The Center for Astrophysical Thermonuclear Flashes

Scaling Relations and Non-Steady Flame Propagation in Type Ia Supernovae

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Asymmetries in Type Ia Supernovae

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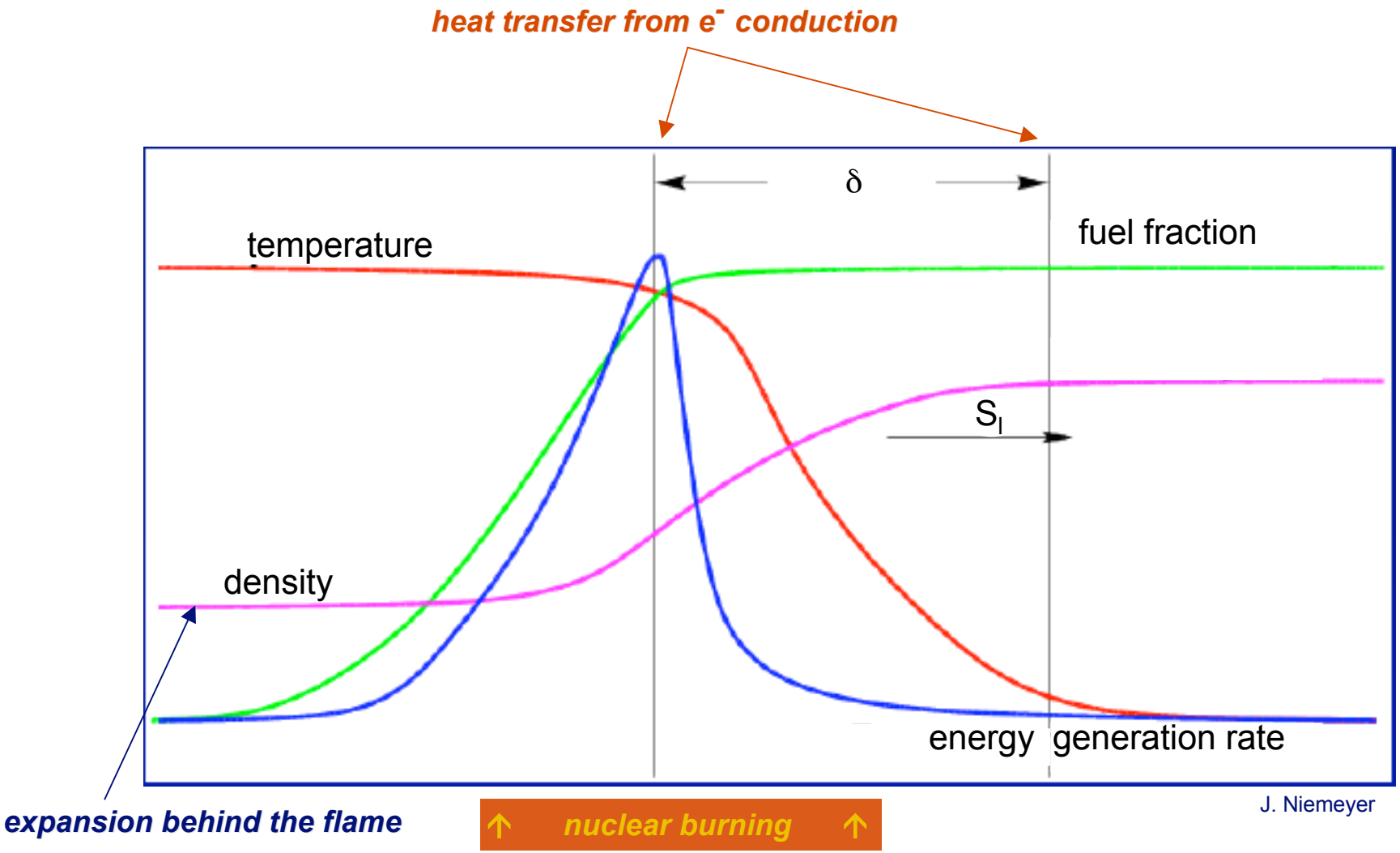
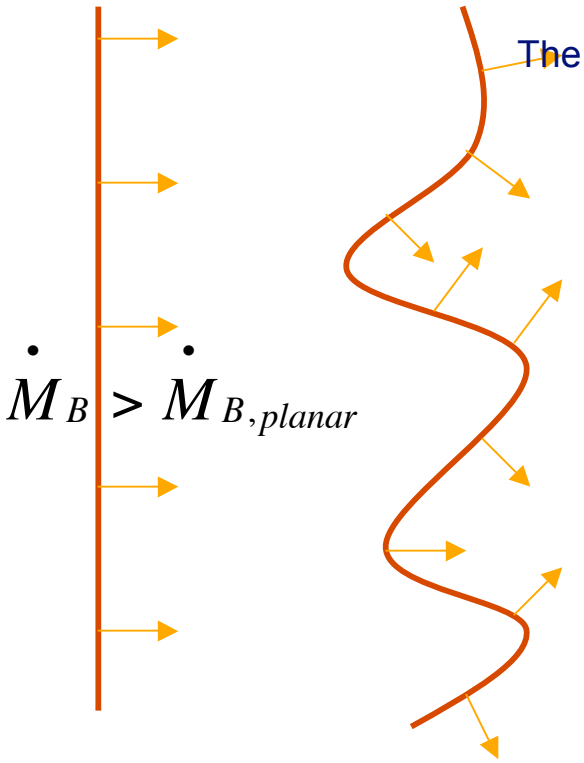
The microscopic physics of thermonuclear flames in Type Ia SNe are well understood



J. Niemeyer

Modifying laminar burning: the role of turbulence



The initially planar flame front propagates normal to itself…

Turbulence tends to distort the flame surface, increasing the surface area.

Turbulent burning speed increases proportionally to the increase in surface area:

$$S_t = \frac{\dot{M}_B}{\rho_{fuel}} \rightarrow S_t = \left(\frac{\Sigma}{\Sigma_0}\right) S_l$$

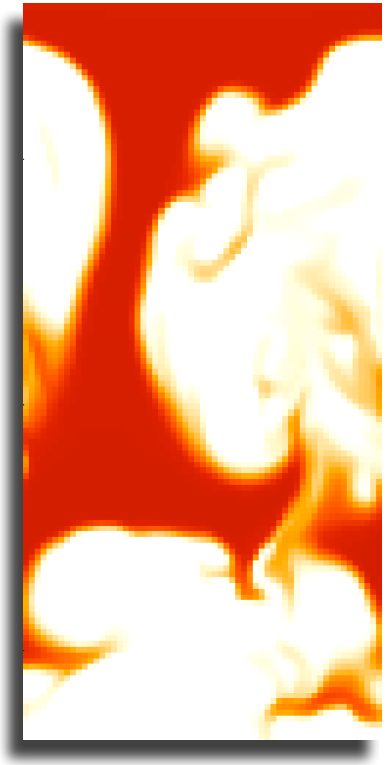
What leads to turbulence?

Rayleigh-Taylor instability

- primary source of turbulence in Type Ia SNe
- growth rate $\propto \lambda^{1/2}$; dominates on large scales
- size of the domain determines the scale (and growth rate) of the largest (and fastest) mode
- Froude number

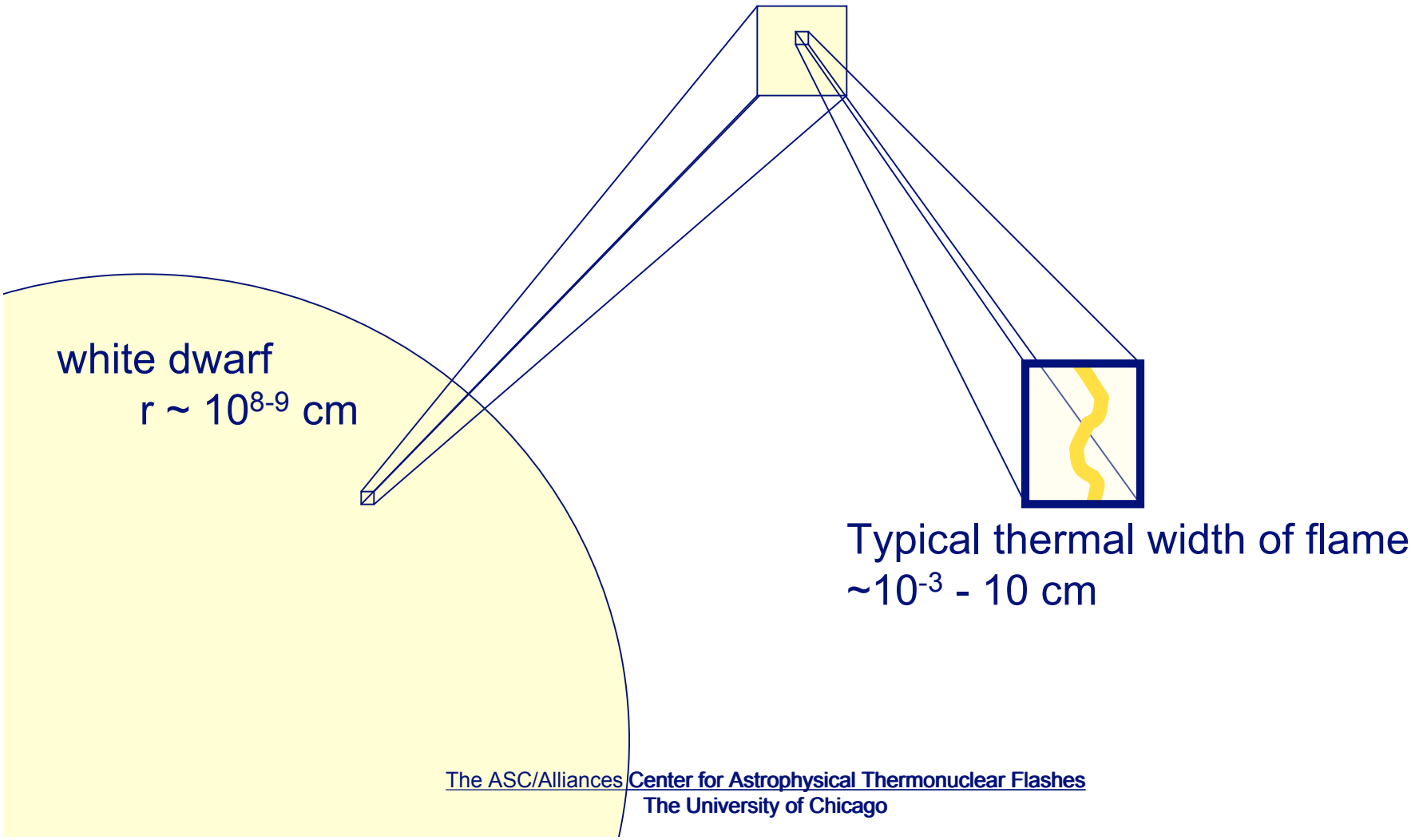
$$F = S_l^2 / gL$$

characterizes the ability of the propagating flame front to smooth out perturbations caused by the R-T instability



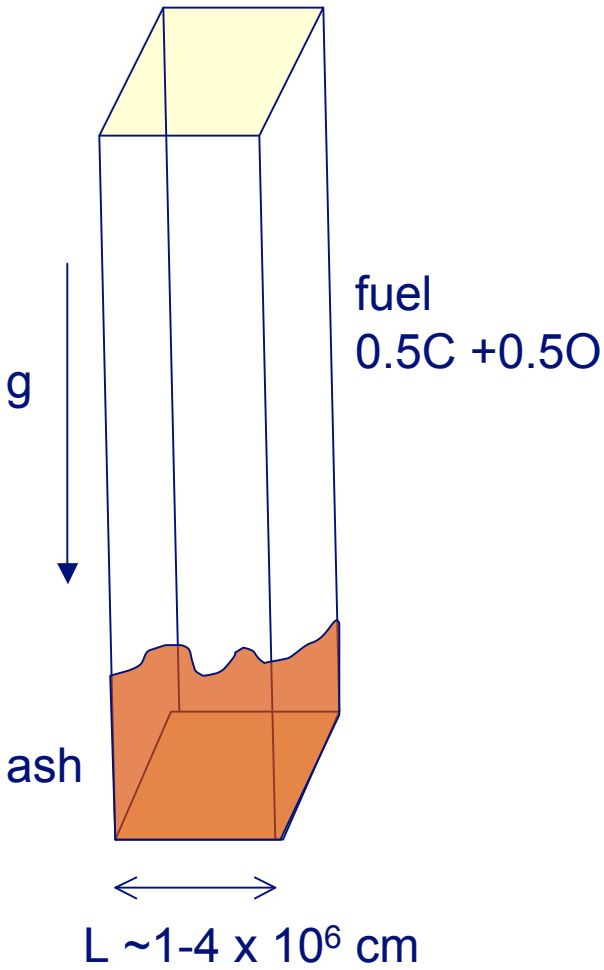
The disparity of scales in the Type Ia problem demands sub-grid modelling

Typical cell in large-scale simulations
$\Delta x \sim 1$ km



Determining a turbulent flame speed

Khokhlov 1995, ApJ 449, 695.



Two dimensionless parameters: F and A

Two velocities: S_l and the R-T characteristic speed

In steady state and with F<<1,

$$S_t = f(A)\sqrt{gL}$$

…independent of the laminar flame speed

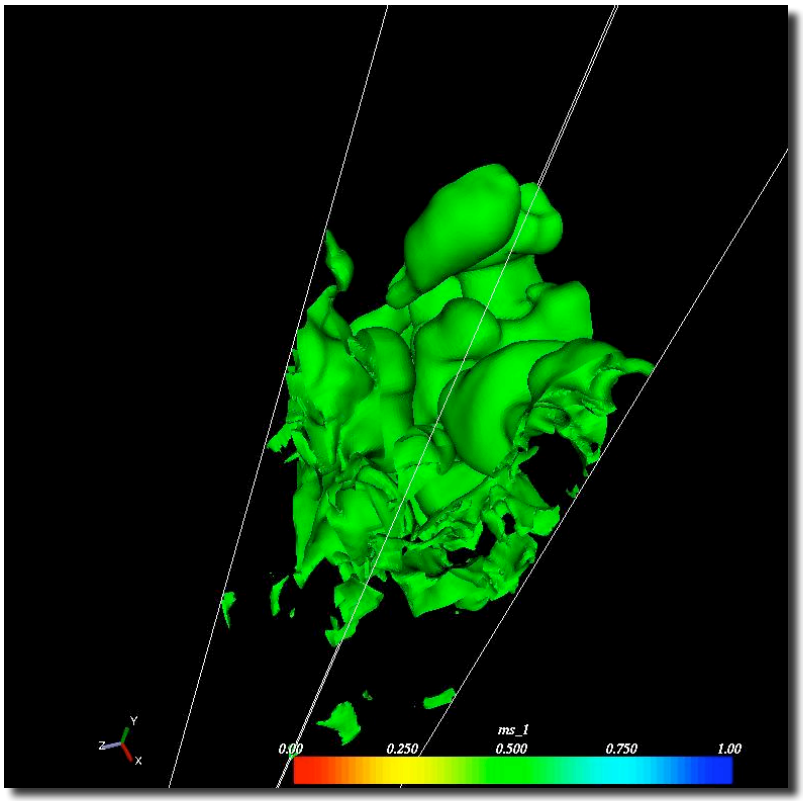
Perform numerical experiments with domains comparable to the cell size from a full-scale SN Ia simulation.

- WD EOS
- g, energy release, S_l, etc. from typical Type Ia SN conditions

The turbulent flame is self-regulating

Steady-state rate of fuel consumption does not depend on small-scale physics:

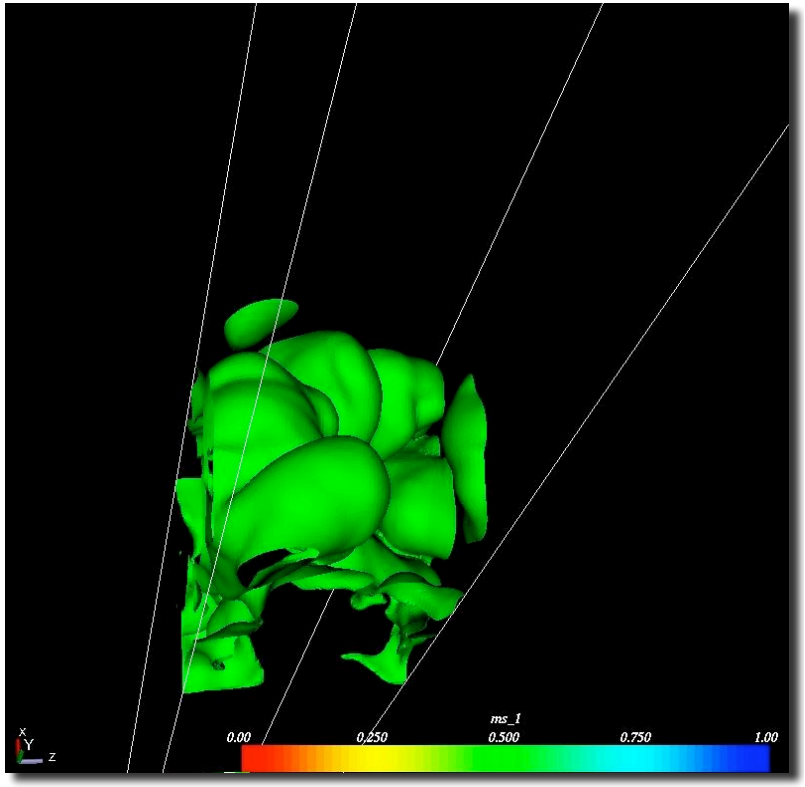
S_l=1.07 x 10^6 cm/s



Σ = 2.84 x 10^{13} cm^2

$$S_t \approx S_l \Sigma$$

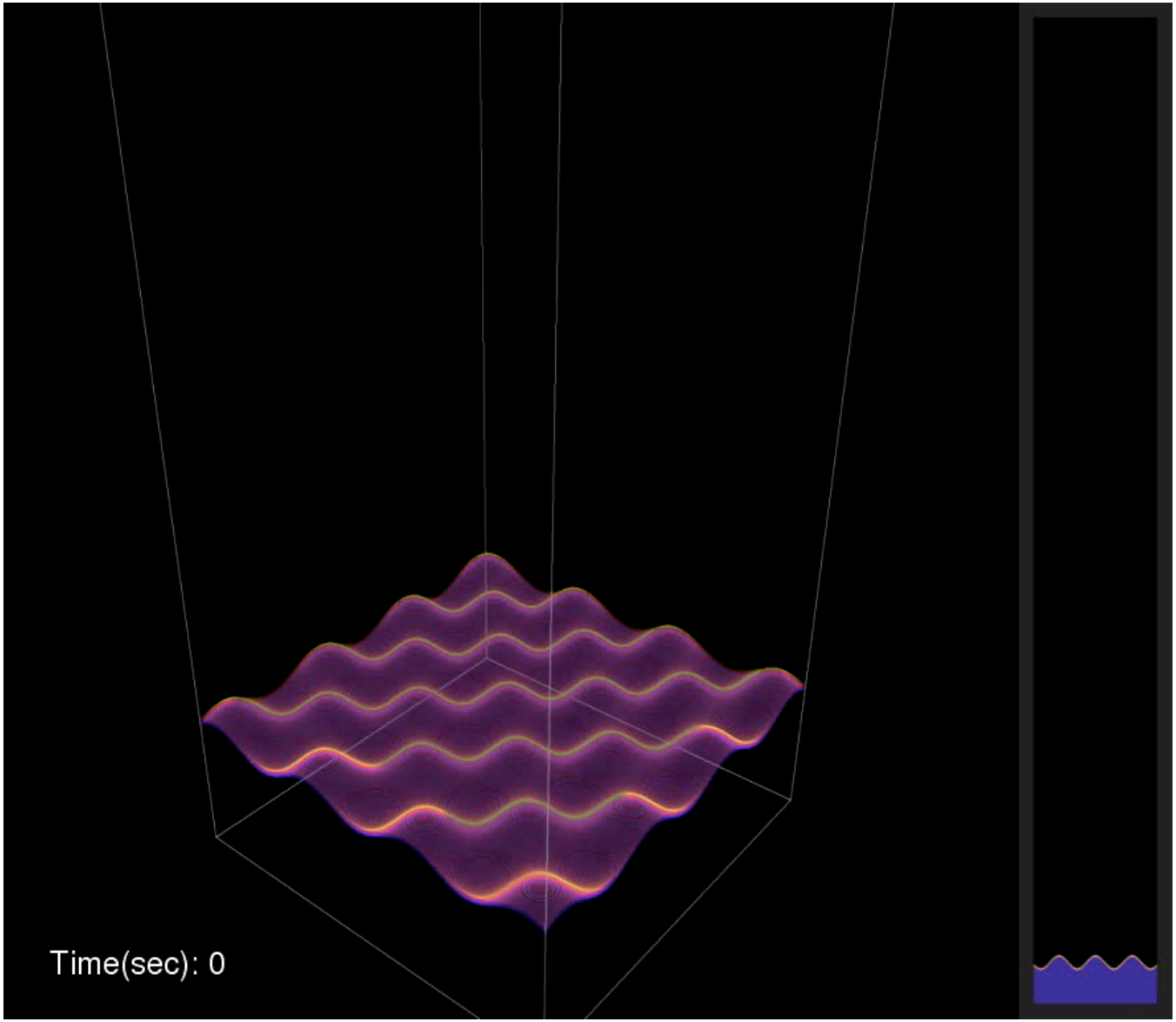
S_l=2.14 x 10^6 cm/s



Σ = 1.41 x 10^{13} cm^2

$$S_t = \alpha\sqrt{gLA}$$

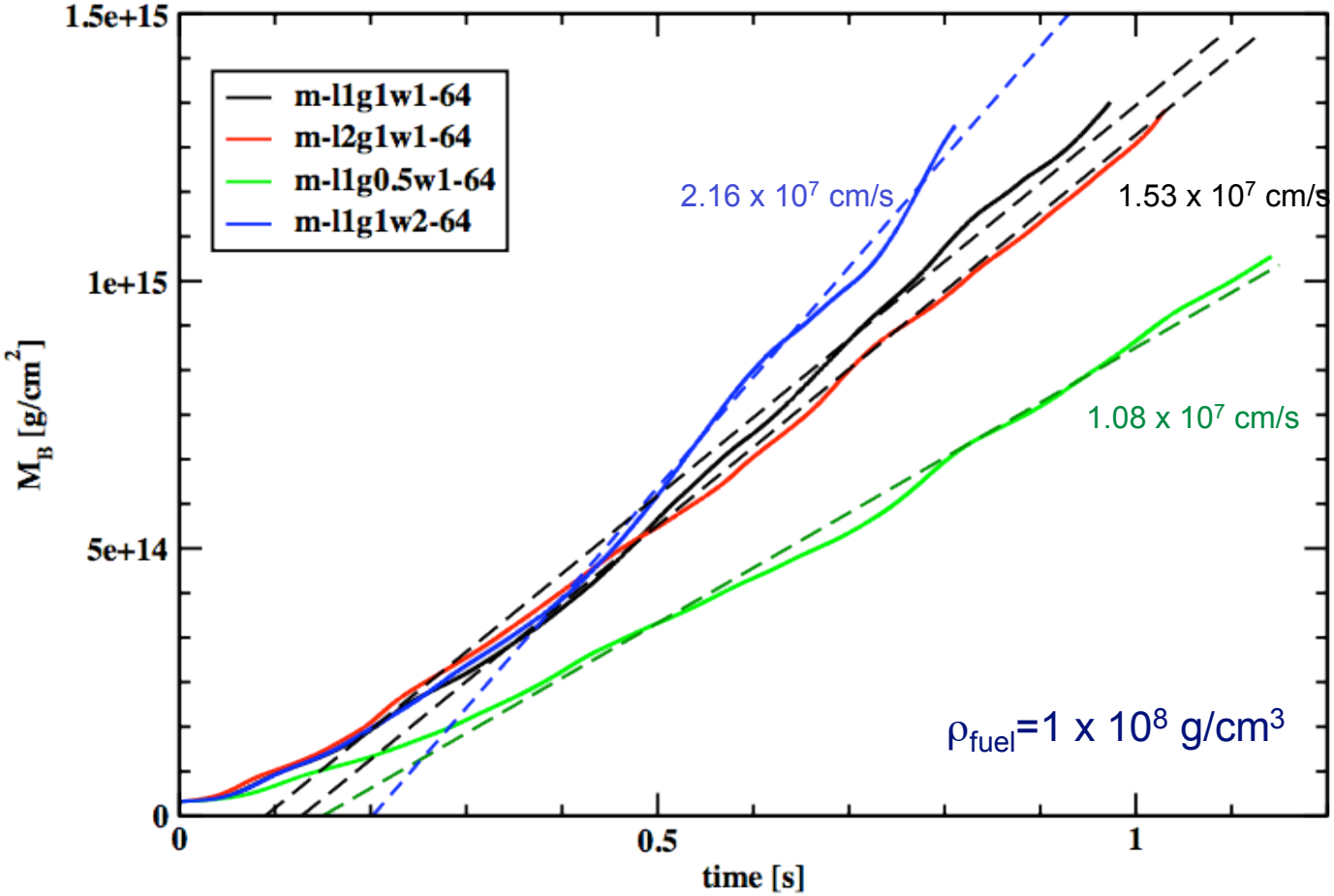
Flame surface: high resolution simulation



All simulations produce turbulent flame speeds in agreement with scaling relation

$$S_t = 0.5\sqrt{AgL}$$

Predictions from scaling law



Turbulent subgrid flame model

$$S = \max(S_l, S_{sub})$$

$$S_{sub} = 0.5\sqrt{Ag\Delta x}$$

Important for making a subgrid model:

- S_t independent of S_l
- $S_t \propto g^{1/2}$
- cell size (Δx) determines turbulent flame speedup

However, this subgrid model depends on the assumption of steady-state velocity perturbations, where the induced turbulence is fully developed.

Non-steady flame propagation

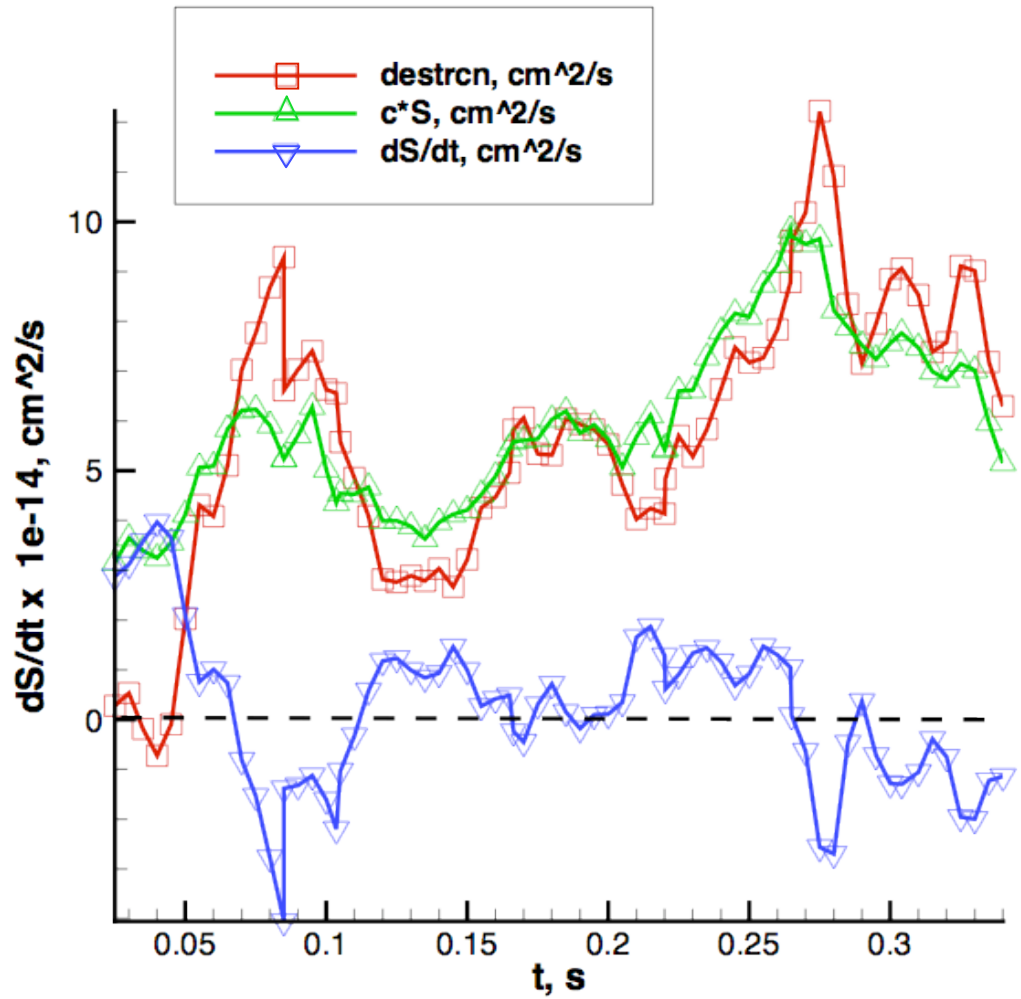
How does the flame surface area change with time?

$$\frac{d\Sigma}{dt} = c\Sigma - dS_l\Sigma^2$$

Khokhlov (1995)

Determination of c and d allows a “non-steady” subgrid model to be formulated

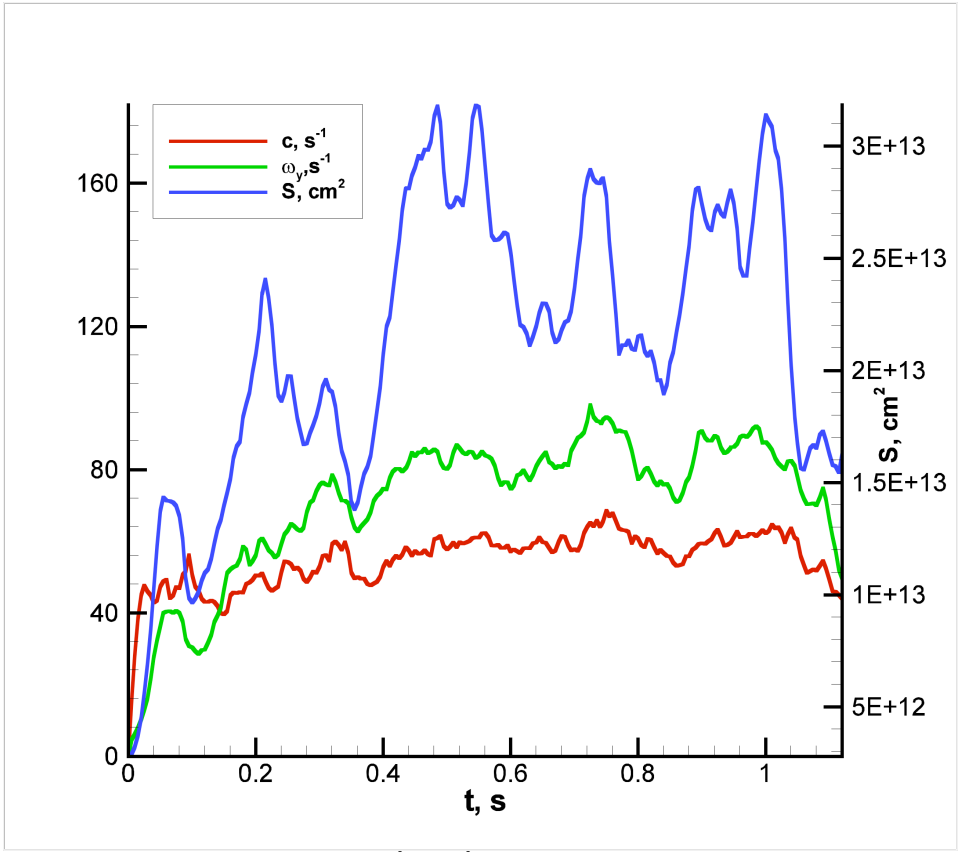
How is d correlated with the turbulent fluid velocities or other flow characteristics?



Correlation between c and y-vorticity

- The creation coefficient c (integrated over the entire flame surface) seems to closely correlate with the average magnitude of the vorticity component in the direction of gravitational acceleration.

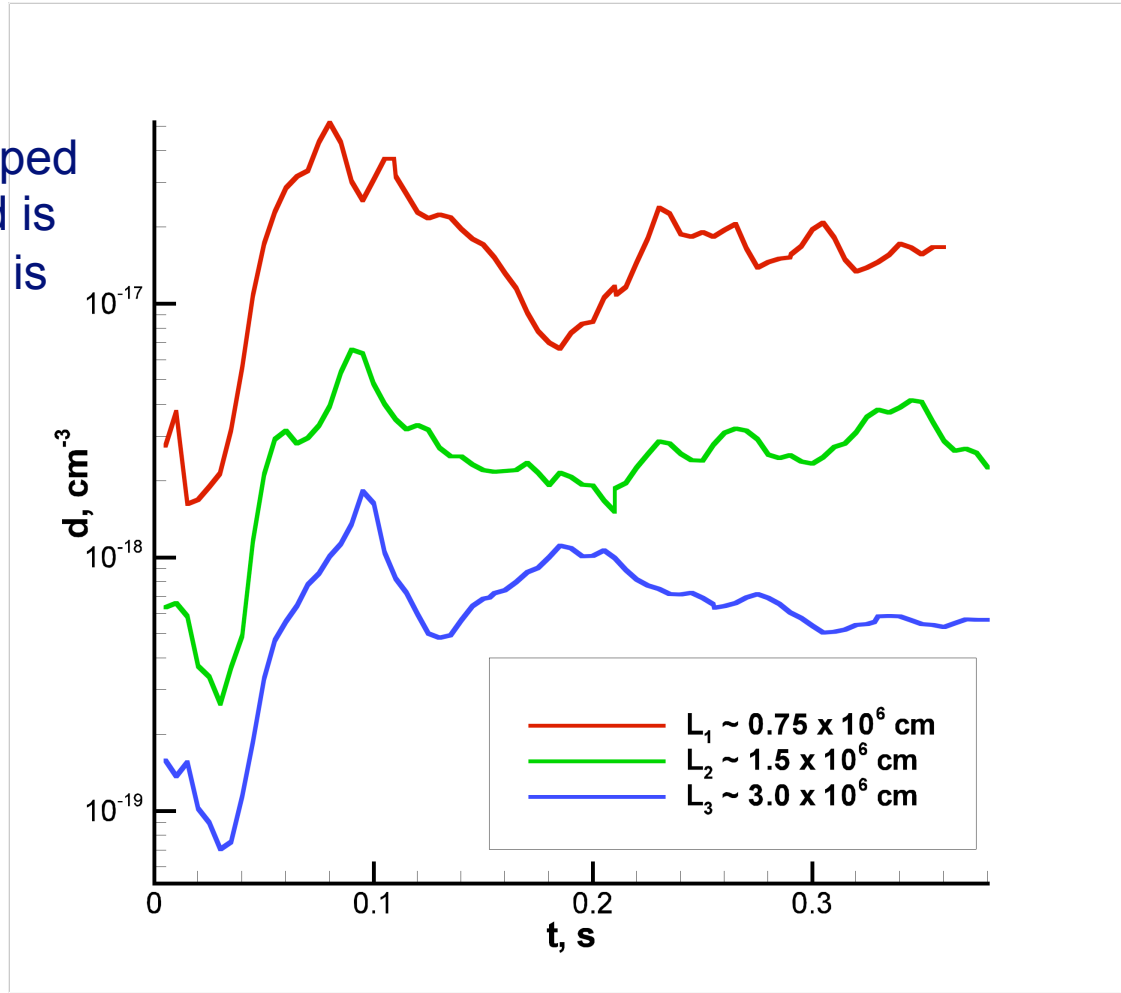
- The higher the y-vorticity, the more wrinkling on the flame surface and the higher the flame surface creation.



$$< c'\omega_y' >= 0.78$$

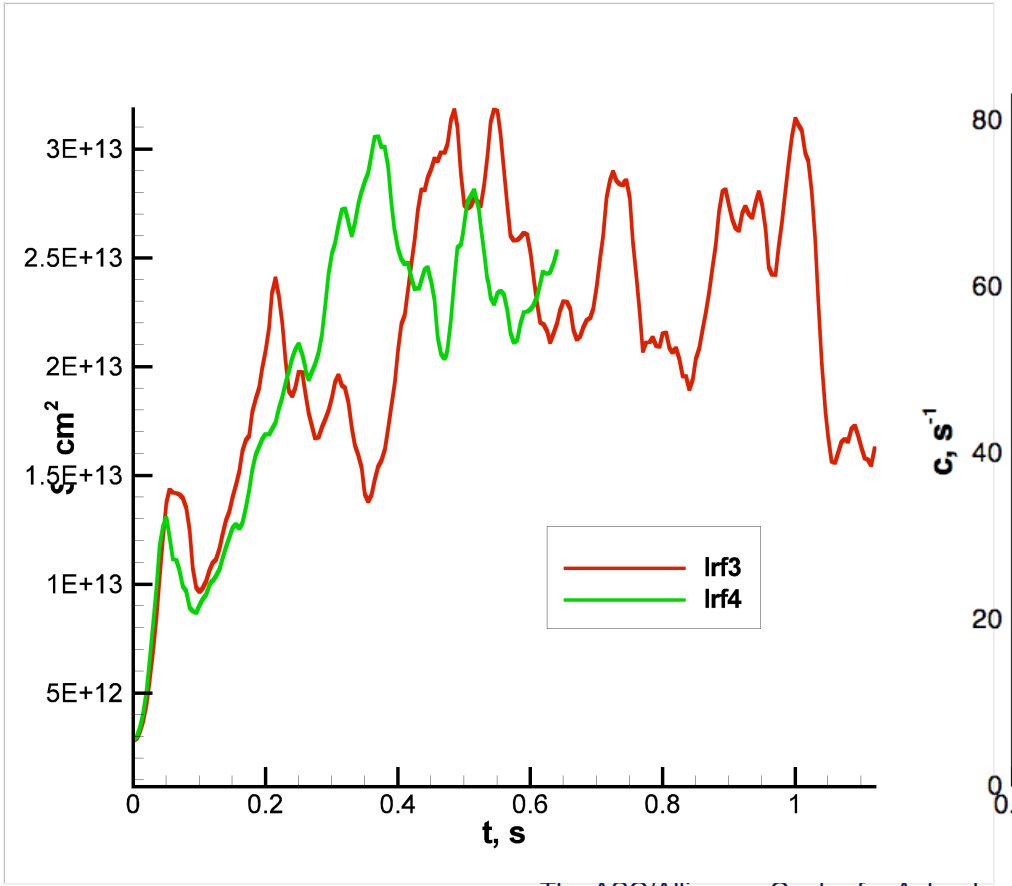
d ~ 1/L^3 in fully developed turbulence

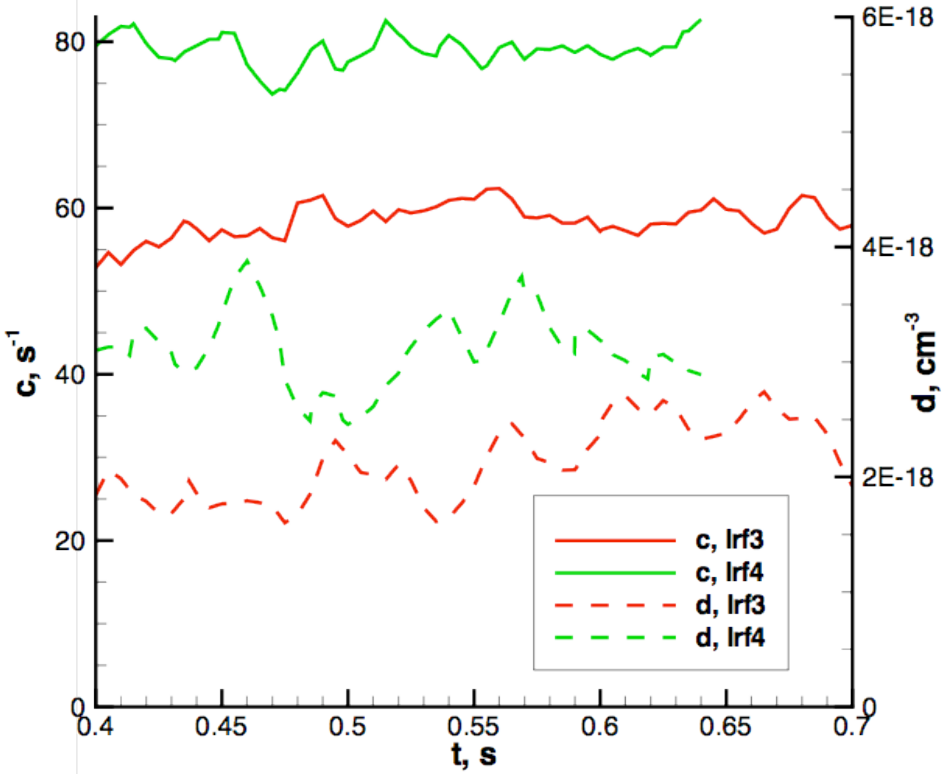
- Destruction coefficient, d, has dimensions of 1/volume
- Evidence that, in fully developed turbulence (here, t ≥ 0.2 s), d is on the order of 1/L^3, where L is the characteristic Rayleigh-Taylor bubble size.



Self-regularization and spatial resolution

- The creation and destruction coefficients appear to increase proportionately as the resolution is increased.
- As a result, the total flame surface, burned mass, etc. converge, as they are dependent on the competition between creation and destruction, even though the creation and destruction coefficient vary directly with spatial resolution.





Conclusions

- Disparity of scales in the Type Ia supernova problem makes subgrid flame modeling necessary
  - Subgrid model controls the transfer of turbulent kinetic energy to internal energy; important for pre-expansion of the WD, rate of bubble mergers, etc.
- Using FLASH, the results of Khokhlov (1995) have been reproduced at much higher resolutions and for a variety of parameter sets spanning much of the important range for large scale simulations.
- More sophisticated and realistic subgrid model will be formulated by examining correlations between fluid flow and flame area creation and destruction
  - Flame surface creation seems to be closely correlated with ω_y
  - Flame surface destruction is directly proportional to the size of the RT bubbles