag{4}$$

or per unit lethargy interval:

$$\tilde{f}_n(u) = \frac{e^{-u}u^{n-1}}{(n-1)!}; \qquad \tilde{f}_0(u) = \delta(u)\ . \tag{5}$$

The latter form is normalized both in u and n, so it is a probability density in u for a given n, and a discrete probability distribution in n for a given u. A representation of Eq. (5) is given in Fig. 1 with dotted lines for n=1 to 8.

Equations (1-5) have been used in the past to calculate the average number of collisions $m(E)$ that a neutron suffers to slow down to a certain energy interval $E+dE$, or the average number $n^*(E)$ to slow down below a certain energy E. This notation was introduced by Cohen [3]. As is well-known, these two numbers are the same for slowing down in hydrogen (and only in hydrogen) – a fact that is surprising in itself:

$$m(E) = n^*(E) = u + 1 \tag{6}$$

where u is the neutron lethargy, defined in Eq. (3). Table I shows the number of collisions needed, on average, to slow down a neutron from E_0=1 to 15 MeV to E=1 eV in hydrogen according to Eq. (6).

Table I. Average number of collisions for neutron slowing down in hydrogen

Incident neutron energy E_0 (MeV)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Average number of collisions *m*(E) (or *n**(E)) to slow down to (or past) E=1 eV. (Eq. (6))	14.8	15.5	15.9	16.2	16.4	16.6	16.8	16.9	17.0	17.1	17.2	17.3	17.4	17.5	17.5

Our derivation consists in developing a simple analytical model to describe the neutron slowing down process in a moderating medium that contains an absorber. In line with the calculations of the mean number of collisions to slow down a neutron to a given energy or past a given energy *E*, starting from energy E_0, the central quantity to be calculated is the probability P_n that the neutron will be absorbed exactly at the *n*th collision. This quantity can be calculated by knowledge of the relevant cross-sections and of the neutron energy spectra at the (*n-1)*th collision.

In our simple model consisting of only hydrogen and boron, capture occurs predominantly on boron-10 and scattering on hydrogen. The total cross section is then $\Sigma_{\mathrm{T}}(E) = \Sigma_c(E) + \Sigma_s(E)$, and the probability of absorption in a collision with energy *E* is given as

$$W(E) \equiv \frac{\Sigma_c(E)}{\Sigma_T(E)} \leq 1 . \qquad (7)$$

We can now calculate recursively both the energy distributions $f_n^{(a)}(E)$ of *n*-times collided neutrons in the presence of absorption, and the probability P_n that the neutron will be captured at the *n*th collision. The latter reads

$$P_n = \int_0^{E_0} f_{n-1}^{(a)}(E) W(E) dE = \int_0^{\infty} \tilde{f}_{n-1}^{(a)}(u) W(u) du ; \qquad (8)$$

whereas the energy spectrum after the *n*th collision, i.e., the energy spectrum of *n*-times collided neutrons, is

$$f_n^{(a)}(E) = \int_0^{E_0} \frac{f_{n-1}^{(a)}(E')}{E'} \left[1 - W(E')\right] dE' ; \qquad (9)$$

or, in the lethargy domain,

$$f_n^{(a)}(u) = \int_0^{u} f_{n-1}^{(a)}(u') \left[1 - W(u')\right] du' . \qquad (10)$$

Note that in Eq. (8) the energy spectrum per unit lethargy interval is used, whereas in Eq. (10) the energy spectrum per unit energy interval appears, where

$$\tilde{f}_n^{(a)}(u) = e^{-u} E_0 f_n^{(a)}(u) . \qquad (11)$$

In particular,

$$P_1 = W(E_0) = W(u = 0) , \qquad (12)$$

and

$$f_1^{(a)}(E) = \frac{1 - W(E_0)}{E_0}; \tag{13}$$

or

$$f_1^{(a)}(u) = \frac{1 - W(0)}{E_0}. \tag{14}$$

From Eqs. (8)-(10), all the quantities P_n and f_n can be calculated recursively. The average number of collisions to capture, $\overline{n}$, is calculated as

$$\overline{n} = \sum_{n=1}^{\infty} n P_n; \tag{15}$$

and all higher-order moments can be calculated in a similar way. It should be noted here that $\overline{n}$ is a tally of the number of neutron collisions that *includes* the capture event. The number of elastic scatterings prior to capture is therefore $\overline{n}$ -1.

