Adaptive Regulation of Amplitude Limited Robot Manipulators with Uncertain Kinematics and Dynamics*

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Abstract

Common assumptions in most of the previous robot controllers are that the robot kinematics and manipulator Jacobian are perfectly known and that the robot actuators are able to generate the necessary level of torque inputs. In this paper, an amplitude-limited torque input controller is developed for revolute robot manipulators with uncertainty in the kinematic and dynamic models. The adaptive controller yields semi-global asymptotic regulation of the task-space setpoint error. The advantages of the proposed controller include the ability to actively compensate for unknown parametric effects in the dynamic and kinematic model and the ability to ensure actuator constraints are not breached by calculating the maximum required torque a priori.

1 Introduction

For a robotic system to interact with and execute tasks in the workspace, a transformation between objects located in the workspace and the robot is typically required. Since the robot is controlled through inputs to the link actuators, the robot kinematics and manipulator Jacobian are used to relate a workspace coordinate system to coordinate systems attached to each actuator. A common assumption in most of the previous robot controllers is that the robot kinematics and manipulator Jacobian are perfectly known and that the robot actuators are able to generate the necessary level of torque inputs. The assumption that the robot kinematics and manipulator Jacobian are perfectly known limits robustness because measurement errors may lead to degraded performance or unpredictable responses. Moreover, this assumption limits the applicability of the controller since many manipulators may have the same kinematic structure but different kinematic parameters. The assumption that the robot actuators are able to generate the necessary level of torque is also limiting since robotic actuators have physical constraints. If the controller commands more torque than the actuators can supply, degraded or unpredictable motion control and thermal or mechanical failure may result.

Based on the need for controllers that take actuator constraints into account, several researchers have proposed amplitude limited controllers (e.g., see [7]-[9], [11], [13], [14], [16], [19], and the references within). Specifically, Santibáñez and Kelly [16], proposed a global asymptotic regulating controller that is composed of a saturated proportional derivative (PD) feedback loop plus an exact model knowledge feedforward gravity compensation term. In [11], the same authors generalized a class of regulators for the control problem given in [16]. Motivated by the research given in [11] and [16], Loria et al. [13] designed an output feedback (OFB) global asymptotic regulating controller; however, exact knowledge of the gravity terms was still required. To provide for robustness, Colbaugh et al. [7], [8] designed full-state feedback (FSFB) and OFB global asymptotic regulating controllers that compensate for uncertainty; however, the control strategy switches between one controller that is used to drive the setpoint error to a small value, and another controller that is used to drive the setpoint error to zero. In [19], a semi-global FSFB adaptive controller was developed that includes an amplitude-limited PD feedback loop plus a feedforward term that adapts for gravity and static friction effects. In [14], Loria et al. designed an exact model knowledge OFB semi-global tracking controller. In [9], Dixon et al. proposed an adaptive FSFB semi-global tracking controller. Each of the previous amplitude limited controllers target a joint-space control objective; hence, to achieve an objective defined in the task-space, the aforementioned controllers would require perfect knowledge of the manipulator forward kinematics and Jacobian.

From a review of literature, it seems few controllers have been developed that target uncertainty in the manipulator forward kinematics and Jacobian. For example in [4]-[6], Cheah et al. developed several approximate Jacobian feedback controllers that exploit a static, best-guess estimate of the manipulator Jacobian to achieve task-space regulation objectives despite parametric uncertainty in the manipulator Jacobian. As reported in

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[3], a drawback of the controllers in [4]-[6] is that the task-space velocity of the robot end-effector is required to be measureable, and the controller in [5] requires the computation of an estimate for the Jacobian inverse. In [3], Cheah et al. resolve these issues by developing a PD controller that exploits a static, best-guess estimate of the manipulator Jacobian to achieve a task-space regulation result.

In this paper, an amplitude-limited torque input controller is developed for revolute robot manipulators with uncertainty in the kinematic and dynamic models. The adaptive controller yields semi-global asymptotic regulation of the task-space setpoint error. As in the recent result in [3], the controller does not require the task-space velocity of the robot end-effector to be measurable, does not require the inverse of the estimated Jacobian to be computed, and does not require exact model knowledge of the robot dynamics. The current result actively compensates for uncertainty in the gravity and static friction effects. In contrast to the use of a static, best-guess estimate as in [3], the controller in this paper also exploits a feedforward control term that actively compensates for the parametric uncertainty in the Jacobian. The advantages of the proposed controller include the ability to actively compensate for unknown parametric effects in the dynamic and kinematic model and the ability to ensure actuator constraints are not breached by calculating the maximum required torque a priori.

