Neutron Slowing Down in a Detector with Absorption

Corresponding author:

Sara Pozzi
Oak Ridge National Laboratory
Oak Ridge, TN 37831-6010, P.O. Box 2008, MS 6010
e-mail: pozzisa@ornl.gov
fax. (865) 576 8380
tel. (865) 574 5699

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Sara A. Pozzi and Imre Pázsit*

Oak Ridge National Laboratory
Oak Ridge, TN 37831-6010 PO Box 2008 Ms6010
e-mail: pozzisa@ornl.gov

(*)Department of Nuclear Engineering, Chalmers University of Technology
SE - 412 96 Göteborg, Sweden
e-mail: imre@nephy.chalmers.se

ABSTRACT

In a recent paper, we presented a simple analytical model to describe the statistics of the number of scattering collisions undergone by fast neutrons as they slow down until they are absorbed. In that study, we assumed that the moderator was infinite and homogeneous, and accounted for scattering and absorption by a single nuclear species. In the present paper, we extend that methodology to the more realistic case of neutron slowing down in a homogeneous mixture. The formulae are derived and evaluated numerically, and the results are found to be in very good agreement with corresponding Monte Carlo simulations. The average value of the number of collisions that a neutron undergoes before being captured is computed. The results for a capture-gated detector composed of hydrogen, carbon, and boron are discussed.

1. INTRODUCTION

In a recent paper [1], we described the process of neutron slowing down via elastic collisions in an infinite, homogeneous sample containing a single scattering species that was doped with an absorbing medium. The simple model was used to obtain the complete statistics of the number of neutron collisions leading to capture. In the present work, we extend that analysis to include the more realistic case of an infinite homogeneous mixture. This model is directly applicable to the study of neutron detectors such as organic scintillators, which are typically made up of a carbon and hydrogen compound.

Recent investigations [2-4] have addressed the use of detectors made of organic scintillators doped with an absorbing medium, such as boron or lithium. The neutron detection occurs in two steps: first the neutron generates a scintillation pulse by multiple scatterings in the scintillating material (hydrogen and carbon); then, the neutron is captured in the absorbing medium (typically lithium or boron). This capture occurs predominantly when the neutron has lost most of its energy in the previous scatterings.

The energy deposited in each individual scattering that a neutron undergoes in the scintillator cannot typically be resolved by the detection system. However, the total light output produced in the scintillator is related to the energy of the incoming neutron by a non-linear relationship. It follows that the entire collision-number statistics is of interest for this application, whose final scope is often to unfold the detector light output into the incoming neutron energy spectrum.

We begin our treatment by considering the “classical” case of neutron slowing down without absorption in an infinite homogeneous mixture, and will extend that treatment to

the case of an infinite homogeneous mixture that contains an absorbing medium. In both cases, the calculation of the number of collisions that a neutron undergoes during the slowing down process requires the knowledge of the neutron spectrum of the *n*-times collided neutrons. The analytical representation of the neutron spectrum is not in closed form, but is based on a recursive formulation in the form of a convolution.

When considering neutron slowing down in a mixture of two (or more) nuclear species, it is natural to ask: how does the slowing down process in a mixture differ from the slowing down process in the individual constituents of the mixture? And furthermore, can the spectrum of the *n*-times collided neutrons be described as some kind of "sum" of the spectra of the *n*-times collided neutrons in pure media consisting of the individual constituents of the mixture? In other words, to what extent are the spectra obtained in the slowing down in the individual species discernible in the "compound spectrum" obtained during the slowing down in the mixture? We will answer these questions in the remainder of this paper.

2. ANALYTICAL MODEL: SLOWING DOWN WITHOUT ABSORPTION

In this Section we will describe the analytical models for neutron slowing down in a single nuclear species and in a mixture of two nuclear species.

2.1 Neutron slowing down in a single nuclear species

The calculation of the number of collisions required to slow down a neutron in hydrogen from energy E_0 to energy $E<E_0$ has been treated extensively [5-8]. The neutron energy distribution following the *n*-th collision is

$$f_n(E) = \frac{1}{E_0(n-1)!} \ln^{n-1}\left(\frac{E_0}{E}\right) \qquad n \geq 1. \tag{1}$$

For n = 0, i.e., for the spectrum of the uncollided neutrons, one can write

$$f_0(E) = \delta(E - E_0)\,. \tag{2}$$

This derivation is of course not restricted to hydrogen, and can be extended to other moderating materials. For A > 1, no simple closed form for the spectra $f_n(E)$, such as (1), can be given. One way to determine $f_n(E)$ is to use a recursion formula of the type

$$f_n(E) = \int_E^{E/\alpha} \frac{f_{n-1}(E')}{(1-\alpha)E'} dE' \tag{3}$$

where $\alpha = (A-1)^2/(A+1)^2$, with $f_0(E)$ given by (2). As is known, the distribution $f_n(E)$ of Eq. (3) is non-vanishing only for $\alpha^n E_0 \leq E \leq E_0$.

