FLEXURAL AND TORSIONAL RESONANCES OF CERAMIC TILES VIA IMPULSE EXCITATION OF VIBRATION

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ABSTRACT

A practice was demonstrated that could independently determine, with high resolution, bulk E, G, and ν of disk, square, hexagonal, and half-hexagonal ceramic tiles or plates. The method combines modal finite element analysis and the flexural and torsional resonance values (measured by impulse excitation of vibration) for a given geometry and material. The consideration of both resonances is important in this practice because ν is able to be explicitly determined as a consequence and its value does not need to be assumed to determine E and G (as would occur when only one of their resonant frequency values is known).

I. INTRODUCTION

Ultrasonic velocity measurement [1] and impulse excitation of vibration (IEV) [2] are two procedures that are used to determine the elastic properties of materials. Each technique has its own advantages and disadvantages. The primary advantage of IEV (*i.e.*, an ability to sample bulk elastic properties of the *whole* tile) provided motivation to explore it further as a means to quantify elastic properties, and to ultimately differentiate and remove those tiles from large populations that have different elastic properties.

The ASTM E494 practice for ultrasonic velocity measurement [1] involves measuring longitudinal and transverse wave velocities in a material whose density and velocity-path thickness are known. The technique's advantages are that it is robust and quick to perform. A disadvantage is only the local volume of material under the longitudinal or transverse sensors is sampled, so if inhomogeneities exist in the bulk ceramic components, then the experimentalist

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may or may not sample them. Some structural ceramics in the past have suffered from possessing inhomogeneities (*e.g.,* density gradients, compositional variations, etc.), so interest exists to utilize a test technique that samples the whole component's volume and generate apparent elastic property values.

The Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Impulse Excitation of Vibration (ASTM C1259 [2]) involves the measurement of resonant frequencies whose values are then used in empirical expressions to determine those elastic properties. The advantage of the technique is the measured resonances are the result of excitation of the *entire* component's volume. This is attractive as it potentially provides a means to non-destructively eliminate components that exhibit statistically different elastic properties from a population. Unfortunately, the component–geometry can sometimes inhibit the ability to measure any two of the three resonance modes (flexure, longitudinal, torsion). For example, it is extremely difficult, if not impossible, to measure longitudinal or torsional resonant frequencies on ASTM C1161 bend bars (*i.e.*, 3x4x50mm). When only one resonant mode is measurable, then one needs to assume a value of one of the three elastic properties (typically ν) in order to determine E (and ultimately G too). Needing to assume a value for one of the three elastic properties is disconcerting especially when one is examining new compositions that are devoid of existing and confidently reported elastic properties.

An IEV practice, inspired by the procedure described in the C1259 Annex for disks, was further developed and demonstrated that independently determines bulk E, G, and ν of square, hexagonal, and half-hexagonal ceramic tiles or plates. The method combines modal finite element analysis and the IEV measurement of flexural and torsional resonance values for a given geometry and material. The evaluation of these properties using the herein described procedure allows one to confidently and non-destructively determine outlier components from a population.

II. EXPERIMENTAL PROCEDURES

This newly developed process for determining E, G, and ν has three steps: modal analysis using finite element analysis (FEA), measurement of resonant frequencies, and combination of their results. The method results in the determined values having high resolution (three significant digits as a minimum) because it is relatively easy to measure densities, tile dimensions, and resonance values with at least four significant digits (*i.e.*, all the inputs used in the E, G, and ν determinations).

The modal analysis of disk, square, hexagon, and half-hexagons (both possible orientations) served two purposes. First, flexure and torsion resonances for each of the listed shapes was determined [3], and were used as inputs for later analysis. Second, the graphical portrayal of the displacement provided guidance on where to support each geometry in order to experimentally capture the flexure and torsion resonances, especially for the cases of the three hexagon-based shapes.

Torsional (f_{tors}) and flexural (f_{flex}) resonances are a function of geometry as well as the density and elastic properties of the material comprising it. Three FEA models were run per geometry and material–because resonant frequencies are multilinearly dependent on only two parameters in this case (*i.e.*, E and ν). E and ν were then varied in a bracketing manner about estimated values for each (where either E or ν was fixed while varying the other). For example, if the ceramic was a

pressureless-sintered SiC, and its E and ν were estimated to be 415 GPa and 0.18, respectively, then the three E - ν combinations for the three modal analyses were (semi-arbitrarily) chosen to be: 440 GPa - 0.12; 440 GPa - 0.24, and; 390 GPa - 0.24. f_{tors} and f_{flex} then multilinearly depended on E and ν.