2 Robot Model

The dynamic model for a rigid n-link, serially connected, direct-drive revolute robot is given as follows [12]

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_s sgn(\dot{q}) = \tau. \tag{1}$$

In (1), $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively, $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ represents gravity effects, $F_s \in \mathbb{R}^{n\times n}$ denotes the constant diagonal static friction coefficient matrix, $sgn(\cdot) \in \mathbb{R}^n$ denotes the vector signum function, and $\tau(t) \in \mathbb{R}^n$ represents the torque input vector. Let $x(t) \in \mathbb{R}^m$ $(m \leq n)$ represent a task-space vector that is related to the robot joint-space as follows

$$x = h(q) \qquad \dot{x} = J(q)\dot{q}(t) \tag{2}$$

where $h(q) \in \mathbb{R}^m$ denotes the differentiable forward kinematics of the manipulator, and $J(q) \triangleq \dfrac{\partial h}{\partial q} \in \mathbb{R}^{m\times n}$ denotes the differentiable manipulator Jacobian.

The dynamic model introduced in (1) has the following properties that are used in the subsequent control development and analysis.

Property 1: The positive definite and symmetric inertia matrix, satisfies the following inequalities

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq m_2 \|\xi\|^2, \qquad \forall \xi \in \mathbb{R}^n \tag{3}$$

where $m_1, m_2 \in \mathbb{R}$ are known positive bounding constants, and $\|\cdot\|$ is the standard Euclidean norm.

Property 2: The time derivative of the inertia matrix and the centripetal-Coriolis matrix satisfy the following skew symmetric relationship

$$\xi^T \left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \qquad \forall \xi \in \mathbb{R}^n. \tag{4}$$

Property 3: The unknown gravitational and static friction terms can be linearly parameterized as follows

$$Y(q,\dot{q})\phi \triangleq G(q) + F_s sgn(\dot{q}) \tag{5}$$

where $\phi \in \mathbb{R}^p$ contains unknown constant parameters, and the regression matrix $Y(q,\dot{q}) \in \mathbb{R}^{n\times p}$ contains measurable functions of the link position and link velocity. Lower and upper bounds denoted by $\underline{\phi}, \overline{\phi} \in \mathbb{R}^p$, respectively, are assumed to be known for each parameter in ϕ as follows

$$\underline{\phi_i} \leq \phi_i \leq \overline{\phi_i} \qquad \forall i = 1, 2, ...p \tag{6}$$

where $\underline{\phi_i}, \overline{\phi_i} \in \mathbb{R}$ denote the i-th component of $\underline{\phi}$ and $\overline{\phi}$, respectively, and $\phi_i \in \mathbb{R}$ denotes the i-th component of ϕ.

Property 4: The time derivative of the inertia matrix, the centripetal-Coriolis matrix, the gravity vector, and the static friction matrix can be upper bounded in the following manner

$$\left\|\dot{M}(q)\right\|_{i\infty} \leq \zeta_m \|\dot{q}\| \quad \|V_m(q,\dot{q})\|_{i\infty} \leq \zeta_c \|\dot{q}\|, \tag{7}$$
$$\|G(q)\| \leq \zeta_g, \quad \|F_s\| \leq \zeta_f.$$

where $\zeta_g, \zeta_f, \zeta_c, \zeta_m \in \mathbb{R}$ are known positive bounding constants, and $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a matrix.

Remark 1 *Since the controller in this paper is developed for revolute robots, the terms $M(q)$, $C(q,\dot{q})$, $G(q)$, and $J(q)$ are bounded for all possible $q(t)$. That is, these terms only depend on $q(t)$ as arguments of bounded trigonometric functions, and*

$$\|J(q)\|_{i\infty} < \delta_1 \tag{8}$$

where $\delta_1 \in \mathbb{R}$ is a known positive constant.

3 Control Development

3.1 Control Objective

As described previously, many robotic tasks are naturally defined in terms of the task-space. Since the robot controller is defined in the joint-space (i.e., as a motor current or a joint torque) the manipulator Jacobian is typically required to relate the task-space to the joint-space.