3. NUMERICAL WORK

Before calculating the moments of the collision number, we consider the energy spectra of *n*-times collided neutrons in the presence of absorption. As the above formulae show, these can be calculated recursively, without calculating the absorption probabilities. In Fig. 1 we show the energy distributions per unit lethargy for the few first collisions. For the sake of comparison, the classical spectra in an absorption-free medium, Eq. (5), are also plotted. The spectra in Fig. 1 show the effect of neutron absorption, which is evident at low energies and with increasing *n*. The spectra with absorption decay faster with increasing lethargy than the classical spectra.

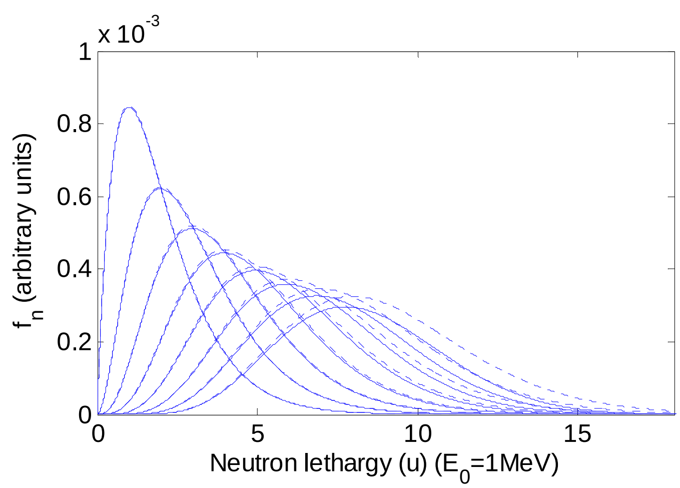

Fig. 1 Neutron energy spectra as a function of neutron lethargy for 1 MeV neutrons after 1, 2… 8 scatterings. Solid lines show spectra with absorption [Eq. (11)], and dotted lines show spectra without absorption [Eq. (5)].

The average number of collisions was calculated for neutrons with initial energies from 1 to 15 MeV in a hypothetical detector consisting of a mixture of hydrogen and boron-10, with approximately 36 atoms of hydrogen for each atom of boron-10. Scattering on boron and absorption on hydrogen were neglected. The scattering and absorption cross sections were taken from the evaluated ENDF/B-VI data, and are shown in Fig. 2 as a function of neutron lethargy, with $E_0 = 15$ MeV. The figures also show the weighting function $W(u)$ defined in Eq. (7).

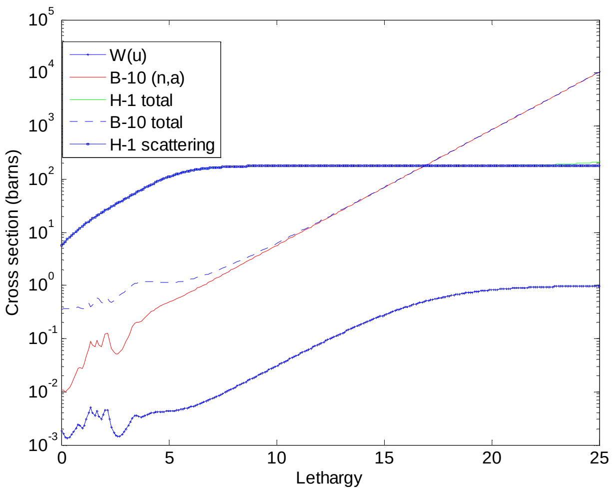

Fig. 2 Hydrogen and boron ENDF/B-VI cross sections and weighting function *W(u)* as a function of neutron lethargy, with E_0=15 MeV.

