Multiscale Interactions in Shallow Water Equations Test Cases

John B. Drake

Oak Ridge National Laboratory
(ORNL)

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Shallow Water Test Case 5c

- Why another variant on Case 5?
  - Multiscale interactions and turbulent diffusion/ dispersion character
  - Asymptotic behavior
  - Relevant Adaptive Grid features
  - Correcting error in out-of-balance initial conditions for case 5
- Initial Conditions
  - Same as Steady Zonal Flow Case 2 (or Case 5)
  - No initial surface geopotential
- Surface forcing: Growing/decaying Mountain
  - Smooth growth over t=[0,1] day
  - Smooth decay over t=[10,11] days
- Duration
  - 60 day simulation
- Diagnostics
  - K.E. Spectrum, Enstrophy Evolution, Triads

$$E_n = \frac{a^2}{4n(n+1)}[\left|\varsigma_n^0\right|^2 + \left|\delta_n^0\right|^2 + 2\sum_{m=1}^{n}\left|\varsigma_n^m\right|^2 + \left|\delta_n^m\right|^2$$

SWE Formulation

- Guo's advective formulation

$$\frac{d\eta}{dt} = -\eta\delta$$

$$\frac{d\delta}{dt} + \nabla^2\phi = R_\delta$$

$$\frac{d\phi}{dt} = -\phi\delta$$

$$R_\delta = \eta(\eta - f) - \vec{k}\cdot v \times \nabla f + v\cdot F - \nabla^2(\phi_s + \frac{v\cdot v}{2})$$

$$F = \frac{1}{\cos\theta}\nabla^2(v\cos\theta) + \frac{2\sin\theta}{a\cos\theta}\begin{pmatrix} -\varsigma \\ \delta \end{pmatrix}$$

Spectral Semi-Implicit Semi-Lagrangian Algorithm

- Three time level semi-Lagrangian Transport method
  - Shape preserving interpolations (Williamson&Rauch, MWR 1989)
  - 2nd order particle tracking
- Spherical Harmonic Transform
  - Approximation of Laplacian and gradient operators
  - Vorticity, divergence inversion for velocities
  - Solution of semi-implicit Helmholz equation
  - Grid stretching per Schmidt conformal transformation
- Asselin time filter (μ=0.06)
- Spectral viscosity (Gelb and Gleeson, MWR 2001) damping of the divergence

$$\delta_n^m = \sigma_n^{SV} \delta_n^m$$

$$\sigma_n^{SV} = \frac{1}{1 + 4\Delta t q_n \dfrac{n^2 (n+1)^2}{aM^3}}$$

$$q_n = \begin{Bmatrix} 0 \quad for \quad n \le n_c \\ \exp\left(-\dfrac{(n-M)^2}{2(n-n_c)^2}\right) \quad for \quad n_c \prec n \le M \end{Bmatrix}$$

$$n_c = 2M^{3/4}$$

Vanishing Viscosity and Spectral Diffusion

- For Barotropic Vorticity Equation
  - Dynamics manifests in resonant triads
  - With a diffusion term, the vanishing viscosity solution is obtained as a limit
  - Is this “the equation we want to solve?”
  - Energy dissapation rate is proportional to enstrophy, Z(t)
  - So, vanishing viscosity implies vanishing energy dissipation
- Spectral diffusion
  - Dependant on resolution (n_c goes to infinity implies viscosity goes to zero)
  - Use Enstrophy evolution to diagnose numerical diffusion of schemes in resolution limit

$$\frac{\partial \varsigma}{\partial t} + J(\psi, \varsigma) = \nu \nabla^2 \varsigma$$