The objective for the controller described in this paper is to regulate the end-effector of a robot manipulator to a desired task-space setpoint using an amplitude limited torque input despite uncertainty in the manipulator Jacobian and the dynamic model. To quantify this objective, a task-space setpoint error denoted by $e(t) \in \mathbb{R}^m$ is defined as follows

$$e \triangleq x - x_d \tag{9}$$

where $x(t)$ was introduced in (2), and $x_d \in \mathbb{R}^m$ denotes the known, constant desired setpoint. As in [3], the subsequent development is based on the assumption that $x(t)$, $q(t)$, and $\dot{q}(t)$ are measurable. Specifically, $q(t)$ and $\dot{q}(t)$ can be obtained from encoder/tachometer sensors, and as in [3], $x(t)$ could be obtained from a camera system.

To facilitate the subsequent amplitude limited control design, a vector function $Tanh(e) \in \mathbb{R}^m$ and a matrix function $Cosh(e) \in \mathbb{R}^{m\times m}$ are defined as follows

$$Tanh(e) \triangleq [\tanh(e_1), \tanh(e_2), \ldots, \tanh(e_m)]^T \tag{10}$$

$$Cosh(e) \triangleq diag\{\cosh(e_1), \cosh(e_2), \ldots, \cosh(e_m)\} \tag{11}$$

where $diag\{\cdot\}$ represents the standard diagonal matrix whose off-diagonal elements are zero. Based on the constraint that the manipulator Jacobian is uncertain, a linear parameterization denoted by $Y_J(q,e)\phi_J$ is defined as follows

$$Y_J(q,e)\phi_J \triangleq J^T(q)Tanh(e) \tag{12}$$

where $Y_J(q,e) \in \mathbb{R}^{n\times p_2}$ contains measurable functions of the link position and task-space setpoint error, and $\phi_J \in \mathbb{R}^{p_2}$ contains the unknown constant parameters contained in the Jacobian matrix. To facilitate the subsequent analysis, an estimate for (12) is developed as follows

$$Y_J(q,e)\hat{\phi}_J \triangleq \hat{J}^T(q)Tanh(e) \tag{13}$$

where $\hat{\phi}_J(t) \in \mathbb{R}^{p_2}$ denotes a subsequently designed parameter estimate. Lower and upper bounds denoted by $\underline{\phi_J}, \overline{\phi_J} \in \mathbb{R}^{p_2}$, respectively, are assumed to be known for each parameter in ϕ_J as follows

$$\underline{\phi_{Jk}} \leq \phi_{Jk} \leq \overline{\phi_{Jk}} \qquad \forall k = 1, 2, \ldots p_2 \tag{14}$$

where $\underline{\phi_{Jk}}$, $\overline{\phi_{Jk}} \in \mathbb{R}$ denote the k-th component of $\underline{\phi_J}$ and $\overline{\phi_J}$, respectively, and $\phi_{Jk} \in \mathbb{R}$ denotes the k-th component of ϕ_J. To facilitate the subsequent design and analysis, the parameter estimation error signals $\tilde{\phi}(t) \in \mathbb{R}^p$ and $\tilde{\phi}_J(t) \in \mathbb{R}^{p_2}$ are defined as follows

$$\tilde{\phi} \triangleq \phi - \hat{\phi} \qquad \tilde{\phi}_J \triangleq \phi_J - \hat{\phi}_J \tag{15}$$

where $\hat{\phi}(t) \in \mathbb{R}^p$ denotes a subsequently designed parameter estimate.

Remark 2 *Based on (8) the following inequality can be developed*

$$\left\|\hat{J}(q)\right\|_{i\infty} < \delta_2. \tag{16}$$

where $\delta_2 \in \mathbb{R}$ denotes a known positive constant (i.e.,$\delta_2 > 0$).

Remark 3 *The following inequalities can be shown to hold for all $e(t) \in \mathbb{R}^m$ and $\dot{q}(t) \in \mathbb{R}^n$ [9]*

$$2\sum_{i=1}^{m} \ln(\cosh(e_i)) \geq \|Tanh(e)\|^2 \geq \tanh^2(\|e\|), \tag{17}$$

$$\|\dot{q}\| + 1 \geq \frac{\|\dot{q}\|}{\tanh(\|\dot{q}\|)}, \tag{18}$$

$$\|Tanh(e)\| \, \|Tanh(\dot{q})\| \leq \|Tanh(e)\|^2 + \|Tanh(\dot{q})\|^2, \tag{19}$$

$$\|Tanh(e)\| \, \|\dot{q}\| \leq \|Tanh(e)\|^2 + \|\dot{q}\|^2, \tag{20}$$

$$\|e\| \geq \|Tanh(e)\|, \tag{21}$$

where $Tanh(\dot{q}) \in \mathbb{R}^n$ is defined as in (10), and $\ln(\cdot)$ denotes the natural logarithm.