In the numerical evaluation of the formulae, it is practical in general to turn to the lethargy variable,

$$u = \ln\left(\frac{E_0}{E}\right). \tag{4}$$

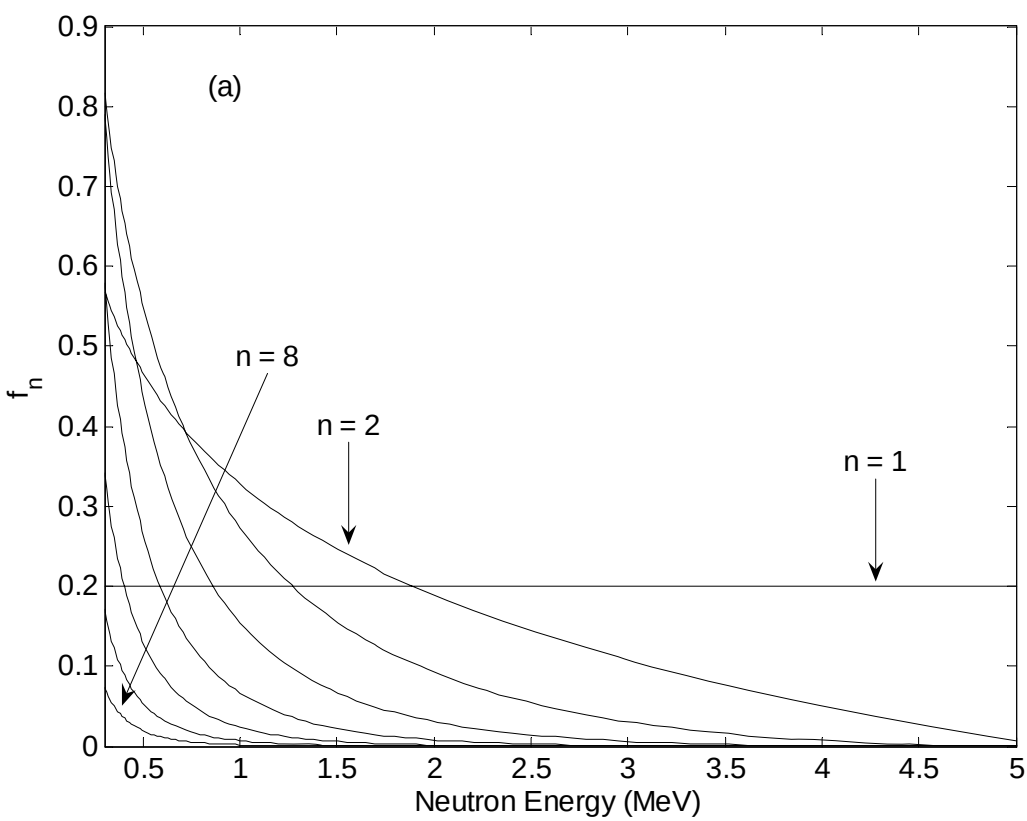
The energy distributions for neutron scattering on hydrogen, as functions of lethargy per unit lethargy interval, can be obtained by a simple substitution of Eq. (4) into Eq. (1), and multiplying the resulting function by dE/du, such that $f_n(u)$ is a density per unit lethargy interval:

$$f_n(u) = \frac{e^{-u} u^{n-1}}{(n-1)!}; \qquad f_0(u) = \delta(u)\,. \tag{5}$$

The above form is normalized both in *u* and *n*, so $f_n(u)$ is a probability density in *u* for a given *n*, and a discrete probability distribution in *n* for a given *u*. For the case $A > 1$ one has

$$f_n(u) = \int_0^q \frac{f_{n-1}^{(a)}(u-u')e^{-u'}}{1-\alpha} du', \tag{6}$$

where $q = -\ln\alpha$. A representation of Eq. (3) is given in Fig. 1 for hydrogen ($\alpha = 0$) and carbon ($\alpha = 0.716$) for n = 1 to 8 and E_0 = 5 MeV.