$$\nabla^2 \psi = \varsigma$$

$$Z(t) = \int \left(\nabla^2 \psi\right)^2$$

$$\varepsilon = \frac{d}{dt} \int \frac{v \cdot v}{2} = \nu Z(t)$$

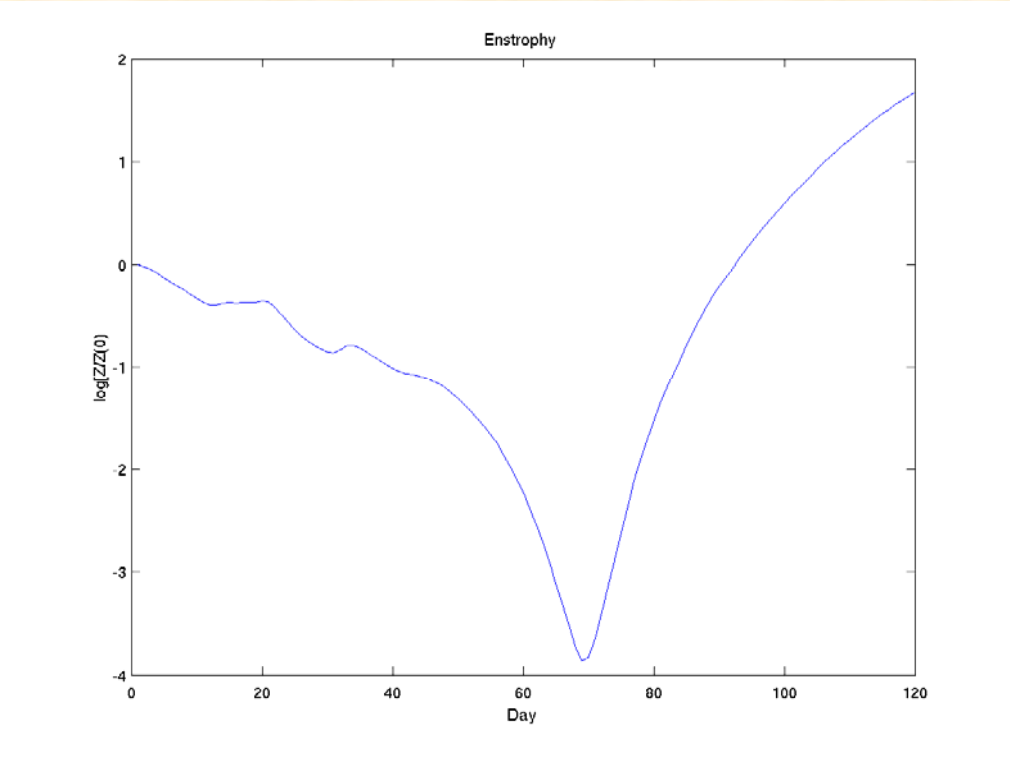
- BVE is a dynamical subsystem of SWE

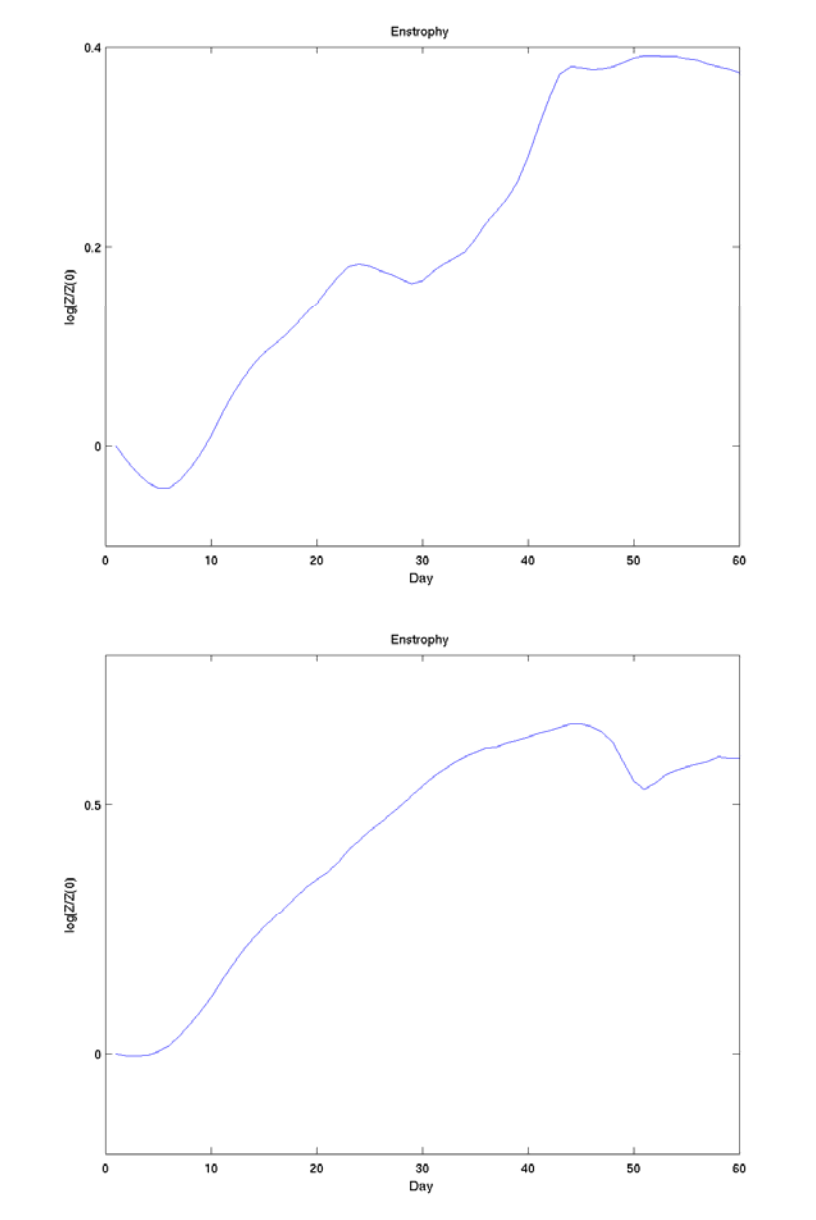
Spectrum from Barotropic Vorticity and Geostrophic Turbulence

- Classical Kolomogorov
  - Homogeneous turbulence theory
  - Energy spectrum with slope k^{-3}
  - Energy – Enstrophy cascade (up-down-up) (Smith et al, JFM, 2002)
- Coherent vortices
  - Arising from forcing (not small perturbations)
  - Energy spectrum k^{-6}
  - "a strong enstrophy cascade is initially present, but as the cascade subsides, significant enstrophy remains trapped in isolated vortices." - (Salmon, GFD, p.226)
- Flow structure of decaying turbulence
  - "stirring of potential vorticity in high latitudes, leads to the predictable polar anticyclone, almost regardless of the detailed nature of the Rossby waves or geostrophic turbulence" – (Rhines, JFM 1974)
  - Structure emerges in first week, full blown by five weeks – (Yoden et al, INC, 1999)
- Enstrophy evolution
  - Finite time blowup (inviscid theoretical)
  - Vanishing viscosity solutions approached by numerical methods (Smolarkiewicz, IJNMF, 2002)

Uniform Resolution Experiments

- T21,T42, T85





T21 Resolution Solution

QuickTime™ and a
decompressor
are needed to see this picture.

T42 Resolution Solution

QuickTime™ and a
decompressor
are needed to see this picture.

T85 Resolution Solution

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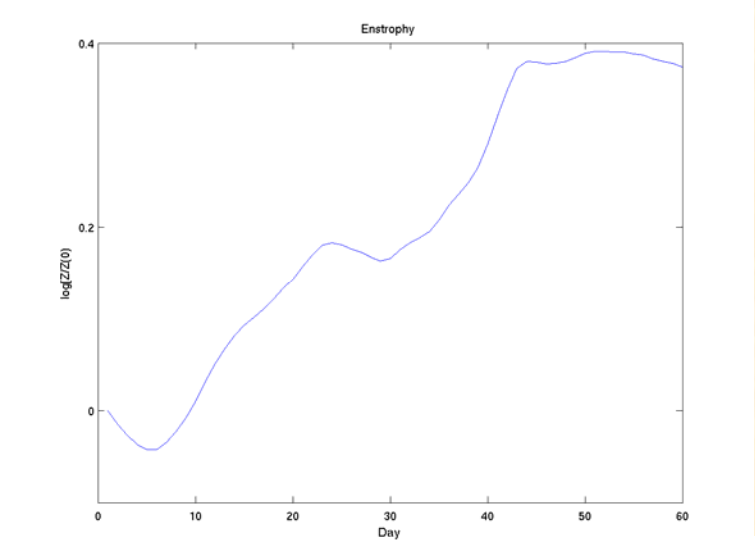
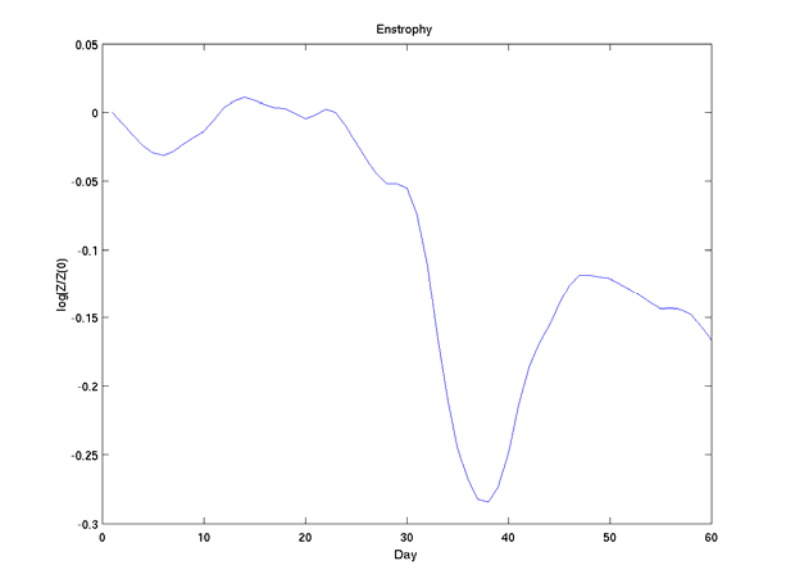
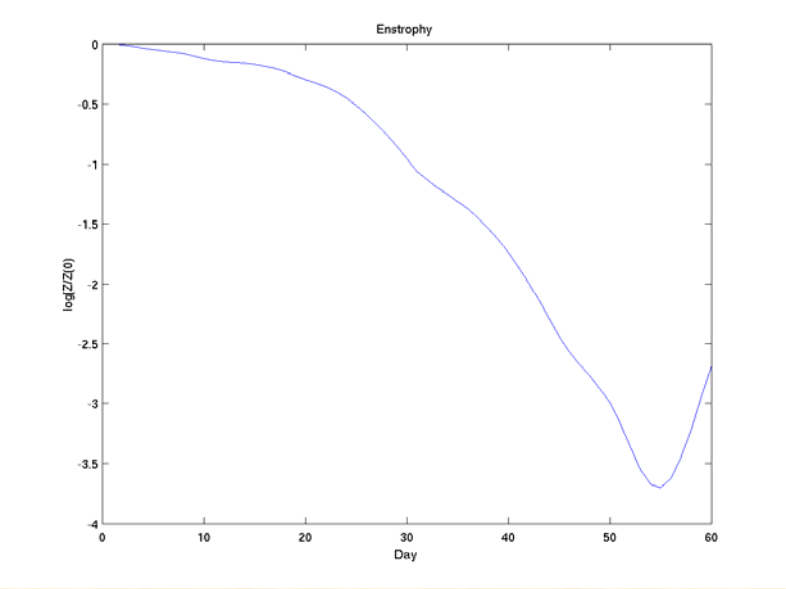
decompressor
are needed to see this picture.