3.2 Closed-Loop Error System

Based on the control objective and the subsequent stability analysis, the following adaptive controller is developed

$$\tau = Y(q,\dot{q})\hat{\phi} - k_p Y_J(q,e)\hat{\phi}_J - k_v Tanh(\dot{q}) \tag{22}$$

where $k_v,\ k_p \in \mathbb{R}$ denote constant control gains. Based on the subsequent stability analysis, the parameter estimates $\hat{\phi}(t)$ and $\hat{\phi}_J(t)$ are generated from the following adaptation laws

$$\dot{\hat{\phi}}_i = proj\{\Omega_{oi}\} \qquad \dot{\hat{\phi}}_{Jk}(t) = proj\{\Omega_{1k}\} \tag{23}$$

where $\Omega_{oi}(q,\dot{q},e)$ and $\Omega_{1k}(q,\dot{q},e)$ denote the i-th and k-th component of $\Omega_o(q,\dot{q},e)$ and $\Omega_1(q,\dot{q},e)$, respectively, $\forall i = 1, 2, \ldots p$ and $\forall k = 1, 2, \ldots p_2$, where the auxiliary terms $\Omega_o(q,\dot{q},e) \in \mathbb{R}^p$ and $\Omega_1(q,\dot{q},e) \in \mathbb{R}^{p_2}$ are defined as

$$\begin{aligned}\Omega_o(q,\dot{q},e) \quad &\triangleq \quad -\Gamma_0 Y^T(q,\dot{q})\left(\dot{q} + \varepsilon Y_J(q,e)\hat{\phi}_J\right) \quad (24)\\ \Omega_1(q,\dot{q},e) \quad &\triangleq \quad -k_p\Gamma_1 Y_J^T(q,e)\dot{q}.\end{aligned}$$

For the adaptation laws given in (23) and (24), $\Gamma_0 \in \mathbb{R}^{p\times p}$ and $\Gamma_1 \in \mathbb{R}^{p_2\times p_2}$ denote constant, diagonal positive definite adaptation gain matrices, $\varepsilon \in \mathbb{R}$ is a positive, constant adaptation weighting gain, and the function $proj\{\cdot\}$ is defined as follows

$$proj\{\Omega_{oi}\} \triangleq \begin{cases} \Omega_{oi} & if \quad \hat{\phi}_i > \underline{\phi_i} \\ \Omega_{oi} & if \quad \hat{\phi}_i = \underline{\phi_i} \quad \text{and} \quad \Omega_{oi} \geq 0 \\ 0 & if \quad \hat{\phi}_i = \underline{\phi_i} \quad \text{and} \quad \Omega_{oi} < 0 \\ 0 & if \quad \hat{\phi}_i = \overline{\phi_i} \quad \text{and} \quad \Omega_{oi} > 0 \\ \Omega_{oi} & if \quad \hat{\phi}_i = \overline{\phi_i} \quad \text{and} \quad \Omega_{oi} \leq 0 \\ \Omega_{oi} & if \quad \hat{\phi}_i < \overline{\phi_i} \end{cases}$$

$$\underline{\phi_i} \leq \hat{\phi}_i(0) \leq \overline{\phi_i} \tag{25}$$

where $\hat{\phi}_i(t)$ denotes the i-th component of $\hat{\phi}(t)$. The $proj\{\Omega_{1k}(q,\dot{q},e)\}$ is defined in the same manner as in

(25) with regard to $\hat{\phi}_{Jk}(t)$. The above projection algorithm ensures that the following inequalities are satisfied (for further details see [1] and [15])

$$\underline{\phi_i} \leq \hat{\phi}_i(t) \leq \overline{\phi_i} \qquad \underline{\phi_{Jk}} \leq \hat{\phi}_{Jk}(t) \leq \overline{\phi_{Jk}}. \tag{26}$$

Based on (23)-(25), the following inequality can also be shown to hold

$$\left\| \dot{\hat{\phi}}_J \right\| \leq \|\Omega_1(q, \dot{q}, e)\| \leq k_p \lambda_{\max}\{\Gamma_1\} \|Y_J(q, e)\|_{i\infty} \|\dot{q}\| \tag{27}$$

where $\lambda_{\max}\{\cdot\}$ denotes the maximum eigenvalue of a matrix. After substituting (22) into (1), the following closed-loop error system can be determined

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} = -Y(q, \dot{q})\tilde{\phi} - k_p Y_J(q, e)\hat{\phi}_J - k_v Tanh(\dot{q}) \tag{28}$$

where (5) and (15) have been utilized.