Torsion and flexure resonances were measured using a commercially available IEV instrument (GindoSonic MK5i, J.W. Lemmens, Inc., St. Louis, MO). Each of the disk, square, hexagon, or half-hexagon tile geometries was supported with hard rubber shims that were positioned at zero (or equal) displacement locations for each respective geometry and resonant mode. Such locations are relatively intuitive for disks and squares; however, the graphical output of the FEA for the full- and half-hexagons guided the experimental shim support placement. The tile was then struck with an impulse hammer and only a repeatable resonant frequency value was recorded. The experiment was then repeated to obtain the other of the two resonant mode frequencies.

The third step involved combining the modal analyses and the experimentally obtained f_{tors} and f_{flex}. Singly, f_{tors} or f_{flex} is not represented by a unique combination of E and ν, but rather a locus of combined E - ν values. However, obtaining and analyzing the combination of the E - ν locus for torsional resonance and the E - ν locus for flexural resonance enables the determination of E and ν that satisfy both resonant modes.

Lastly, ultrasound velocity measurements were made in all the tiles IEV-examined tiles and the Young's Modulus values of both test methods were compared. Longitudinal and shear sensors (Panametrics, Inc., Waltham, MA) were used to measure the velocities (Matec Instruments, Northborough, MA) of both and E was determined using ASTM E494 [1].

III. RESULTS & DISCUSSION

Two E - ν loci representations result upon combination of the modal analyses and experimentally obtained f_{tors} and f_{flex}; an example of which is shown in Fig. 1. The intersection of the two loci represents a unique E - ν combination that satisfies both modal resonance modes. That combination is concluded to be the bulk E and ν of the material comprising the tile and its analysis symbolizes the utility of the study's procedure.

A limitation of only knowing one of the two loci is described in reference to Fig. 1. If only one resonant mode can be exploited, then one needs to estimate the Poisson's ratio in order to determine E; this is typically the case for bend bars (a common test coupon in the structural ceramic community) as indicated in ASTM C1259 [2].

Graphical illustrations for the two resonant modes for each of the five explored geometries appear in Figs. 2-6. The resulting E - ν combination for each that satisfied both measured resonances appears in the caption of each figure. Additionally, the Annex in ASTM C1259 [2] describes a method that determines E for disks through the analysis of both flexure and torsional resonances. An E = 383 GPa was determined using ASTM C1259 for the disk shown in Fig. 2; this matches the E determined using the herein described test method and verified by ultrasonic measurements.

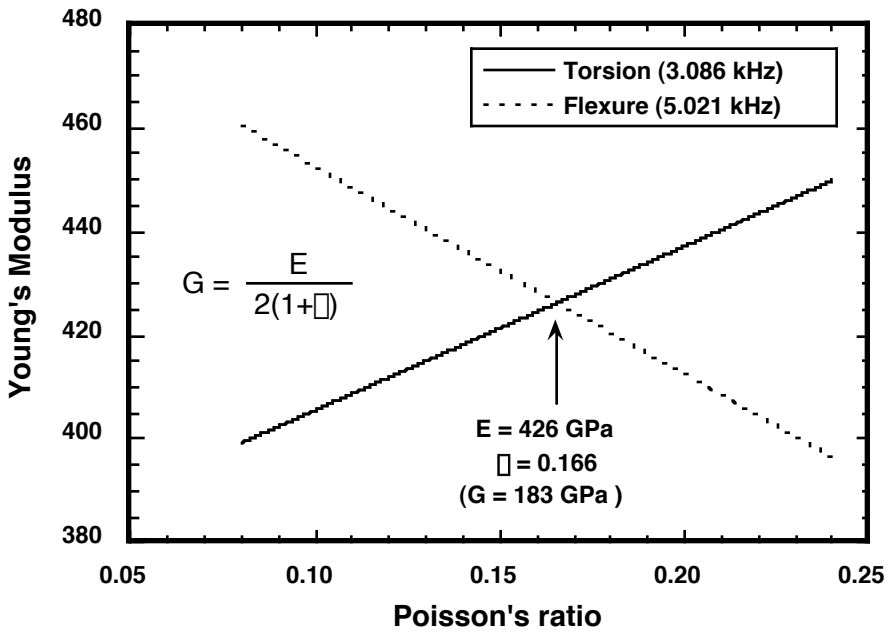

Figure 1. Example of Young's Modulus - Poisson's ratio loci for torsional and flexural resonant isofrequencies for a 100.8 x 100.8 x 4.03 mm Hexoloy® SA SiC tile (ρ = 3.16 g/cc).

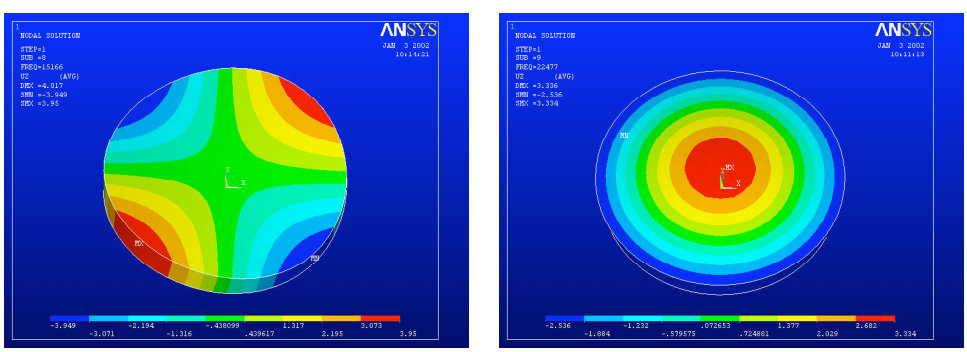

Figure 2. Torsional (left) and flexural resonant states for a CAP3 AD995 Al_2O_3 disk. Determined elastic properties: E = 383 GPa, ν = 0.238, and G = 155 GPa.

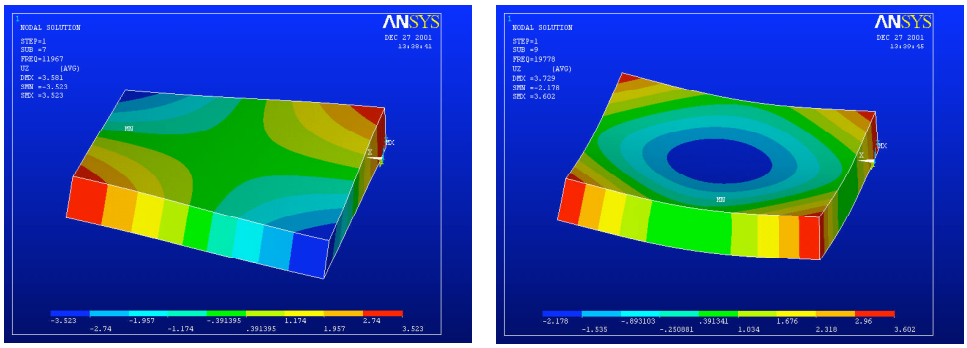

Figure 3. Torsional (left) and flexural resonant states for a Hexoloy® SA SiC square plate. Determined elastic properties: E = 423 GPa, □ = 0.162, and G□□82□GPa.

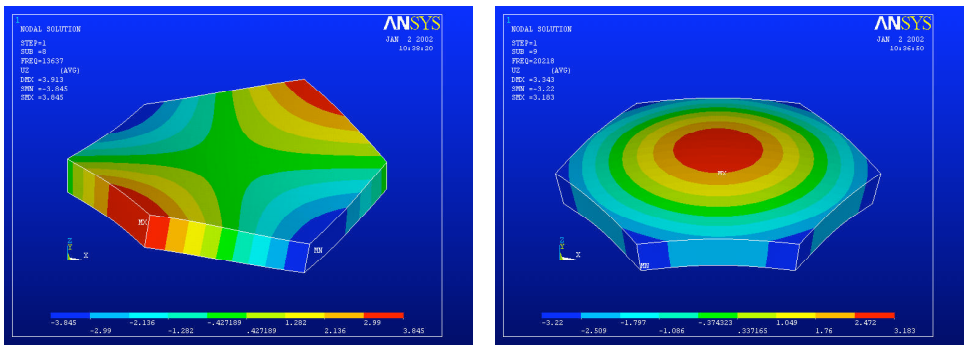

Figure 4. Torsional (left) and flexural resonant states for a SiC-SC-1R SiC hexagon plate. Determined elastic properties: E = 451 GPa, □ = 0.161, and G□□94□GPa.

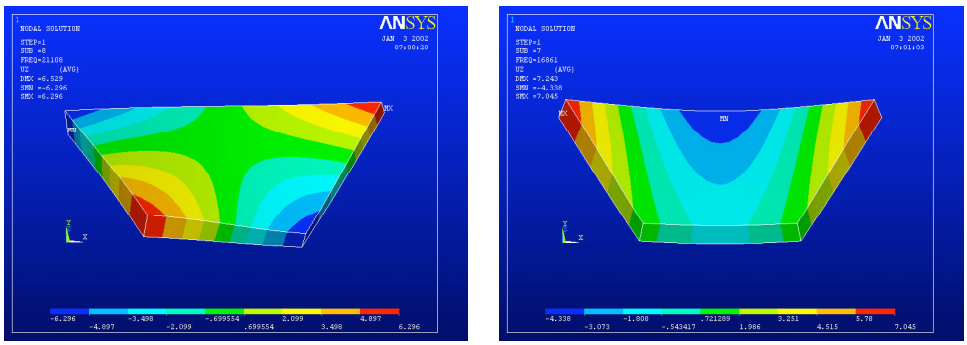

Figure 5. Torsional (left) and flexural resonant states for a CAP3 AD995 Al_2O_3 half-hexagon (trapezoid). Determined elastic properties: E = 362 GPa, □□□.208, and G = 150 GPa.

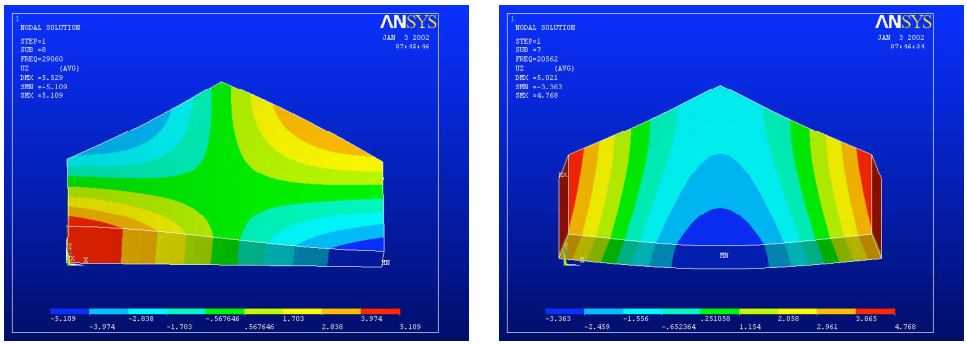

Figure 6. Torsional (left) and flexural resonant states for a CAP3 AD995 Al_2O_3 half-hexagon (house-shape). Determined elastic properties: E = 386 GPa, ν = 0.154, and G = 167 GPa.

Numerous ceramics were explored using IEV and the ultrasonic velocity method. The Young's modulus, as measured by both techniques, are listed in Table 1. The differences in E measured by each technique is 2% or less for disks, squares, full hexagons, and house-shaped half-hexagons. The difference was relatively large (5.8%) for the trapezoidal half-hexagon; this high value was attributed to its actual geometry not being sufficiently close to the idealized half-hexagon geometry modeled in the FEA. This illustrates a limitation to the technique described in the study; namely, if the geometry is not sufficiently modeled by the FEA, then uncertainty will be introduced into the reported values of the elastic properties.

If the ceramic in a tile or plate is homogeneous, then the E measured with IEV and ultrasonic velocity techniques should match. However, if the ceramic is not homogeneous, then the two measured E values may or may not match depending on the nature of what volume was sampled by the longitudinal and shear sensors used with the ultrasonic velocity technique. A match of E from the two techniques would be fortuitous in this situation. The advantage of the IEV technique is that the resonance is a consequence of the *whole* material state of the component making it an effective way to identify outliers in a population.

IV. SUMMARY

High resolution, bulk E, G, and ν for numerous ceramics fabricated in shapes of disks, squares, hexagons, and half-hexagons were obtained. The employed method combined modal finite element analysis and the flexural and torsional resonance values measured by impulse excitation of vibration. The utilization of both resonances in the analysis allowed E, ν, and G to be independently determined.

Table I. Young's Modulus Comparison of Values Generated With Impulse Excitation (E_{IEV}) and Ultrasound (E_{Ultr}).

Material	Plate Shape	E_{IEV} (GPa)	E_{Ultr} (GPa)	% Diff.
CAP3 AD995 Al_2O_3	Disk	383	385	0.4
PAD B_4C	Square	446	451	1.1
Ceralloy 147-31N Si_3N_4	Square	304	307	1.0
Hexoloy® SA SiC	Square	423	431	2.0
PAD SiC-B SiC	Square	450	455	1.1
PAD SiC-N SiC	Square	451	452	0.2
PAD WC	Square	699	698	0.0
TiB_2	Square	569	560	1.6
CAP3 AD995 Al_2O_3	Hexagon	382	375	1.9
SiC-SC-1R SiC	Hexagon	451	455	1.0
CAP3 AD995 Al_2O_3	Half-hexagon (trapezoid)	362	384	5.8
CAP3 AD995 Al_2O_3	Half-hexagon (house-shape)	386	383	0.7

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[3] ANSYS, Release 5.6, Canonsburg, PA.