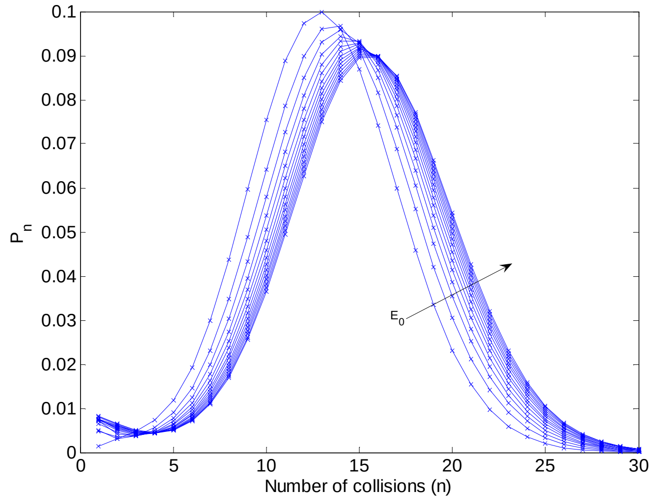

Fig. 3. P_n [Eq.(8)] as a function of the number of neutron collisions *n*.

The calculated values of the P_n for various values of E_0 from 1 to 15 MeV, in 1-MeV steps, are shown in Fig. 3. The values of $\bar{n}$, calculated with Eq. (15), are given in Table II.

Table II. Average number of collisions to neutron capture.

Incident neutron energy E_0 (MeV)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Average number of collisions *(n)* obtained with analytical model	13.1	13.7	14.1	14.3	14.5	14.6	14.8	14.9	15.0	15.1	15.1	15.2	15.3	15.3	15.4
Average number of collisions (*n*) simulated with MCNP-PoliMi	13.1	13.7	14.1	14.3	14.5	14.6	14.7	14.9	14.9	15.0	15.1	15.2	15.2	15.3	15.3

The results obtained with the analytical model were compared with Monte Carlo simulations, performed with the MCNP-PoliMi code [8] and a specifically designed post-processing code. In order to make a proper comparison, a large detector with dimensions 100 by 100 by 100 cm was simulated, so leakage did not occur: all the neutrons slowed down and were captured inside the detector. To perform a meaningful comparison with the numerical model described previously, the Monte Carlo histories that included neutron scattering on boron or absorption by hydrogen were not considered in the analysis performed with the post-processor. These histories were few compared with the number of histories comprising scatterings on hydrogen and final capture on boron.

The results are shown in Fig. 4, and the data are reported in Table II. Figure 4 shows excellent agreement between the Monte Carlo simulations and the simple model calculations. The relative difference between the model and the simulations is within 0.5% on the entire energy range.

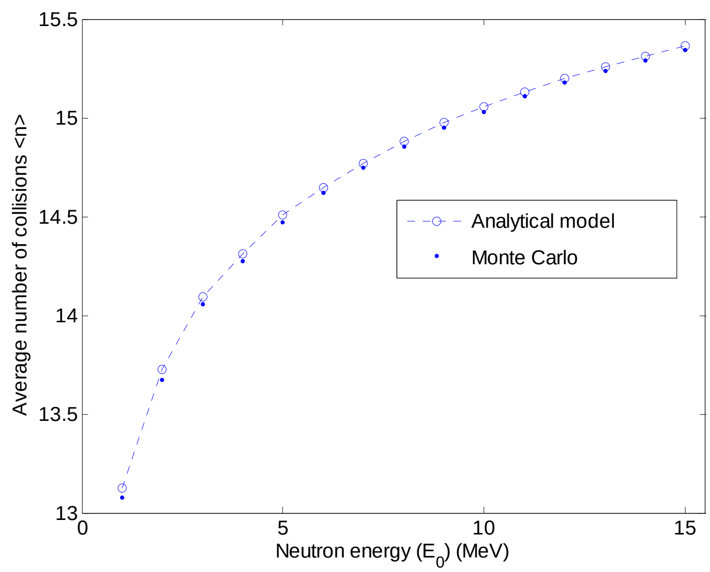

Fig. 4. Average number of neutron collisions to capture as a function of incident neutron energy (MeV) for a large detector comprising hydrogen and boron.

The question might be raised whether these results can be converted into simpler formulae, based on the classical result of the number of collisions $m(E)$ [or $n^*(E)$] required to slow down to (or past) a certain energy E, which was discussed earlier.