- **Assumption:** Based on (13) and (23)-(25), the following inequalities are assumed to be valid [3]

$$\delta_3 \|\xi\|^2 < \xi^T \hat{J}^T(q)\hat{J}(q)\xi \quad \forall \xi \in \mathbb{R}^n \tag{29}$$

  where $\delta_3 \in \mathbb{R}$ denotes a known positive constant (i.e., $\delta_3 > 0$).

Remark 4 *The time derivative of the linear parameterization $Y_J(q, e)\hat{\phi}_J(t)$ is given by the following expression*

$$\begin{aligned} \frac{d}{dt}\left(Y_J(q, e)\hat{\phi}_J(t)\right) &= \frac{\partial}{\partial q} Y_J(q, e)\dot{q}\hat{\phi}_J(t) \\ &+ \frac{\partial}{\partial e} Y_J(q, e)J(q)\dot{q}\hat{\phi}_J(t) + Y_J(q, e)\dot{\hat{\phi}}_J(t) \end{aligned} \tag{30}$$

where (2) and (9) were utilized. After taking the norm of (30), the following expression can be obtained

$$\begin{aligned} \left\| \frac{d}{dt}\left(Y_J(q, e)\hat{\phi}_J(t)\right) \right\| &\leq \left\| \frac{\partial}{\partial q} Y_J(q, e)\dot{q} \right\|_{i\infty} \left\| \hat{\phi}_J(t) \right\| \\ + \left\| \frac{\partial}{\partial e} Y_J(q, e)J\dot{q} \right\|_{i\infty} \left\| \hat{\phi}_J(t) \right\| &+ \|Y_J(q, e)\|_{i\infty} \left\| \dot{\hat{\phi}}_J(t) \right\|. \end{aligned}$$

Since $q(t)$ and $e(t)$ appear as arguments of bounded trigonometric functions in $Y_J(q, e)$, the following inequalities can be developed

$$\|Y_J(q, e)\|_{i\infty} \leq \zeta_{J1} \qquad \left\| \frac{\partial}{\partial q} Y_J(q, e)\dot{q} \right\|_{i\infty} \leq \zeta_{J2} \|\dot{q}\| \tag{31}$$

$$\left\| \frac{\partial}{\partial e} Y_J(q, e)J\dot{q} \right\|_{i\infty} \leq \zeta_{J3}\delta_1 \|\dot{q}\| \tag{32}$$

where δ_1 was defined in (8), and $\zeta_{J1}, \zeta_{J2}, \zeta_{J3} \in \mathbb{R}$ denote known positive constants. The inequalities in (26), (27), (31), and (32) can now be used to formulate the following upper bound

$$\left\| \frac{d}{dt}\left(Y_J(q, e)\hat{\phi}_J(t)\right) \right\| \leq \zeta_J \|\dot{q}\| \tag{33}$$

where $\zeta_J \in \mathbb{R}$ denotes a known positive constant defined as follows

$$\zeta_J \triangleq \max\left\{\zeta_{J2}\bar{\phi}_{Jk}, \zeta_{J3}\delta_1\bar{\phi}_{Jk}, k_p\lambda_{\max}\{\Gamma_1\}\zeta_{J1}^2\right\}. \tag{34}$$

4 Stability Analysis

Theorem 1 *Given the robotic system defined by (1) and (2), the control torque input given in (22), along with the adaptation law given in (23)-(25) ensures semi-global asymptotic regulation of the task-space error in the sense that*

$$\lim_{t\to\infty} \|e(t)\| = 0. \tag{35}$$

The result in (35) is valid, provided the control gains k_p and k_v given in (22)-(24), and the adaptation weighting gain ε defined in (24) are chosen to satisfy the following sufficient conditions

$$k_p > k_v \frac{\delta_2}{\delta_3} > 0, \tag{36}$$

$$\varepsilon < \min\left\{\frac{m_1}{2\delta_2 m_2}, \frac{k_p}{2\delta_2 m_2}, \frac{1}{2\delta_2}\right\}, \tag{37}$$

and

$$\frac{k_v(1 - 2\varepsilon\delta_2)}{2\varepsilon\zeta_\chi} \geq \left[\sqrt{\frac{\lambda_2(0)}{\frac{1}{2}m_1 - \varepsilon\delta_2 m_2}} + 1\right]^2 \tag{38}$$

where m_1 and m_2 are defined in (3), δ_2 and δ_3 were defined in (16) and (29), respectively, $\zeta_\chi \in \mathbb{R}$ denotes a subsequently defined, positive bounding constant, and $\lambda_2(t) \in \mathbb{R}$ denotes a subsequently defined positive bounding function.