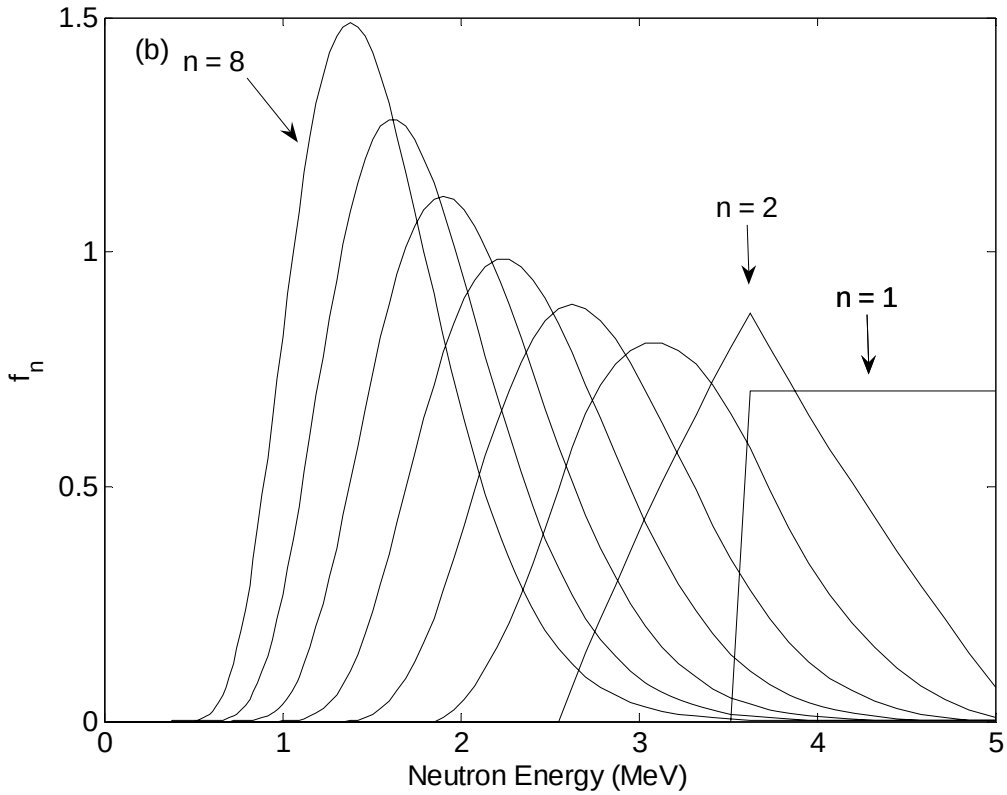


Fig. 1. Neutron spectra ($f_n(E)$) after *n* scatterings on hydrogen (a) and carbon (b), (Eq. (3)), for *n*=1 to 8 for E_0=5 MeV.

2.2 Neutron slowing down in a mixture

In the case of a mixture of more than one nuclear species, the neutron energy spectra are calculated by taking into account the relevant macroscopic scattering cross sections of the constituents of the mixture. In the following, we will consider the two elements hydrogen and carbon, because these are the typical constituents of organic scintillators. However, the formulation can be easily extended to other elements, or to mixtures containing more than two components. To keep the analytical formulation as simple as possible, we only consider elastic scattering. In the case of scattering on carbon, this is an

approximation when the neutron energy is above the threshold for inelastic scattering, which is approximately 4.4 MeV.

Denote the (constant) probabilities of elastic scattering on hydrogen and carbon as

$$c_H = \frac{\Sigma_{s,H}}{\Sigma_T} \quad \text{and} \quad c_C = \frac{\Sigma_{s,C}}{\Sigma_T},$$

with $\Sigma_T = \Sigma_{s,H} + \Sigma_{s,C}$.

Then, by extending Eq. (6) for the case of two nuclear species, the energy spectrum of *n*-times collided neutrons is given by

$$f_n(u) = c_H \int_0^\infty f_{n-1}(u-u')e^{-u'}du' + c_C \int_0^{\min(q,u)} \frac{f_{n-1}(u-u')e^{-u'}}{1-\alpha} du', \quad n \geq 2. \tag{7}$$

Naturally, for the first term on the r.h.s. of Eq. (7), we could have used the compact analytical form given in Eq. (5). However, for the case with absorption, which is treated in Section 3, no analytical solution similar to Eq. (5) can be given, whereas the form given in Eq. (7) can be easily modified to account for absorption.

The initialisation for Eq. (7) is $f_0(u) = \delta(u)$, and

$$f_1(u) = c_H e^{-u} + c_C \frac{e^{-u} H(q-u)}{1-\alpha}, \tag{8}$$

where $H(u)$ is the unit step function: the second part of (8) is different from zero only for *u* between *0* and *q*.

