On the nature of pairing in the two-dimensional t-J model

Th.A. Maier [1,*]

Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6164

Abstract

Using an extended dynamical cluster approximation we study the nature of superconductivity in the two-dimensional t-J model. Short-ranged correlations within a finite cluster are treated explicitly, while the effects of longer-ranged physics are incorporated on the mean-field level with self-consistent Fermionic and Bosonic fields. To solve this cluster problem, we employ an non-crossing approximation which allows for transitions to the symmetry-broken state. At sufficiently low temperatures we find a stable superconducting solution with d-wave order parameter. Upon pairing, the exchange energy is lowered, consistent with an exchange-based pairing mechanism.

Key words:
t-J model, superconductivity, dynamical cluster approximation

The nature of pairing in high-temperature superconductors (HTSC) remains one of the most important unsolved problems. Early in the history of HTSC it was realized that electronic correlations play a crucial role in these systems. Hence, models describing the behavior of itinerant correlated electrons, in particular the Hubbard model and its strong-coupling limit, the t-J model,

$$H = -t \sum_{\langle ij \rangle, \sigma} \tilde{c}^\dagger_{i\sigma} \tilde{c}_{j\sigma} + \frac{J}{2} \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j) \qquad (1)$$

with $\tilde{c}_{i\sigma} = c_{i\sigma}(1 - n_{i\bar{\sigma}})$, were proposed to capture the generic physics of HTSC [1].

Evidence for superconductivity in the 2D t-J model at $T = 0$ has recently been observed in finite size numerical calculations for relatively large systems and physically relevant values of J/t [2]. Here we use an extended Dynamical Cluster Approximation (DCA) to address this problem. The motivation for this study is two-fold. First, the DCA is built for the thermodynamic limit and hence is generally complementary to finite size calculations [3]. Second, we are interested in the origin of pairing in the 2D t-J model.

In the DCA [3] the N-site lattice is divided into N/N_c clusters, each of size N_c and with a reciprocal space denoted by $\mathbf{K}$. Assuming that correlations are short-ranged, i.e. within the N_c-site cluster, the self-energy $\Sigma(\mathbf{k}, \omega)$ is approximated by the cluster self-energy $\Sigma(\mathbf{K}, \omega)$. By coarse-graining, i.e. averaging the single-particle Green function over the N/N_c wave-vectors of the superlattice, the original lattice is then mapped onto an effective periodic cluster of size N_c. The cluster is embedded in a self-consistent mean-field which is determined by equating the cluster Green function with the coarse-grained Green function. Since the model (1) contains the non-local interaction J, the spin-susceptibility is also treated self-consistently, in an analogous way to the Green-function. The mean-field hence consists of a Fermionic host accounting for the hopping t of electrons between clusters, and a Bosonic host that represents the effects of the fluctuating magnetic fields induced by J. Thus, for $N_c = 1$, the algorithm reduces to the extended dynamical mean field theory [4], while for larger N_c non-local correlations

* Corresponding Author: Phone: (865) 576-3597, Fax: (865) 241-0381, Email: maierta@ornl.gov

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within the cluster are incorporated. To solve this effective cluster problem, we employ an extended non-crossing approximation [5] extended to include the effects of the Bosonic field (see [6] for $N_c = 1$) and to allow for a transition to the superconducting state.

We performed simulations for the cluster size $N_c = 4$, the smallest possible that can capture d-wave pairing. We use t for the energy scale and set $J = 0.3t$. As evidenced by the finite anomalous coarse-grained Green function $\bar{F}(\mathbf{K}, \omega) = \langle\langle \tilde{c}_{\mathbf{K}\uparrow}; \tilde{c}_{-\mathbf{K}\downarrow} \rangle\rangle$, a typical result for which is shown in Fig. 1A, we do find superconductivity in the 2D t-J model in the thermodynamic limit. The $\mathbf{K}$-dependence of $\bar{F}$ is consistent with a $d_{x^2-y^2}$-symmetry of the order parameter. As shown in Fig. 1B, the critical temperature T_c has a maximum $T_c^{max} \approx 0.0377t \approx 110$K (for $t = 0.25$eV) at the optimal doping $\delta \approx 0.20$. Due to the breakdown of the NCA technique at very low temperatures, the phase-diagram cannot be extended beyond the region shown.

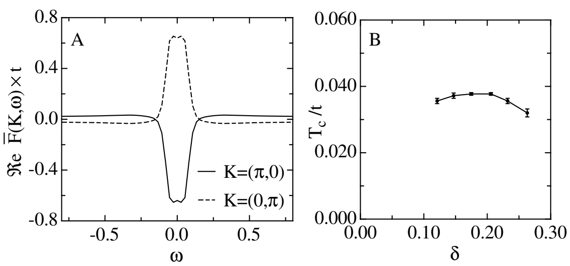

Fig. 1. (A) Real part of the coarse-grained anomalous Green function at 20% doping and $T = 0.0348t < T_c$. (B) Doping dependence of the critical temperature T_c.

To explore the nature of pairing in our simulations, we plot in Fig. 2 the exchange energy E_{xc} obtained from the spin-susceptibility $\chi(\mathbf{q}, \omega)$ via the fluctuation-dissipation theorem,

$$E_{xc} = \frac{3}{\pi} \frac{1}{N} \sum_{\mathbf{q}} J(\mathbf{q}) \int d\omega \frac{\Im m \chi(\mathbf{q}, \omega)}{1 - e^{-\beta\omega}} , \tag{2}$$

where $J(\mathbf{q})$ is the Fourier-transform of the exchange interaction and $\beta = 1/T$. Consistent with the exchange-based pairing mechanism proposed by Scalapino and White [7], we find that the exchange energy in the superconducting state is reduced compared to the normal state, while the energy coming from the kinetic-like (first) term in the t-J model, Eq. (1), slightly increases upon pairing (not shown). Since the exchange interaction J has mixed kinetic and Coulombic origins, it is important to note that these BCS-like results are not inconsistent with our previous results for the Hubbard model where pairing is accompanied by a lowering of the electronic kinetic energy [8].

According to Eq. (2), the gain in exchange energy may be related to an increase in the spectral weight of

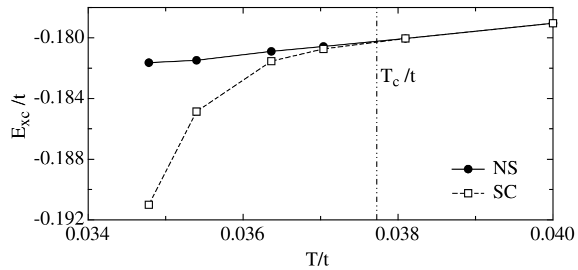

Fig. 2. Exchange energy as a function of temperature at 20% doping across the superconducting transition in the normal (NS) and superconducting (SC) states.

the spin-susceptibility near $\mathbf{Q} = (\pi, \pi)$ since $J(\mathbf{q}) < 0$ for $\mathbf{q}$ near $\mathbf{Q}$. Although our data for the coarse-grained

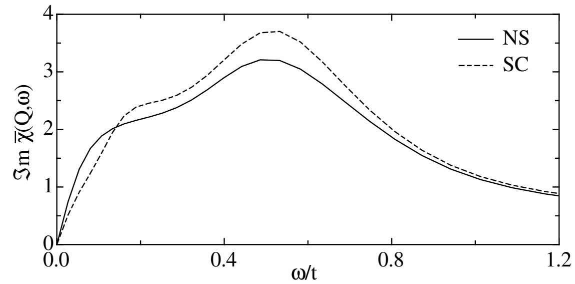

Fig. 3. Imaginary part of the coarse-grained dynamic spin-susceptibility in for $\mathbf{Q} = (\pi, \pi)$ at T=0.034t and 20% doping in the superconducting (SC) and normal (NS) states.

spin-susceptibility in Fig. 3 does not display the narrow ~ 40 meV resonance peak observed in experiments, we find that the reduction in exchange energy indeed originates in an increase of spectral weight in the region near $\mathbf{Q} = (\pi, \pi)$ consistent with experiments [9].

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