Proof: Let $V(t) \in \mathbb{R}$ denote the following the nonnegative function

$$\begin{aligned} V(t) \quad \triangleq \quad & \frac{1}{2}\dot{q}^T M(q)\dot{q} + \varepsilon Tanh^T(e)\hat{J}(q)M(q)\dot{q} \\ & + \sum_{i=1}^{m} k_p \ln(\cosh(e_i)) + \frac{1}{2}\tilde{\phi}^T\Gamma_0^{-1}\tilde{\phi} + \frac{1}{2}\tilde{\phi}_J^T\Gamma_1^{-1}\tilde{\phi}_J. \end{aligned} \tag{39}$$

Based on (3), (17), (19), (20), and (39), the Raleigh-Ritz Theorem [10] can be used to bound $V(t)$ by the following inequalities

$$\lambda_1(t) \leq V(t) \leq \lambda_2(t). \tag{40}$$

In (40), the positive function $\lambda_1(t) \in \mathbb{R}$ is defined as follows

$$\begin{aligned} \lambda_1(t) \quad \triangleq \quad & \left(\frac{1}{2}m_1 - \varepsilon\delta_2 m_2\right)\|\dot{q}(t)\|^2 \\ & + \sum_{i=1}^{m}(k_p - 2\varepsilon\delta_2 m_2)\ln(\cosh(e_i(t))) \\ & + \frac{1}{2}\lambda_{\min}\{\Gamma_0^{-1}\}\left\|\tilde{\phi}(t)\right\|^2 \\ & + \frac{1}{2}\lambda_{\min}\{\Gamma_1^{-1}\}\left\|\tilde{\phi}_J(t)\right\|^2, \end{aligned} \tag{41}$$

and the positive function $\lambda_2(t)$ introduced in (38) and (40) is defined as

$$\begin{aligned}\lambda_2(t) &\triangleq \left(\frac{1}{2}+\varepsilon\delta_2\right) m_2 \|\dot{q}(t)\|^2 \\ &+\sum_{i=1}^{m}(k_p+2\varepsilon\delta_2 m_2)\ln(\cosh(e_i(t))) \\ &+\frac{1}{2}\lambda_{\max}\left\{\Gamma_0^{-1}\right\}\left\|\tilde{\phi}(t)\right\|^2 \\ &+\frac{1}{2}\lambda_{\max}\left\{\Gamma_1^{-1}\right\}\left\|\tilde{\phi}_J(t)\right\|^2.\end{aligned} \tag{42}$$

Based on (41), it is straightforward that if ε is selected according to (37), then $\lambda_1(t) \geq 0$; hence, from (40) $V(t) \geq 0$.

After taking the time derivative of (39), the following simplified expression can be obtained

$$\begin{aligned}\dot{V}(t) = & -\dot{q}^T Y\tilde{\phi} - \varepsilon Tanh^T(e)\hat{J}(q)Y\tilde{\phi} \\ & -k_p\dot{q}^T Y_J\tilde{\phi}_J - k_v\dot{q}^T Tanh(\dot{q}) + \varepsilon\chi \\ & -\varepsilon k_p Tanh^T(e)\hat{J}(q)\hat{J}^T(q)Tanh(e) \\ & -\varepsilon k_v Tanh^T(e)\hat{J}(q)Tanh(\dot{q}) \\ & -\tilde{\phi}^T\Gamma_0^{-1}\dot{\hat{\phi}} - \tilde{\phi}_J^T\Gamma_1^{-1}\dot{\hat{\phi}}_J\end{aligned} \tag{43}$$

where (4), (12), (13), (15), and (28) were utilized, and the auxiliary term $\chi(t) \in \mathbb{R}$ is defined as follows

$$\begin{aligned}\chi \triangleq & \frac{d}{dt}\left(Y_J(q,e)\hat{\phi}_J(t)\right) M(q)\dot{q} \\ & +Tanh^T(e)\left(\hat{J}(q)\left(\dot{M}(q) - V_m(q,\dot{q})\right)\dot{q}\right).\end{aligned} \tag{44}$$

Based on the form of (44) and the properties of $Tanh(\cdot)$ defined in (10), the expressions in (3), (7), (16), and (33) can be utilized to show that

$$\|\chi\| \leq \zeta_\chi \|\dot{q}\|^2 \tag{45}$$

where the positive constant ζ_χ introduced in (38) and (45) is defined as

$$\zeta_\chi \triangleq \max\left\{\zeta_J m_2, \delta_2\zeta_m, \delta_2\zeta_c\right\}. \tag{46}$$