Stretched Grid Experiments

- T42 (c=1,2, 3)







Conclusions

- For coarse resolution (T21 and T42), the simulated SWE solution does not allow the k^{-3} cascade to set up.
- A k^{-6} cascade is shown associated with large coherent structures in coarse resolution
- At T85 the k^{-3} spectrum is evident by 60 days.
- Enstrophy evolution, and hence energy dissipation rate, reflects a blow up of the solution for T21 resolution after 60 days.
- Stretched grid solutions show improvements in spectrum and enstrophy evolution for moderate stretching (c=2) but significant degradation for (c=3) at T42.
- The proposed test case 5c is interesting for asymptotic properties and decaying turbulence
- Time dependant / spatially varying features make it suitable for exercising adaptive mesh and multiresoltuion methods

Phoenix

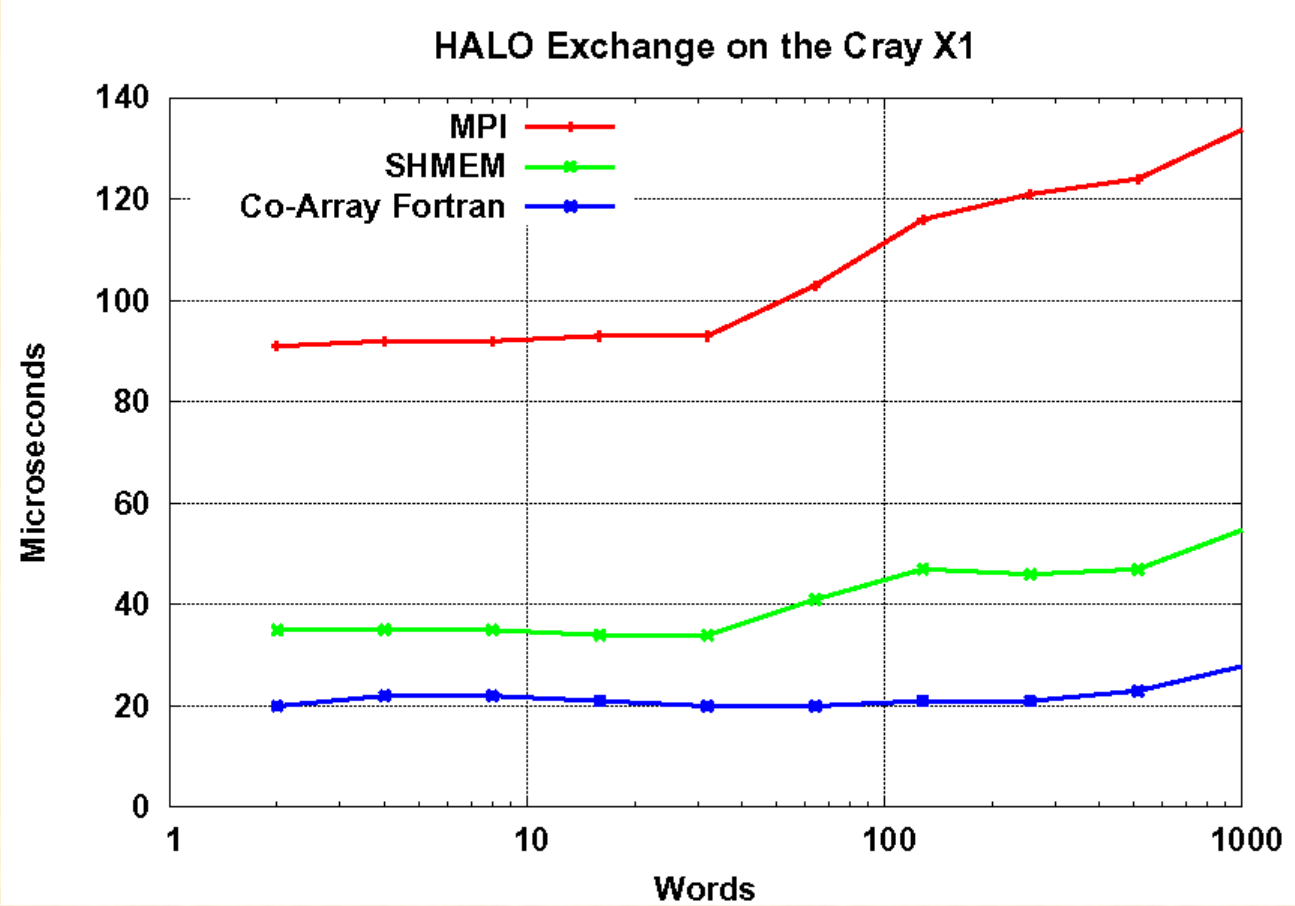
Cray X1 with 128 SMP nodes

- 4 Multi-Streaming Processors (MSP) per node
- 4 Single Streaming Processors (SSP) per MSP
- Two 32-stage 64-bit wide vector units running at 800 MHz and one 2-way superscalar unit running at 400 MHz per SSP
- 2 MB Ecache per MSP
- 16 GB of memory per node

for a total of 512 processors (MSPs), 1024 GB of memory , and 6.4 TF/s peak performance.