In particular, one may ask what energy level must be chosen for the average number of collisions required to reach that energy, or to slow past that energy, to equal the average number of collisions to absorption calculated by Eq. (13). From Eq. (6), one obtains that this energy can be calculated by setting

$$\bar{n} = u + 1, \tag{16}$$

from which one obtains the energy as

$$E = e^{-(\bar{n}-1)} E_0 . \tag{17}$$

For E_0 = 1 MeV, Table II gives $\bar{n} = 13$. This number includes the capture event, so we must subtract 1 from it, and from Eq. (17) we obtain E = 16.7 eV. Similarly, for E_0 = 10 MeV, one obtains E = 22.6 eV. This shows that it is not possible to use one single energy as the slowing down limit and apply the classical formula to calculate from it the average number of collisions to absorption; however, as a rough estimate, one can take the neighborhood of 18 eV as the equivalent slowing down limit. This energy is significantly larger than the most likely energy of the thermal distribution, and is also higher than the (obviously arbitrary) upper limit of the thermal region, which is usually set as 1 eV. The above threshold energy, about 18 eV, can, on the other hand, be compared with the average energy of the neutrons prior to absorption, which can also be calculated using Monte Carlo. In the present model it varies between 400 and 600 eV with neutron initial energies of 1 to 15 MeV ,which is actually significantly higher than the hypothetical limit of slowing down to give the same average number of collisions.

Finally, we mention that the classical formula can be used as a rule of thumb for estimating the difference (as opposed to the absolute value) between the number of collisions needed for absorption with source neutrons of two different energies. To slow down to any given lower energy E from initial energies E_1 and $E_2 < E_1$ requires $u_1 + 1$ and $u_2 + 1$ collisions, respectively. Hence the difference in the number of collisions is $u_1 - u_2$. For E_1 = 10 MeV and E_2 = 1 MeV this gives a difference of $\ln(10)$ = 2.3 collisions. From the model (and also from the more accurate Monte-Carlo simulations) this difference is approximately 2 collisions. Hence the classical formula remains suitable to give a rough estimate of the differences between the number of collisions to absorption from different starting energies, at least in this particular case.

4. CONCLUSIONS

The simple model introduced in this paper yields the complete statistics of the number of collisions leading to capture in a homogeneous mixture of a scattering and absorbing material. We obtained excellent agreement between the model results and corresponding Monte Carlo simulations. A deeper understanding of the collision-number-resolved statistics of the slowing down process was gained, which will be used in the future to interpret measurements in the field of fissile material accounting. The result is also useful for the basic understanding of the neutron slowing down process. The model can be extended to include neutron scattering on elements other than hydrogen, neutron multiplication, and transport in an inhomogeneous medium.

5. REFERENCES

1. L. Ruby, "Dancoff's Solution for the Number of Collisions Necessary to Slow Down," *Nucl. Sci. Eng.*, **86**, 110 (1984).

2. H. Van Dam, "A Difference of at Least Three Collisions," *Nucl. Sci. Eng.*, **89**, 365 (1985)

3. E. R. Cohen, "How Many Collisions to Slow Down a Neutron," *Nucl. Sci. Eng.*, **89**, 99, (1985).

4. P. Reuss, "Additional Remarks on the Number of Collisions Required to Slow Down a Neutron," *Nucl. Sci. Eng.*, **93**, 105 (1986).

5. S. A. Pozzi, R. B. Oberer, and J. S. Neal, "Analysis of the Response of Capture-Gated Organic Scintillators," proceedings of the IEEE Nuclear Science Symposium, October 16-22, 2004, Rome, Italy.

6. Judith F. Briesmeister, Editor, "MCNP -- A General Monte Carlo N-Particle Transport Code," Los Alamos National Laboratory report LA-13709-M (2000).

7. J. T. Mihalczo, J. A. Mullens, J. K. Mattingly, and T. E. Valentine, "Physical Description of Nuclear Materials Identification System (NMIS) Signatures," *Nuclear Instruments and Methods in Physics Research A,* **450** *2-3*, 531 (2000).

8. S. A. Pozzi, E. Padovani, and M. Marseguerra, "MCNP-PoliMi: A Monte Carlo Code for Correlation Measurements," *Nuclear Instruments and Methods in Physics Research A,* **513,** *3*, 550 (2003).

9. K. N. Mukhin, E*xperimental Nuclear Physics* (Hungarian translation) (1985).

10. I. V. Kurchatov, L. A Artsimovich, et al., *Sov. Phys. JETP* **5**, 659 (1965) (In Russian).