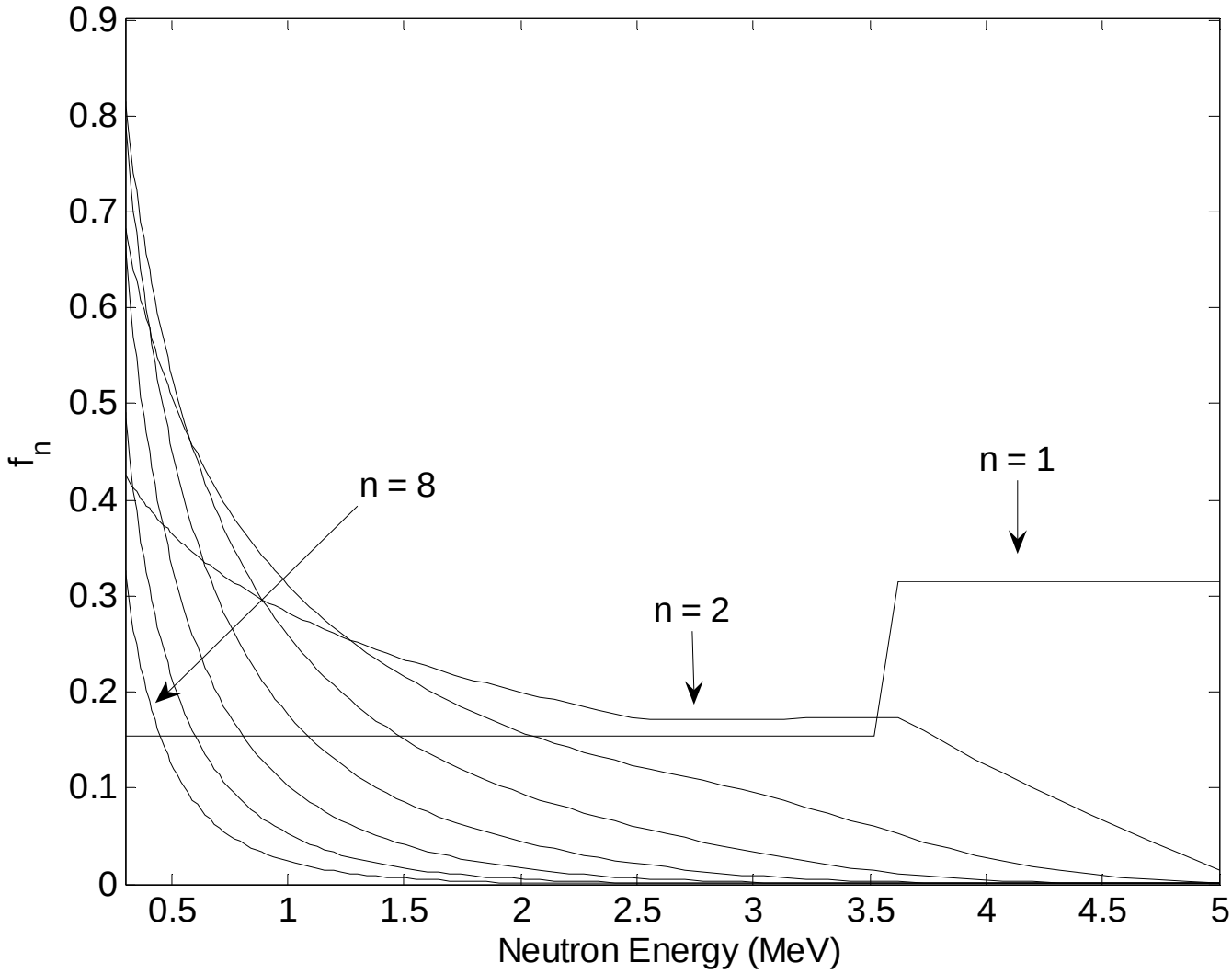


Fig. 2. Neutron spectra ($f_n(E)$) after *n* scatterings on a hydrogen and carbon mixture, (Eq. (7)), for n=1 to 8 for E_0=5 MeV.

Figure 2 shows that the individual constituent spectra can be discerned in the energy spectrum of the mixture only up to the third collision, approximately. For higher collision numbers, the spectra look qualitatively very similar to the spectra obtained by slowing

down in hydrogen alone. This effect can be explained by an increasing "mixing" of the alternative collisions on hydrogen and carbon. Specifically, the first collision is either on hydrogen or carbon, hence the spectrum is a pure weighted sum of the individual spectra, as described in Eq. (8). For the second generation, i.e. n = 2, if c_H and c_C are equal, then the fraction of neutrons that collided twice on either hydrogen or carbon is 25%, hence the spectrum of scattering on carbon alone is still discernible. However, the probability of scattering on just one species decreases rapidly with increasing n. Indeed, the neutrons that have collided at least once on hydrogen (their relative number increases with each scattering) cease to have a spectrum which extends only to a finite region, i.e. between $\alpha^n E_0$ and E_0. Even if a subsequent collision of such a neutron occurs on carbon, the spectrum will now extend down to $E = 0$, therefore resembling to the spectrum of scattering on hydrogen alone. This effect can also be described in the following way: any scattering on hydrogen will "smear out" the smaller energy decrease given by scatterings on carbon. Moreover, $c_H(E) > c_C(E)$ for all values of E of interest, and hence the "collision mixing" effect described above favours the elimination of sequences of scatterings on carbon only. With increasing collision numbers, this effect further enhances the process of the spectra being more and more resembling to the spectra of slowing down on hydrogen alone.

3. ANALYTICAL MODEL: SLOWING DOWN IN A MIXTURE WITH CAPTURE

We now consider the case of neutron slowing down in a mixture of more than one nuclear species when an absorber is present. Consider the energy-dependent cross-sections $\Sigma_{s,H}(E)$ (hydrogen scattering cross section), $\Sigma_{s,C}(E)$ (carbon scattering cross section), and $\Sigma_{a,B}(E)$ (boron absorption cross-section).

The total cross section is then

$$\Sigma_T\left(E\right)=\Sigma_{s,H}\left(E\right)+\Sigma_{s,C}\left(E\right)+\Sigma_{a,B}\left(E\right). \tag{9}$$

In Eq. (9), the assumption is that scattering occurs only in hydrogen and carbon, and absorption occurs only in the absorbing species, boron. As before, inelastic scattering is not taken into account.

The probability that a neutron having energy E undergoes a scattering collision on hydrogen and carbon is equal to

$$c_H\left(E\right)=\frac{\Sigma_{s,H}\left(E\right)}{\Sigma_T\left(E\right)} \quad \text{and} \quad c_C\left(E\right)=\frac{\Sigma_{s,C}\left(E\right)}{\Sigma_T\left(E\right)},$$

respectively. It follows that $c_H(E)+c_C(E)<1$, and the probability of capture in the absorbing species can be defined as

$$W(E)=1-\{c_H(E)+c_C(E)\}.$$

The neutron energy spectra of the n-times collided neutrons can then be expressed recursively as follows:

$$f_n^{(a)}(u) = \int_0^{\infty} f_{n-1}^{(a)}(u-u')e^{-u'}c_H(u-u')du' + \int_0^{\min(q,u)} \frac{f_{n-1}(u-u')c_C(u-u')e^{-u'}}{1-\alpha}du' . \quad (10)$$

Eq. (10) can be readily evaluated numerically with the initialization

$$f_1(u) = c_H(0)e^{-u} + c_C(0)\frac{e^{-u}H(q-u)}{1-\alpha} .$$

The representation of the neutron energy spectra (Eq. (10)) resulting from *n* neutron scatterings is shown in Fig. 3 for *n* up to 8. A comparison with Fig. 2 shows that when absorption is taken into account, the spectra have a shape similar to the absorption-free case, but their amplitude is smaller, especially in the low-energy tail. This is the effect of the depletion of the neutron population due to absorption.

The effect of the absorption can be quantified by considering the integrals of the neutron spectra with respect to energy as a function of the number of scatterings, *n*. These integrals, $F_n = \int_0^{\infty} f_n(E)dE$, are shown in Fig. 4. On average, after approximately 13 scatterings only half of the neutrons survive in the detector.