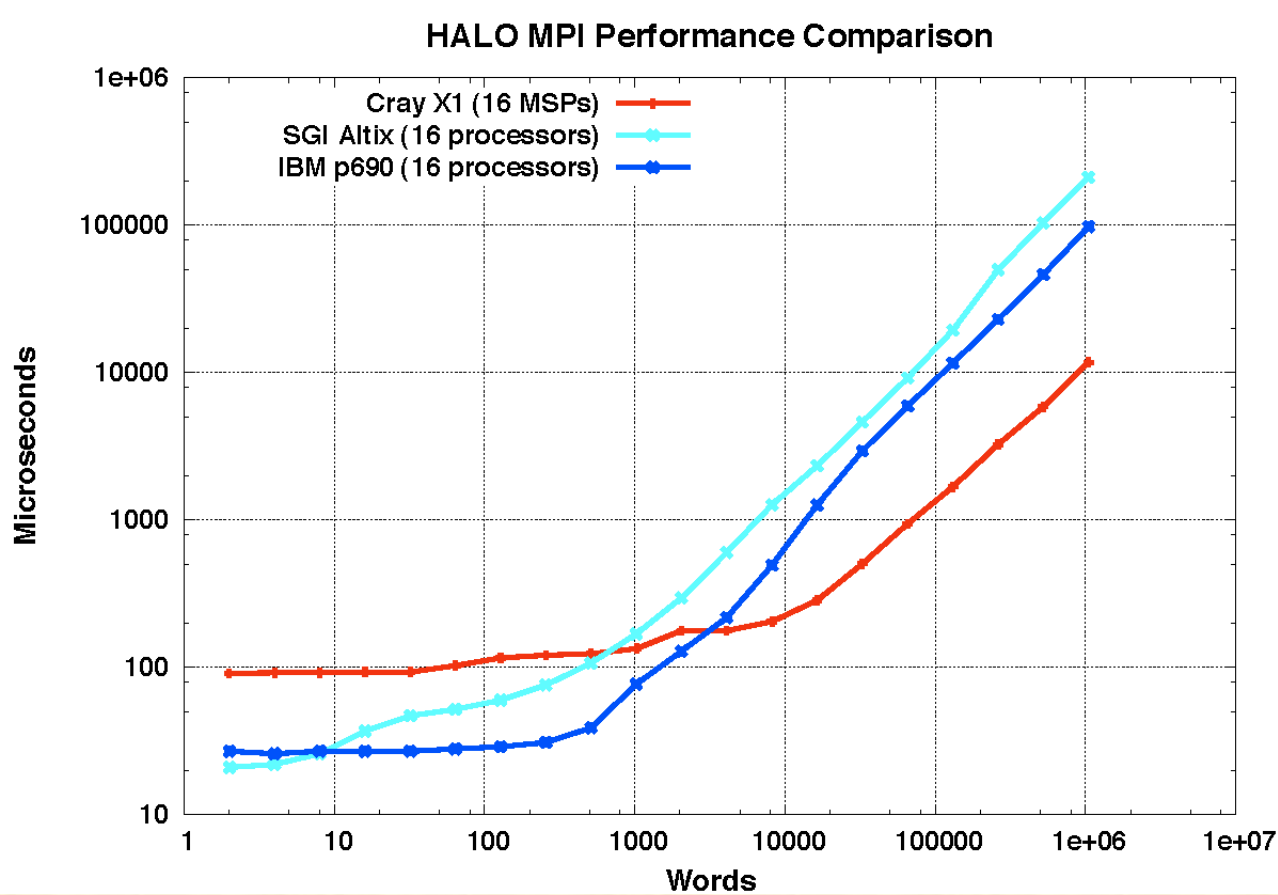
HALO Exchange Paradigm Comparison

Comparing performance of MPI, SHMEM, and Co-Array Fortran implementation of Allan Wallcraft's HALO benchmark on 16 MSPs. SHMEM and Co-Array Fortran are substantial performance enhancers for this benchmark.