After utilizing (16), (19), (21), (24), (29), and (45), the following expression can be obtained

$$\begin{aligned}\dot{V}(t) \leq & -\varepsilon k_p\delta_3\|Tanh(e)\|^2 - k_v\|Tanh(\dot{q})\|^2 \\ & +\varepsilon k_v\delta_2\left(\|Tanh(e)\|^2 + \|Tanh(\dot{q})\|^2\right) + \varepsilon\zeta_\chi\|\dot{q}\|^2 \\ & +\tilde{\phi}^T\Gamma_0^{-1}\left(\Omega_o - \dot{\hat{\phi}}\right) + \tilde{\phi}_J^T\Gamma_1^{-1}\left(\Omega_1 - \dot{\hat{\phi}}_J\right)\end{aligned} \tag{47}$$

By utilizing (23)-(25), the following advantageous expression[1] can be developed for the upper bound of (47)

$$\begin{aligned}\dot{V}(t) \leq & -\tfrac{k_v}{2}\|Tanh(\dot{q})\|^2 - \varepsilon(k_p\delta_3 - k_v\delta_2)\|Tanh(e)\|^2 \\ & -\left(\tfrac{k_v}{2} - \varepsilon k_v\delta_2\right)\|Tanh(\dot{q})\|^2 + \varepsilon\zeta_\chi\|\dot{q}\|^2.\end{aligned} \tag{48}$$

[1] For more details on how the projection algorithm allows one to proceed from (47) to (48), the reader is referred to [17].

Provided that the condition given in (36) and the following inequality are both satisfied

$$-\left(\frac{k_v}{2} - \varepsilon k_v\delta_2\right)\|Tanh(\dot{q})\|^2 + \varepsilon\zeta_\chi\|\dot{q}\|^2 \leq 0 \tag{49}$$

the expression in (48) can be used to prove that $\dot{V}(t) \leq 0$. To facilitate further analysis, (18), (40), and (41) are used to obtain the following sufficient condition for (49)

$$\frac{(k_v - 2\varepsilon k_v\delta_2)}{2\varepsilon\zeta_\chi} \geq \left(\sqrt{\frac{V(t)}{\frac{1}{2}m_1 - \varepsilon m_2}} + 1\right)^2. \tag{50}$$

If the conditions in (36), (37), and (50) are satisfied, the inequality in (48) can be used to obtain the following inequality

$$\dot{V}(t) \leq -\beta\|\psi\|^2 \tag{51}$$

where $\beta \in \mathbb{R}$ is a positive constant, and $\psi(t) \in \mathbb{R}^{m+n}$ is given by

$$\psi \triangleq \begin{bmatrix} Tanh^T(e) & Tanh^T(\dot{q}) \end{bmatrix}^T. \tag{52}$$

From (51), it is clear that $\dot{V}(t) \leq 0$; therefore,

$$V(z(t),t) \leq V(z(0),0) \leq \lambda_2(z(0),0) \qquad \forall t \geq 0 \tag{53}$$

where $\lambda_2(t)$ was defined in (42), and $z(t) \in \mathbb{R}^4$ is given by

$$z \triangleq \begin{bmatrix} \|\dot{q}\|^2 & \sum_{i=1}^{m}\ln(\cosh(e_i)) & \left\|\tilde{\phi}\right\|^2 & \left\|\tilde{\phi}_J\right\|^2 \end{bmatrix}^T. \tag{54}$$

Based on (53), the final sufficient condition for (50) can be expressed by the inequality in (38). For more details on the above semi-global stability argument, the reader is referred to [2], where a similar type of argument was utilized for a different problem.