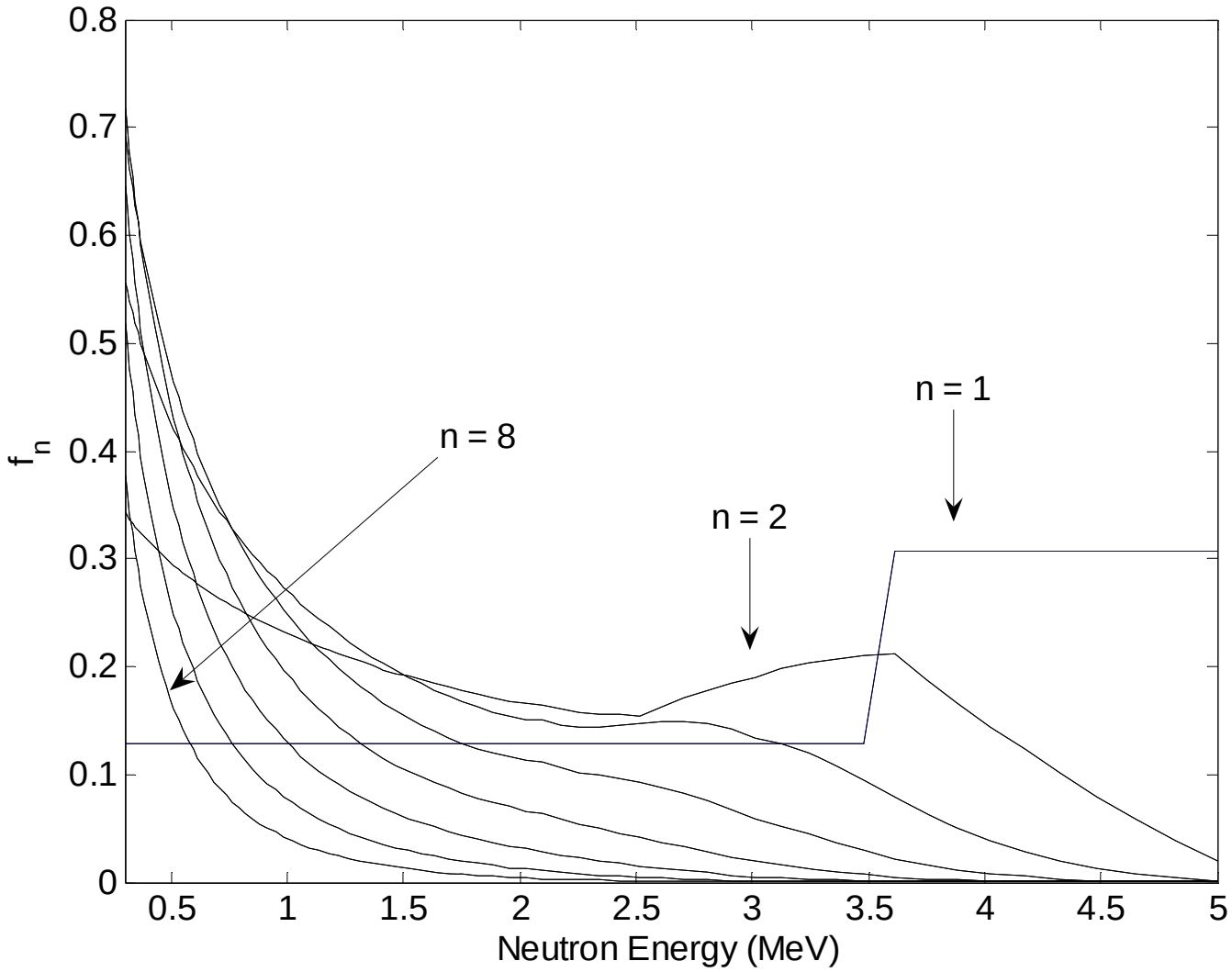


Fig. 3. Neutron spectra ($f_n(E)$) after n scatterings in a carbon, hydrogen, and boron detector (Eq. (10)), for n=1 to 8 for E_0=5 MeV.

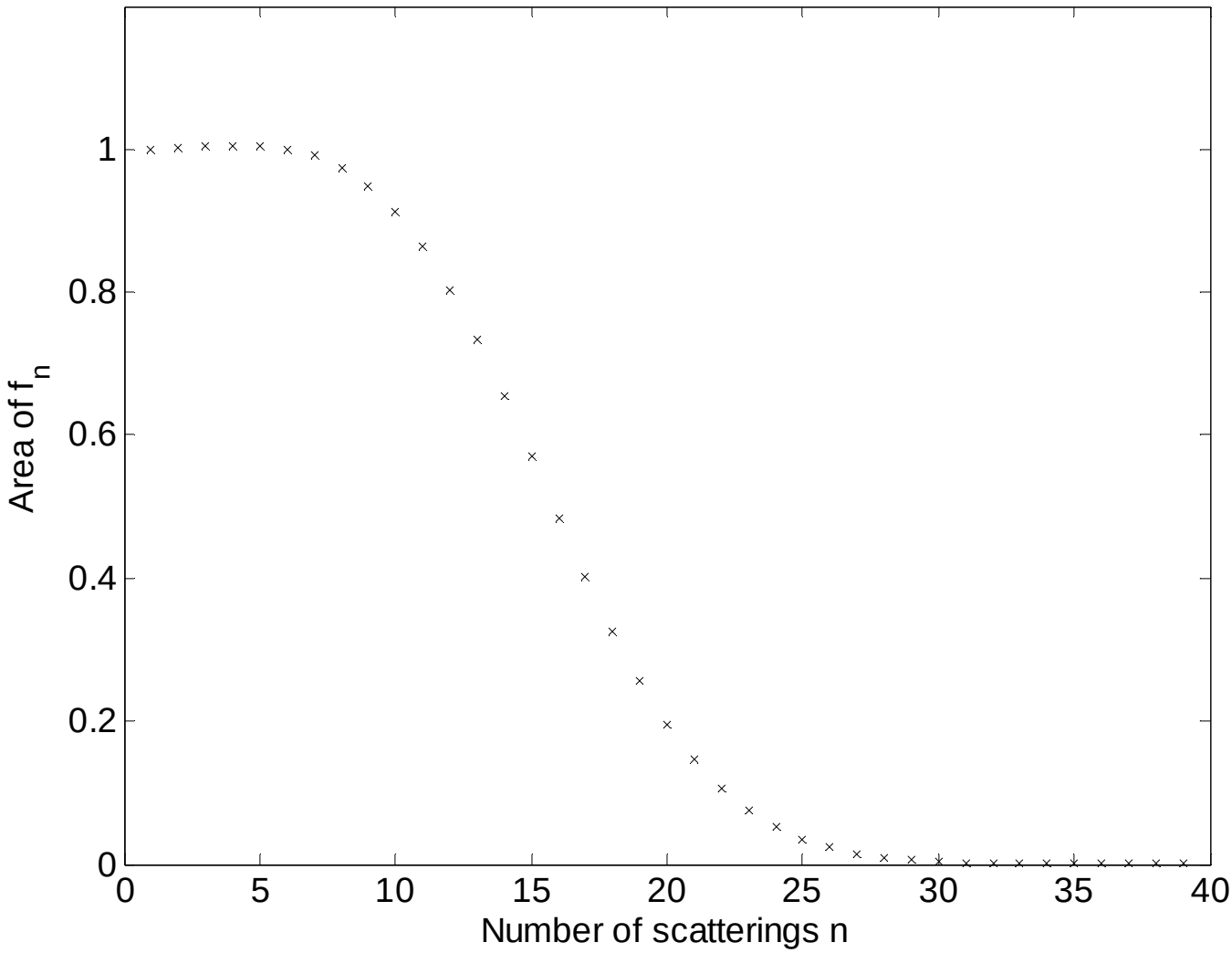


Fig. 4. The integrals $F_n = \int_0^\infty f_n\left(E\right)dE$, as a function of *n*.