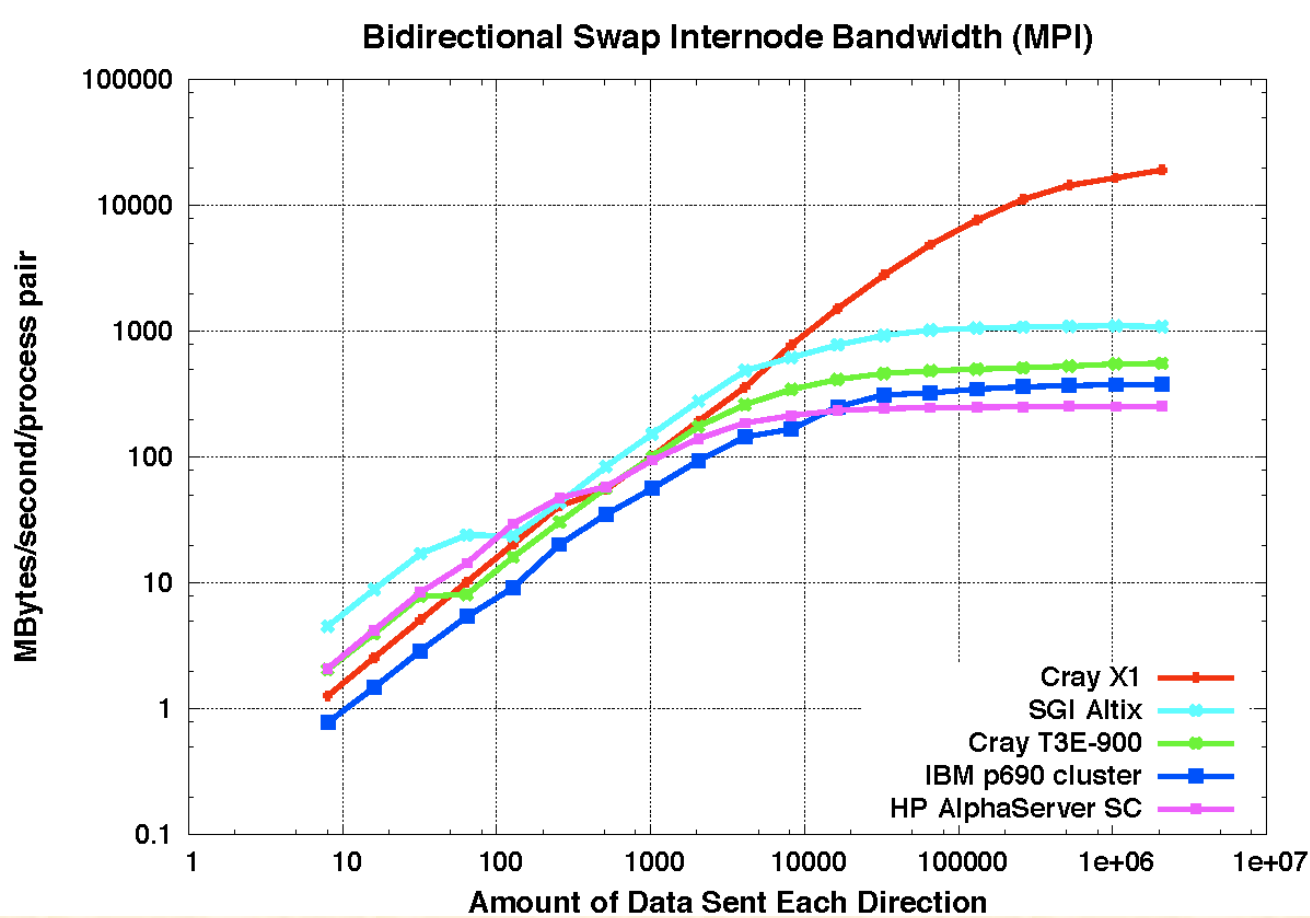
HALO Exchange Platform Comparisons

Comparing HALO performance using MPI on 16 MSPs of the Cray X1 and 16 processors of the IBM p690 (within a 32 processor SMP) and the SGI Altix (within a 128 processor SMP). Achievable bandwidth is much higher on the X1. For small halos, the p690 and Altix MPI HALO performance is between the X1 SHMEM and Co-Array Fortran HALO performance.



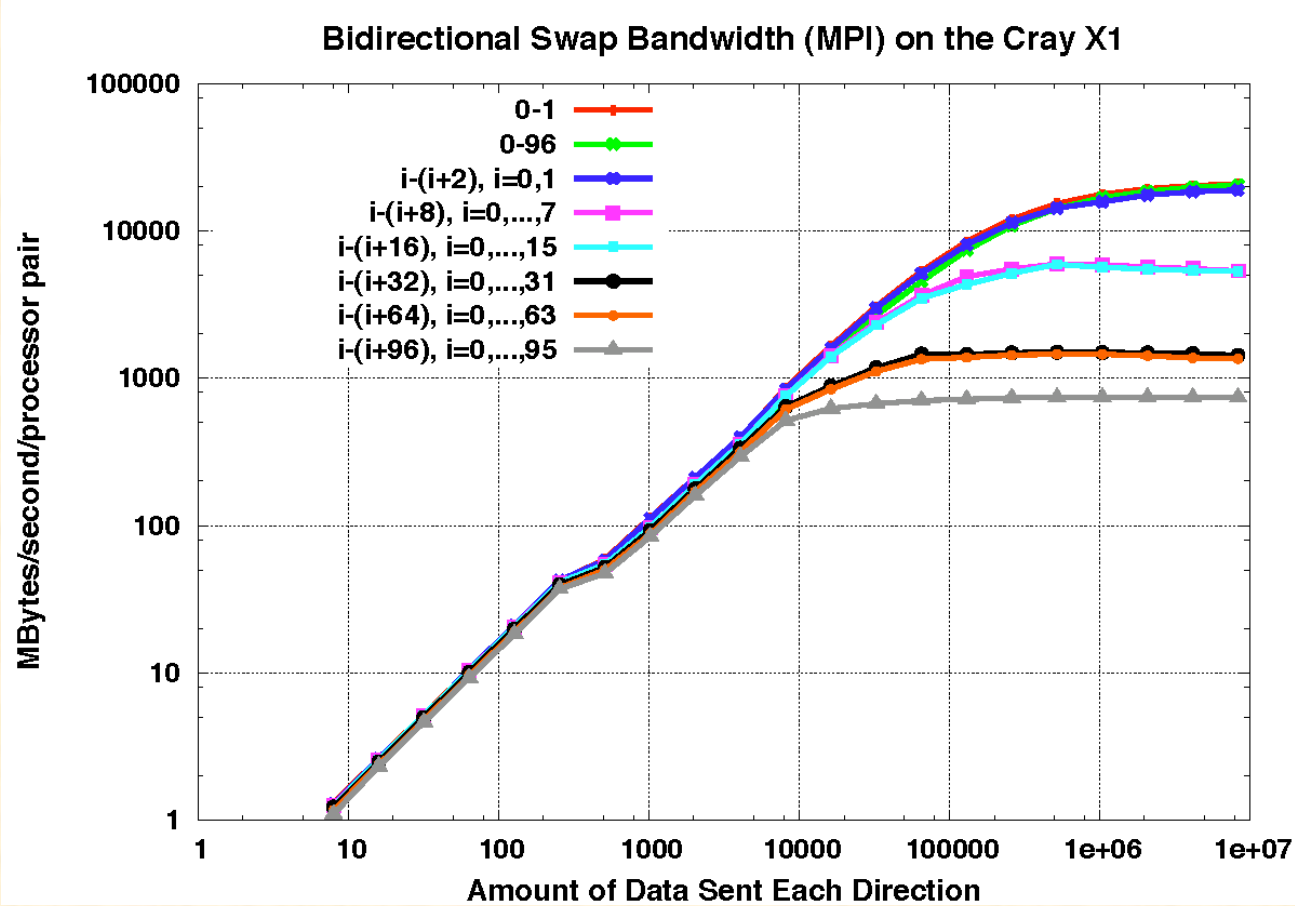
MPI SWAP Platform Comparisons

Comparing performance of SWAP for different platforms. Experiment measures bidirectional bandwidth between two processors in different SMP nodes. For the Altix we used processors 96 and 192. The significant advantage in bandwidth on the X1 is important for a number of important applications.



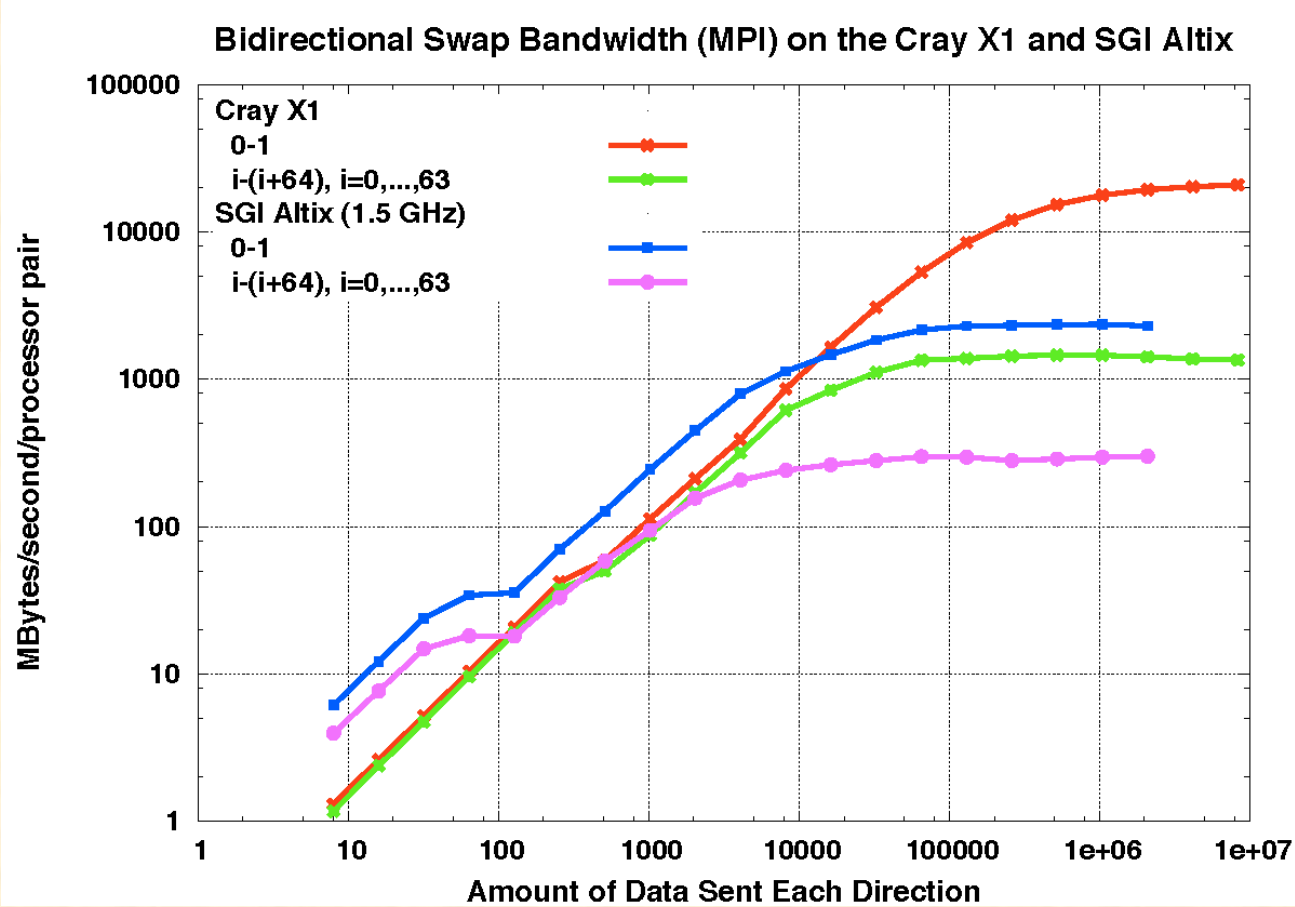
MPI SWAP Contention Evaluation

Comparing performance of SWAP for different communication patterns. Each experiment measures the per processor pair bandwidth when some number of pairs are swapping data simultaneously. For example, i-(i+8),i=0,..,7 means that processor 0 is swapping data with processor 8, processor 1 with processor 9, etc.



MPI SWAP Contention Evaluation

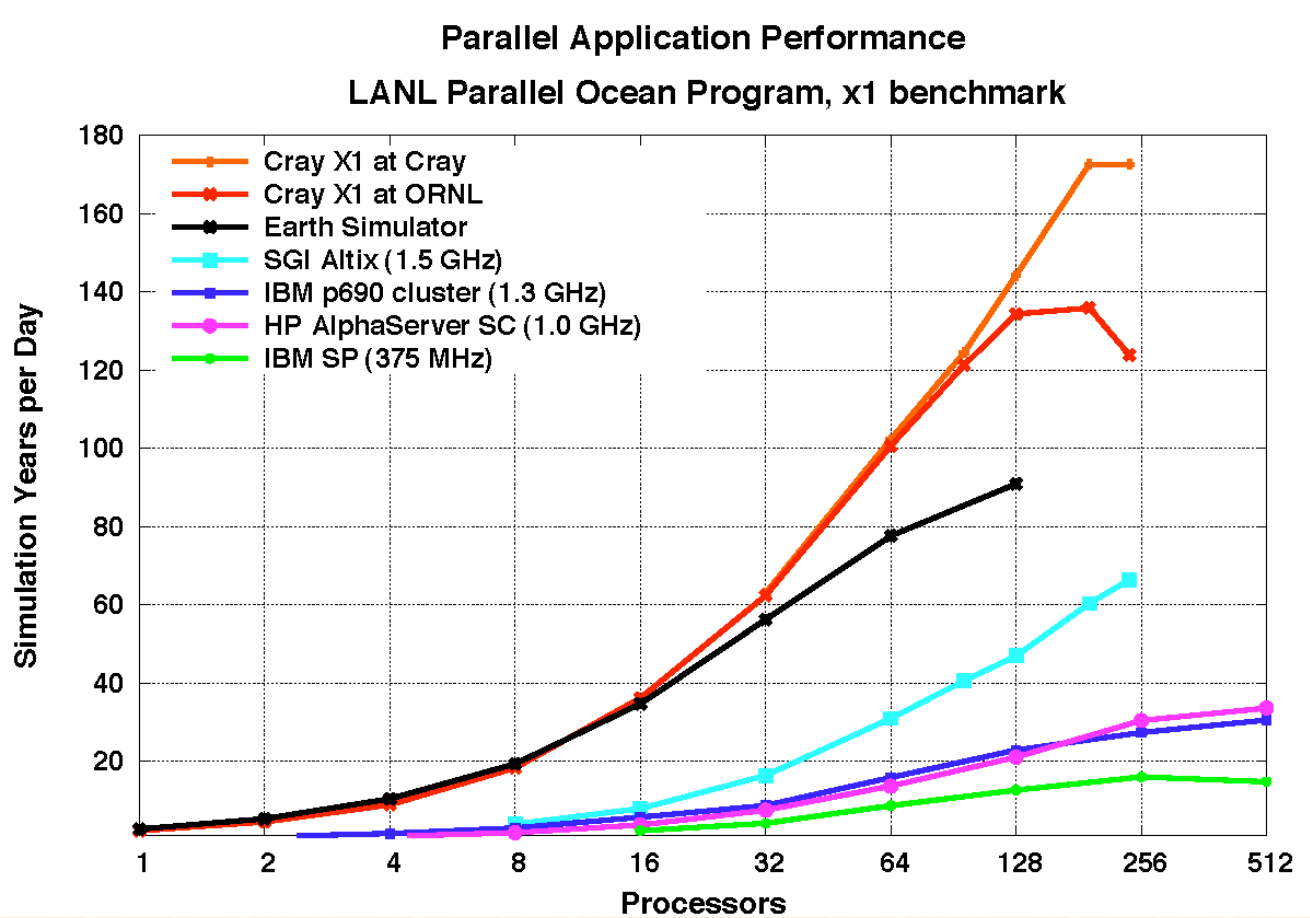
Comparing performance of SWAP for different communication patterns on the X1 and the Altix. Contention also degrades performance on the Altix, and the X1 retains its advantage.



POP Simulation Rate

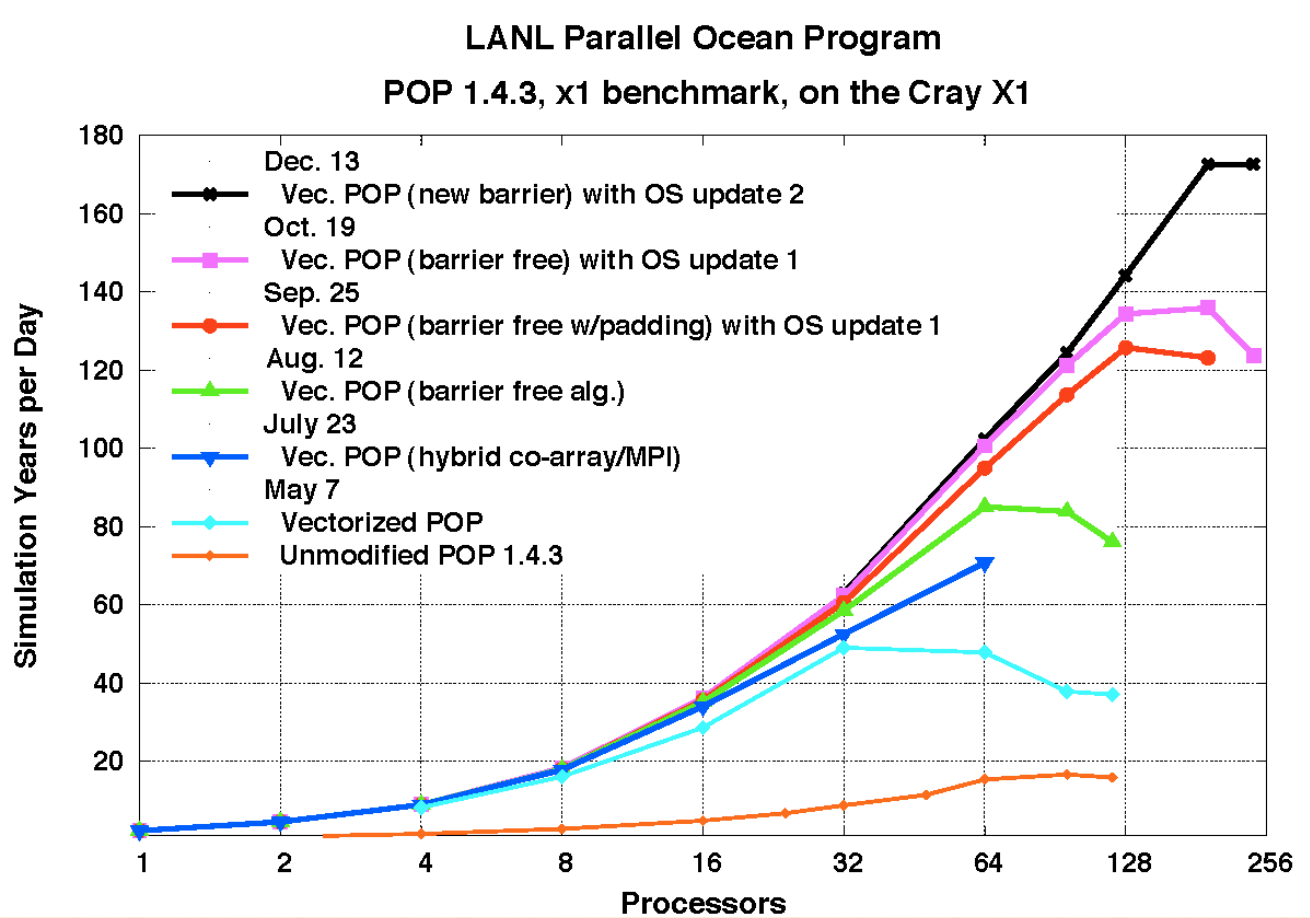
Comparing performance and scaling across platforms.

- Earth Simulator results courtesy of Dr. Y. Yoshida of the Central Research Institute of Electric Power Industry
- IBM SP results courtesy of Dr. T. Mohan of Lawrence Berkeley National Laboratory
- X1 at Cray results courtesy of J. Levesque of Cray, Inc. Cray system using beta versions of system software.



POP Performance Evolution on the X1

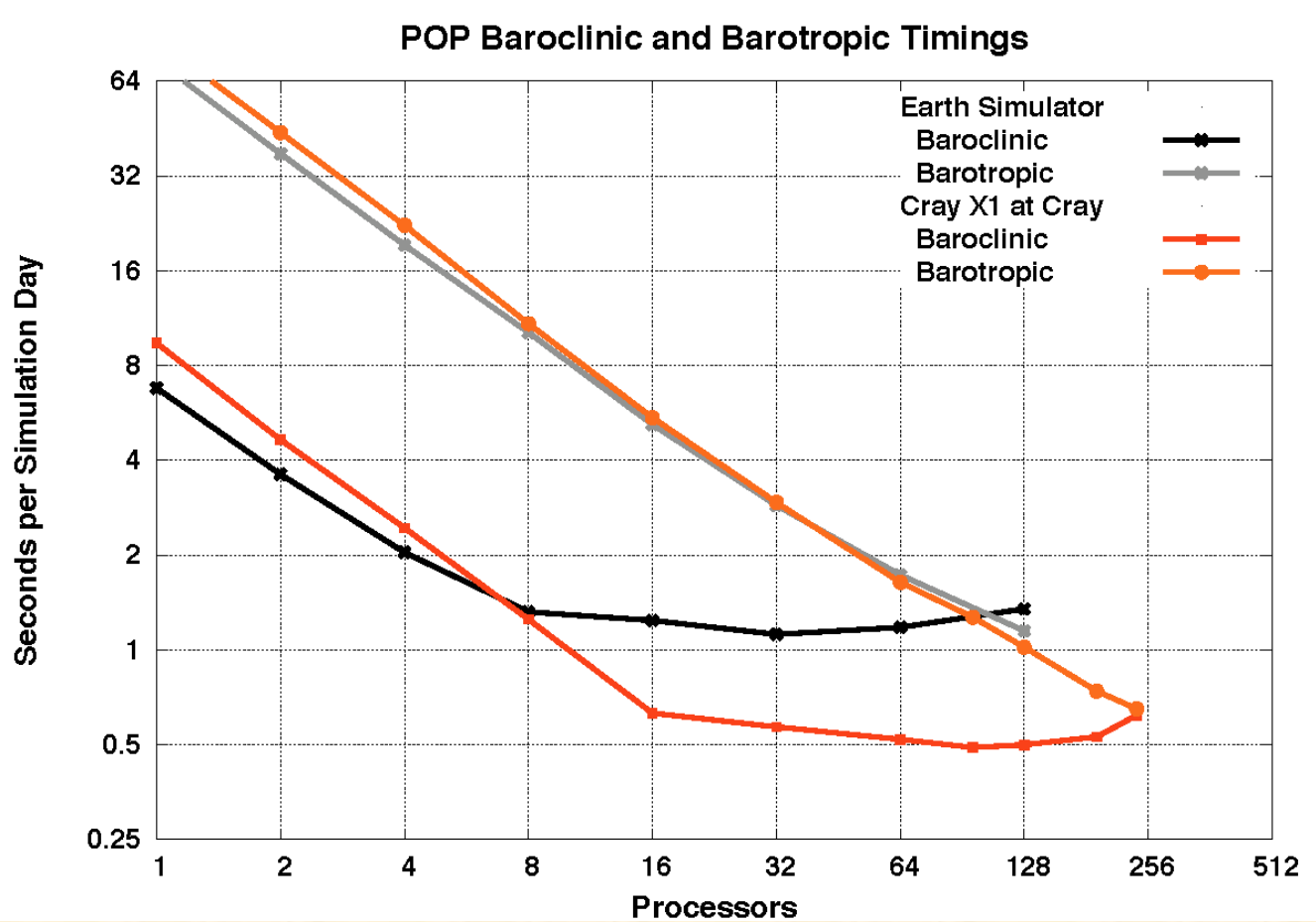
Comparing performance and scaling on the X1 over time. While the majority of recent improvements are due to performance improvements in the OS and other system software, many of these were motivated by the parallel algorithm analysis and optimizations.



POP Performance Diagnosis vs. ES40

Cray X1 at Cray
Communication-bound for more than 240 processors, with communication costs just starting to increase.

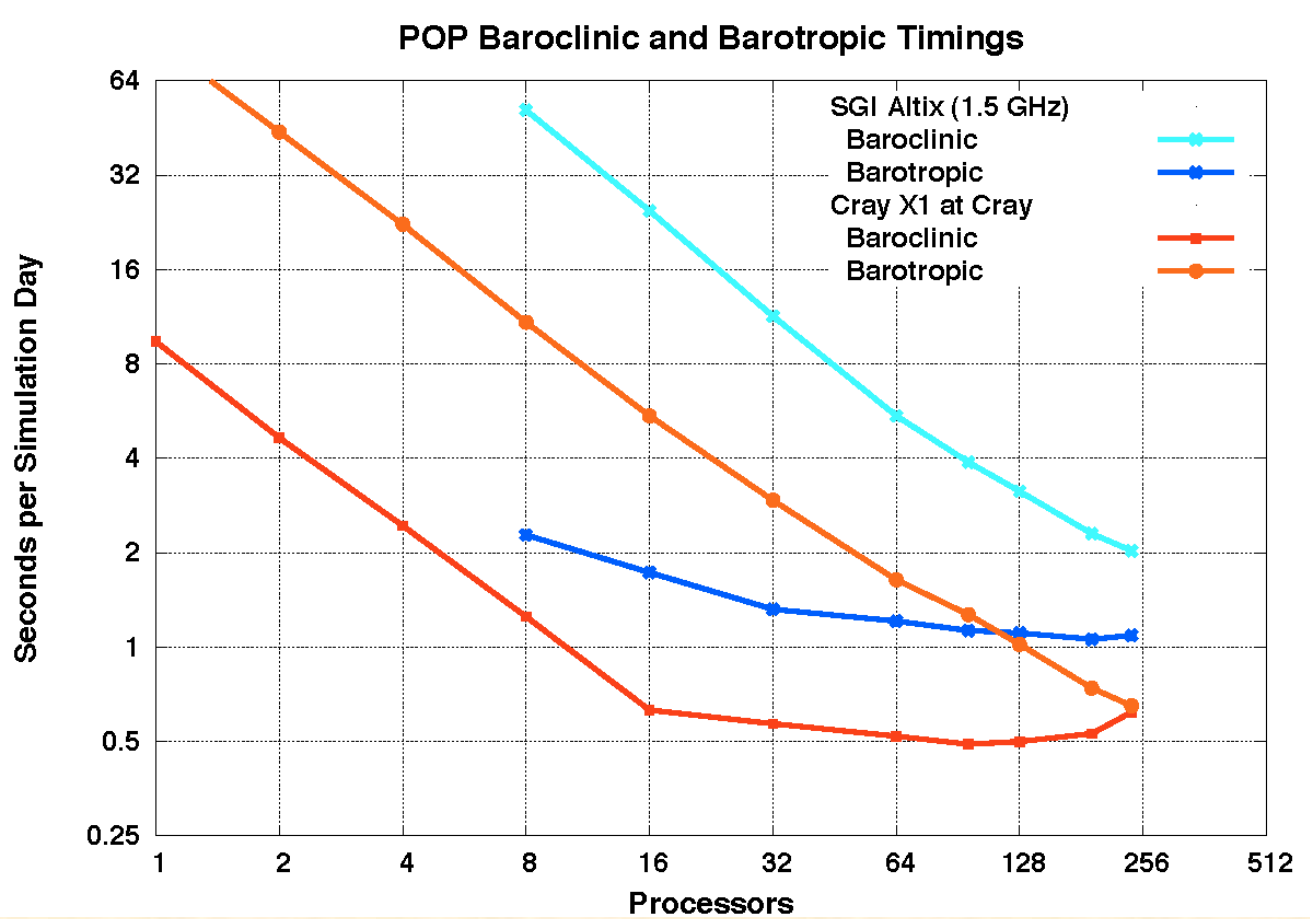
Earth Simulator
Communication-bound for 128 processors. Better vector performance for large granularity, but worse performance compared to X1 for small granularity (shorter vectors).



POP Performance Diagnosis vs. Altix

Cray X1 at Cray
- Communication-bound for more than 240 processors, with communication costs just starting to increase.

SGI Altix
- Not yet communication bound. Using MPI point-to-point and collectives for barotropic. Initial experiments with SHMEM do not show significant improvement.



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