From (53) it is clear that $V(t) \in \mathcal{L}_\infty$; hence, $\dot{q}(t)$, $e(t)$, $\tilde{\phi}(t)$, $\tilde{\phi}_J(t), \psi(t) \in \mathcal{L}_\infty$. Since $e(t) \in \mathcal{L}_\infty$, and the desired setpoint is constant, it is clear that $x(t) \in \mathcal{L}_\infty$. Since the development is directed at revolute robots, $q(t)$ only appears in $h(q)$ in (2) as an argument of bounded trigonometric functions; hence, it is typically unclear how the boundedness of $q(t)$ can be proven. However, the boundedness of $q(t)$ is typically not a concern since $q(t)$ only appears as an argument of bounded trigonometric functions in the controller. From (6), (15), (23)-(25), and the preceding arguments, it is clear that $\dot{\hat{\phi}}(t)$, $\hat{\phi}(t)$, $\dot{\hat{\phi}}_J(t)$, $\hat{\phi}_J(t)$, $\tau(t) \in \mathcal{L}_\infty$. Moreover, (2) and the facts that $\dot{q}(t)$, $J(q) \in \mathcal{L}_\infty$ can be used to prove that $\dot{x}(t)$, $\dot{e}(t) \in \mathcal{L}_\infty$; hence, $e(t)$ is uniformly continuous. From (51), (52), and the properties of the hyperbolic tangent, it is clear that $\dot{q}(t)$, $e(t) \in \mathcal{L}_2$ [10]. Since $e(t)$, $\dot{e}(t) \in \mathcal{L}_\infty$ and $e(t) \in \mathcal{L}_2$, Barbalat's Lemma [18] can be invoked to conclude the result in (35). □

Remark 5 *An important advantage of the proposed adaptive controller given by (22)-(25) is that it can be upper bounded in terms of a priori known terms as follows*

$$\|\tau\| \leq \|Y\|_{i\infty} \left\|\overline{\phi}\right\| + k_p \|Y_J\|_{i\infty} \left\|\overline{\phi}_J\right\| + k_v. \tag{55}$$

Furthermore, the adaptive weighting gain ε can be selected arbitrarily small to satisfy the conditions given in (36)-(38); hence, the magnitude of k_v can be made arbitrarily small.

5 Conclusion

An amplitude limited controller was developed for robot manipulators despite uncertainty in the dynamic and kinematic models. The adaptive controller yields semi-global asymptotic regulation of the task-space setpoint error. The advantages of the proposed controller include the ability to actively compensate for unknown parametric effects in the dynamic and kinematic model and the ability to ensure actuator constraints are not breached by calculating the maximum required torque a priori.

References

[1] M. Bridges, D. M. Dawson, and C. Abdallah, "Control of Rigid-Link Flexible-Joint Robots: A Survey of Backstepping Approaches," *Journal of Robotic Systems,* Vol. 12, pp. 199-216 (1995).

[2] T. Burg, D. Dawson, and P. Vedagarbha, "A Redesigned DCAL Controller without Velocity Measurements: Theory and Demonstration," *Robotica*, Vol. 15, pp. 337-346 (1997).

[3] C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate Jacobian Control for Robots with Uncertain Kinematics and Dynamics," *IEEE Trans. on Robotics and Automation*, Vol. 19, No. 4, pp. 692-702 (2003).

[4] C. Cheah, S. Kawamura, and S. Arimoto, "Feedback Control for Robotic Manipulators with Uncertain Kinematics and Dynamics," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Leuven, Belgium, May 1998, pp. 3607-3612.

[5] C. Cheah, S. Kawamura, and S. Arimoto, "Feedback Control for Robotic Manipulators with an Uncertain Jacobian Matrix," *Int. Journal of Robotic Systems*, Vol. 12, No. 2, pp. 119-134 (1999).

[6] C. Cheah, S. Kawamura, S. Arimoto, and K. Lee, "PID Control for Robotic Manipulator with Uncertain Jacobian Matrix," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Detroit, Michigan, May 1999, pp. 494-499.

[7] R. Colbaugh, E. Barany, and K. Glass, "Global Regulation of Uncertain Manipulators Using Bounded Controls," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Albuquerque, New Mexico, April 1997, pp. 1148-1155.

[8] R. Colbaugh, E. Barany, and K. Glass, "Global Stabilization of Uncertain Manipulators Using Bounded Controls," *Proc. of the IEEE American Control Conf.*, Albuquerque, New Mexico, June 1997, pp. 86-91.

[9] W. E. Dixon, M. S. de Queiroz, D. M. Dawson, and F. Zhang, "Tracking Control of Robot Manipulators with Bounded Torque Inputs," *Robotica*, Vol. 17, pp. 121-129 (1999).

[10] W. E. Dixon, A. Behal, D. M. Dawson, and S. Nagarkatti, *Nonlinear Control of Engineering Systems: A Lyapunov-Based Approach*, Birkhäuser Boston, 2003.

[11] R. Kelly and V. Santibáñez, "A Class of Global Regulators with Bounded Control Actions for Robot Manipulators," *Proc. of the IEEE Conf. Decision and Control*, Kobe, Japan, Dec. 1996, pp. 3382-3387.

[12] F. Lewis, C. Abdallah, and D. