On the nature of pairing in the two-dimensional t-J model

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BCS and high-T_c superconductors

- **BCS superconductors**
  - Condensation energy ∝ ion kinetic energy (Chester, Phys. Rev. 1956)
  - Link: pairing ↔ electron-ion interaction
  - Scattering of electrons by phonons → attractive potential
- **High-T_c cuprates**
  - Antiferromagnetic spin correlations important
  - Condensation energy ∝ change in exchange energy (Scalapino & White, PRB 1998)
  - Pairing mechanism ?

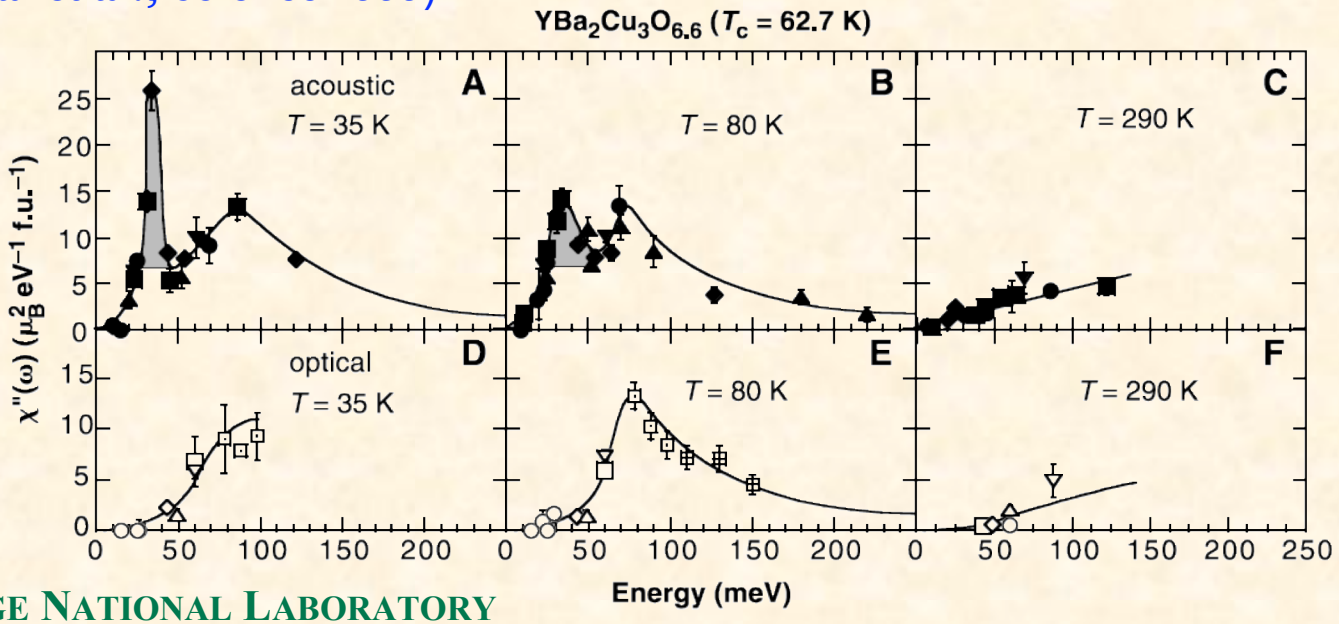
Neutron scattering: Exchange-based pairing?

- **Exchange energy ↔ Magnetic excitations**

$$J\left\langle \vec{S}_i \cdot \vec{S}_j \right\rangle = 3\int \frac{d^2q}{2\pi}\int \frac{d\omega}{\pi} S\left(\vec{q},\omega\right)\cos\left[\vec{q}\cdot\left(\vec{x}_i - \vec{x}_j\right)\right]$$

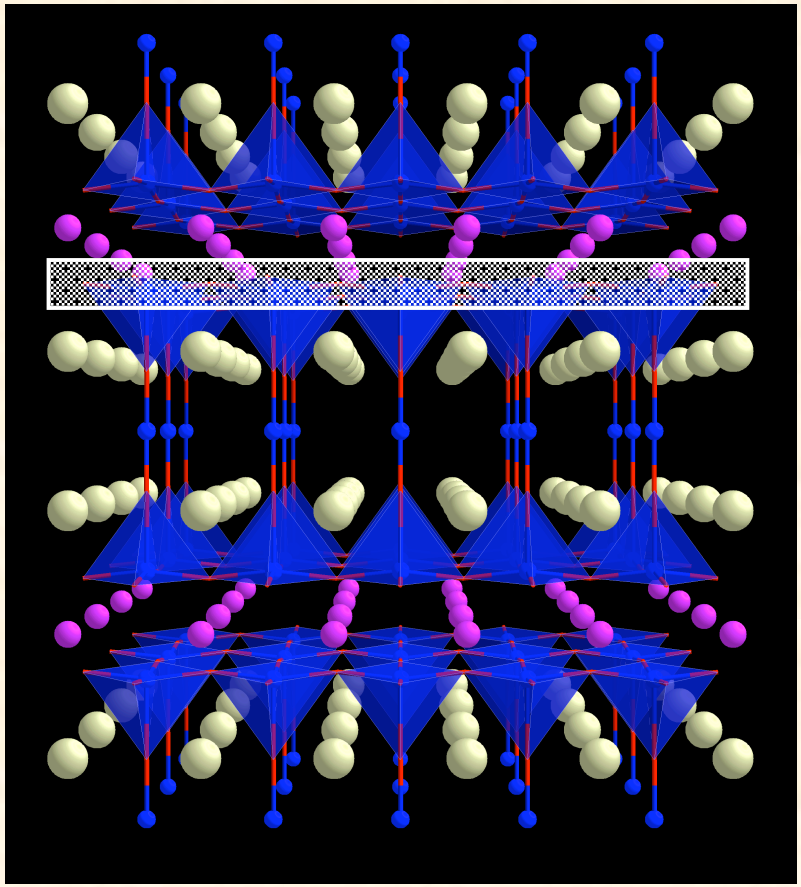
- **Neutron scattering: resonance peak**

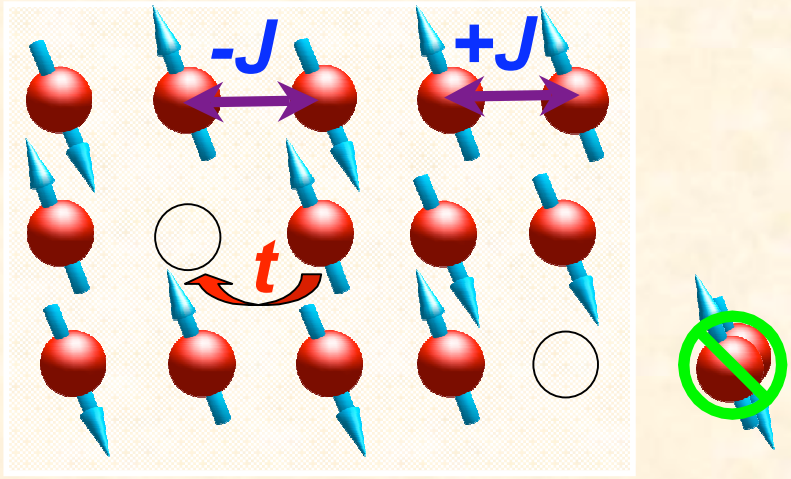
(Dai et al., Science 1999)



Two-dimensional t-J model

- **Layered structure**





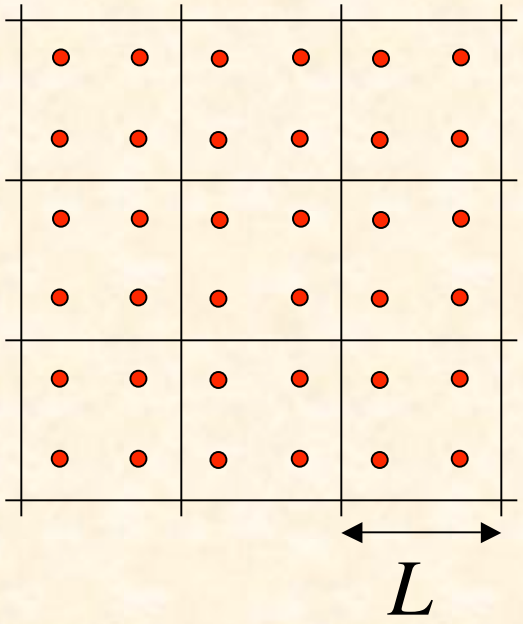
(Zhang and Rice, PRB 1988)

$$H = -t \sum_{\langle ij \rangle, \sigma} \tilde{c}^{\dagger}_{i\sigma} \tilde{c}_{j\sigma} + \frac{J}{2} \sum_{\langle ij \rangle} \left[\vec{S}_i \cdot \vec{S}_j - \frac{n_i n_j}{4} \right]$$

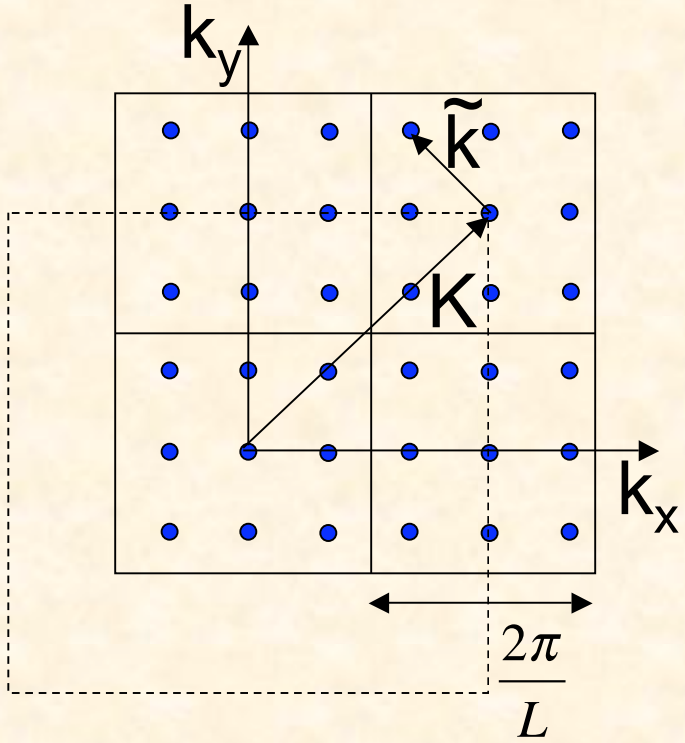
✓ **Superconducting groundstate**
(Sorella et al., PRL 2002)

DCA: Coarse-graining

- **Divide lattice into clusters of size $N_c = L \times L$**



- **1st Brillouin zone**



- **Neglect correlations between clusters →**
  Self-energy: $\Sigma\left(K+\tilde{k},\omega\right) \approx \Sigma\left(K,\omega\right)$

DCA: Locator expansion

- **Green function**

$$G\left(K+\tilde{k}\right)=g(K)+g(K)\delta t\left(K+\tilde{k}\right)G\left(K+\tilde{k}\right)$$

$$g(K,z)=\left[z-\bar{\varepsilon}_K-\Sigma(K,z)\right]^{-1}$$

$$\bar{\varepsilon}_K=\frac{N_c}{N}\sum_{\tilde{k}}t\left(K+\tilde{k}\right)$$

$$\delta t\left(K+\tilde{k}\right)=t\left(K+\tilde{k}\right)-\bar{\varepsilon}_K$$

$$\Sigma(K)=F\left[\bar{G}(K)\right];\ \bar{G}(K)=\frac{N_c}{N}\sum_{\tilde{k}}G\left(K+\tilde{k}\right)$$

- **Susceptibility**

$$\chi(Q+\tilde{q})=\chi_c(Q)-\chi_c(Q)\delta J(Q+\tilde{q})\chi(Q+\tilde{q})$$

$$\chi_c(Q,z)=\left[M(Q,z)+\bar{J}_Q\right]^{-1}$$

$$\bar{J}_Q=\frac{N_c}{N}\sum_{\tilde{q}}J(Q+\tilde{q})$$

$$\delta J(Q+\tilde{q})=J(Q+\tilde{q})-\bar{J}_Q$$

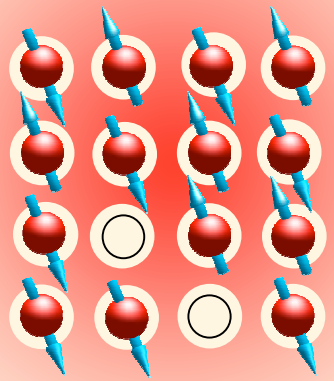
$$M(Q)=F\left[\bar{\chi}(Q)\right];\ \bar{\chi}(Q)=\frac{N_c}{N}\sum_{\tilde{q}}\chi(Q+\tilde{q})$$

DCA: effective cluster problem

- **Self-consistency**

$$\bar{G}(K,\omega) = \frac{N_c}{N}\sum_{\tilde{k}} \frac{1}{\omega - \bar{\varepsilon}_K - \Sigma(K,\omega) - \delta t\left(K+\tilde{k}\right)} = \frac{1}{\omega - \bar{\varepsilon}_K - \Sigma(K,\omega) - \Gamma(K,\omega)}$$

$$\bar{\chi}(Q,\omega) = \frac{N_c}{N}\sum_{\tilde{q}} \frac{1}{M(Q,\omega) + \bar{J}(Q) + \delta J(Q+\tilde{q})} = \frac{1}{M(Q,\omega) + \bar{J}(Q) + \Theta(Q,\omega)}$$

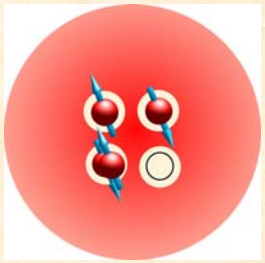


NCA: $\Sigma = F[\Gamma,\Theta]$

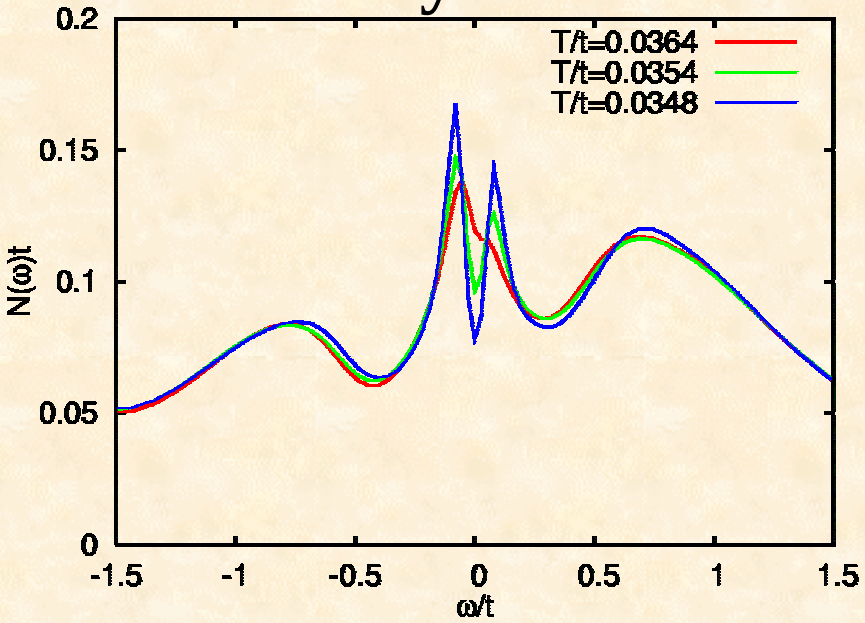
$M = F[\Gamma,\Theta]$

4-site cluster: pairing

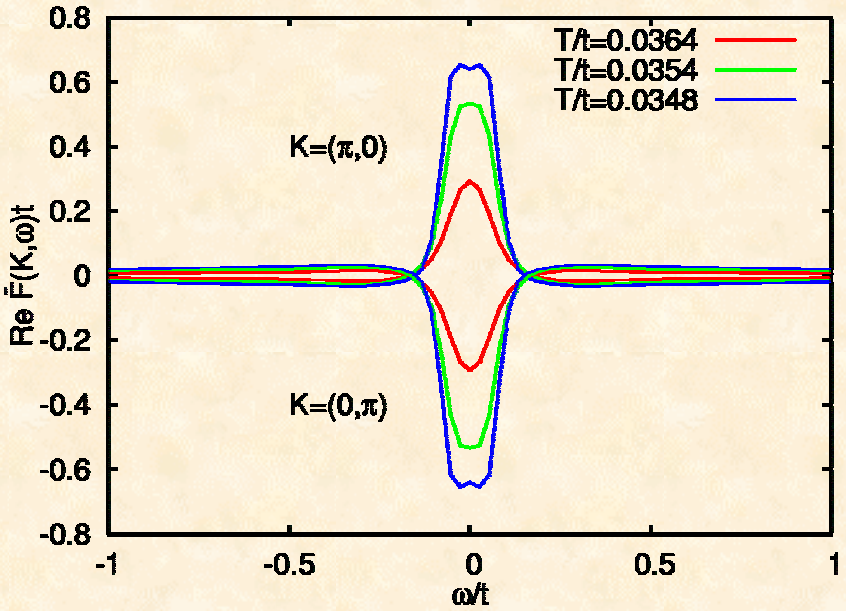
J=0.3t, 20% doping



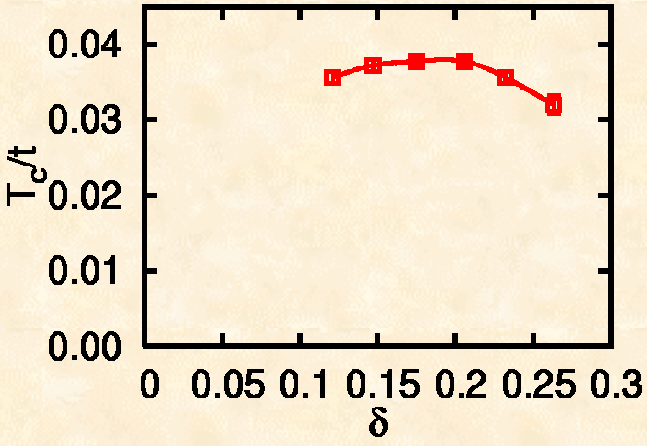
Density of states



Anomalous Green function



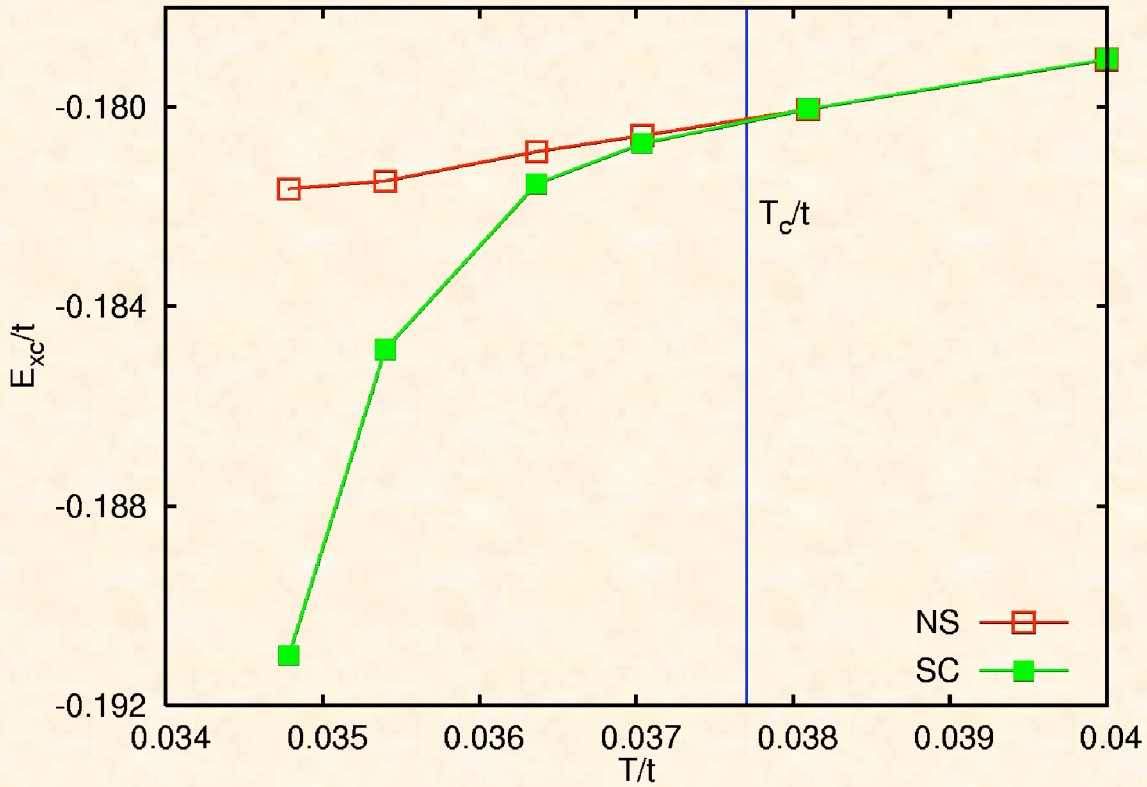
Phase-diagram



Exchange energy driven pairing

J=0.3t, 20% doping

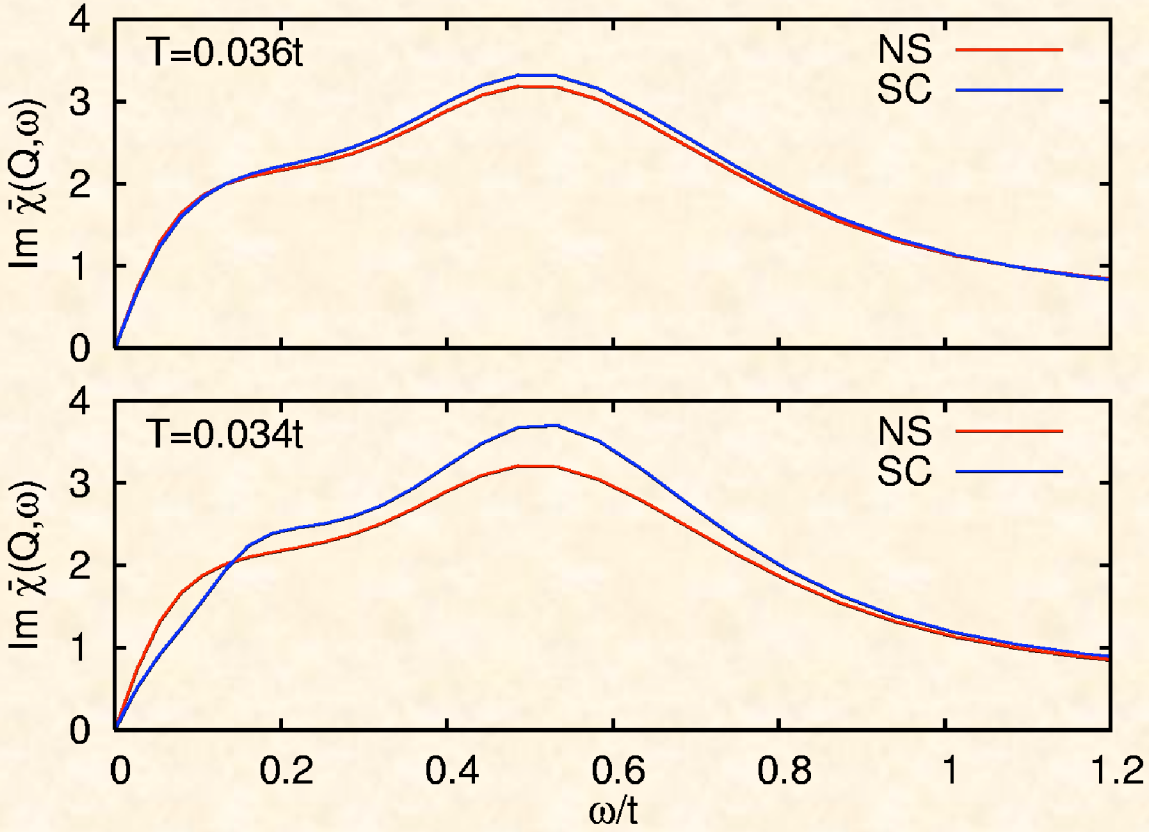
- **Exchange-energy** $E_{xc} = \frac{3}{\pi}\frac{1}{N}\sum_q J(q)\int d\omega \frac{\mathrm{Im}\,\chi(q,\omega)}{1-e^{-\beta\omega}}$



Enhanced magnetic response at Q=(π,π)

J=0.3t, 20% doping

- **Magnetic susceptibility**



Summary and Outlook

- **Short-ranged correlations → DCA**
- **d-wave superconductivity in 4-site cluster**
- **Pairing driven by exchange energy → AF spin correlations important**
- **Enhanced magnetic response at $Q=(\pi,\pi)$**

- **Superconductivity in larger clusters?**
- **LDA+DCA: Material specific simulations**