The quantity of interest is the probability that a neutron be absorbed exactly at the *n*-th collision. This probability is given by

$$P_n = \int_0^\infty f_{n-1}^{(a)}(u)W(u)du \,. \tag{11}$$

The quantities P_n are normalized to one, i.e. $\sum_n P_n = 1$, because the medium is infinite and therefore the only possible fate for the neutrons is capture, after some number n of collisions. The P_n's are shown in Fig. 5 for initial neutron energies from 1 to 10 MeV, in 1 MeV increments. The P_n's can be used to calculate the statistics of the number of collisions in a detector made up of a mixture of more than one scattering species and when an absorber is present, as will be shown in Section 4.

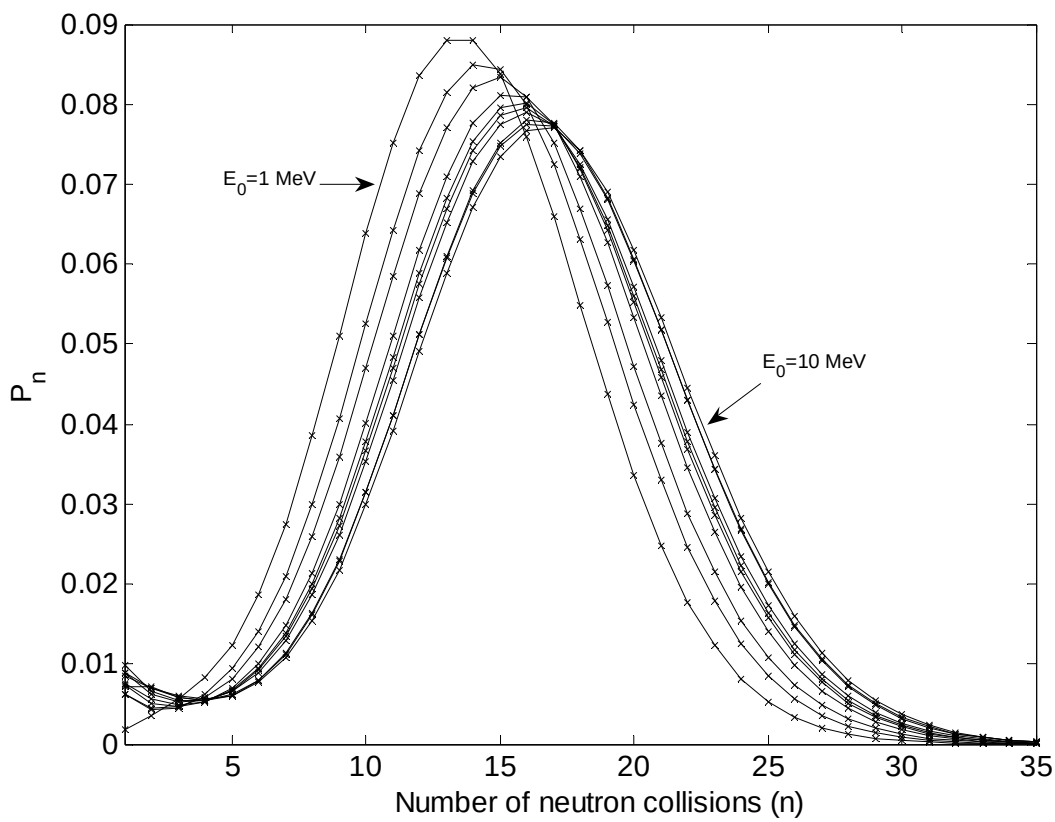


Fig. 5. Probability of neutron capture following n collisions (P_n's) (Eq. (11)) for E_0 = 1 to 10 MeV.

4. COMPARISON WITH MONTE CARLO SIMULATIONS

The Monte Carlo code MCNP-PoliMi was used to simulate a large number of neutron histories in the scattering and absorbing medium. Tallies were performed to compare the Monte Carlo results with the results from the analytical formulation presented in Sections 2 and 3. The simulations were performed for a large spherical detector, having radius 250 cm. The neutron source was placed at the center of the detector, emitting neutrons isotropically. The simulations were performed for monoenergetic neutrons, with initial energy E_0 equal to 1 to 15 MeV, in 1 MeV increments, for 10^5 neutron histories. Each of these simulations took approximately 30 seconds to complete on a Pentium IV, 3.3 GHz machine. The composition of the detector is shown in Table 1.

Table 1. Composition of the detector.

No. of H atoms per cm^3	4.98×10^{22}
No. of C atoms per cm^3	2.86×10^{22}
No. of ^{10}B atoms per cm^3	0.243×10^{22}

Leakage did not occur because all neutrons were eventually captured in the detector. Most of these captures occurred in the boron. The quantities P_n were determined by binning the neutron histories according to the number of scatterings that the neutron had undergone before being captured, and by dividing this quantity by the total number of histories. The information on the number of scatterings in a neutron history is one of the outputs given in the MCNP-PoliMi output file.

The probabilities P_n's calculated by Monte Carlo are compared to the same quantities calculated with the analytical formula, Eq. (11). The result is shown in Fig. 6 for the case of E_0 =1 MeV. As it can be seen, there is very good agreement between the Monte Carlo result and the evaluated analytical formula.

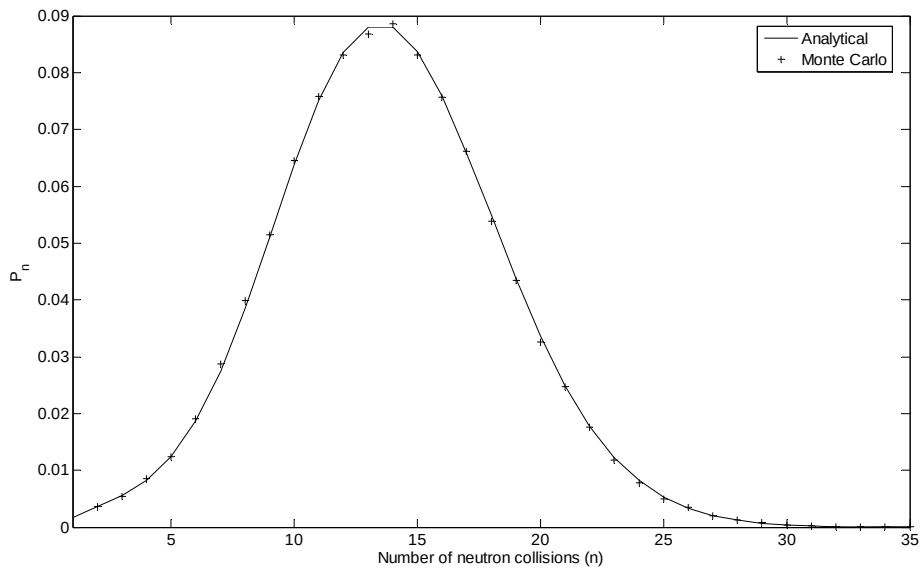


Fig. 6. Probability P_n of neutron capture after n scatterings in a carbon, hydrogen, and boron detector, Eq. (11), for E_0 =1 MeV.

The average number of collisions to capture, $\overline{n}$, is calculated as

$$\overline{n} = \sum_{n=1}^{\infty} n P_n \ , \tag{12}$$

and all higher-order moments can be calculated in a similar way. It should be noted here that $\overline{n}$ is a tally of the number of neutron collisions that *includes* the capture event. The

number of elastic scatterings prior to capture is therefore $\bar{n}$-1. The average number of neutron collisions, $\bar{n}$, is given in Table 2 for E_0=1 to 15 MeV.

Table 2. Average number of collisions, $\bar{n}$, to capture in an infinite detector

Incident neutron energy E_0 (MeV)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Average number of collisions, $\bar{n}$, obtained with analytical model	13.9	14.6	15.1	15.7	15.9	15.9	16.1	16.5	16.5	16.6	16.9	17.0	17.2	17.1	17.3
Average number of collisions, $\bar{n}$, simulated with MCNP-PoliMi and, shown in parenthesis, σ_n	13.8 (0.01)	14.6 (0.01)	15.1 (0.01)	15.6 (0.01)	15.7 (0.01)	15.8 (0.01)	15.9 (0.01)	16.2 (0.01)	16.1 (0.01)	16.2 (0.01)	16.3 (0.01)	16.4 (0.01)	16.5 (0.01)	16.5 (0.01)	16.5 (0.01)

Table 2 shows that there is a slight deviation of the analytical model from the Monte Carlo as the incident neutron energy increases. This deviation can be explained by the neglect of neutron inelastic scattering on carbon in the analytical model. The effect of inelastic collisions is a faster slowing down and hence a smaller number of collisions to capture.

The results of Table 2 show that the average number of collisions to capture in case of slowing down in a mixture of hydrogen and carbon is greater than the average number of collisions to capture in a medium comprised of hydrogen alone. This latter case was treated

in Ref. [1]. The increase is less than one collision for low initial neutron energies, and more than one collision for high initial neutron energies.

5. CONCLUSIONS

The results presented in this paper yield the complete statistics of the number of collisions leading to capture in an infinite homogeneous mixture of two scattering species and of one absorbing species. This result can be easily generalized to the case of more scattering or absorbing species. We obtained very good agreement between the results from the analytical model and the corresponding Monte Carlo simulations. This derivation is useful for the basic understanding of the neutron slowing down process in a scintillation detector doped with an absorber.

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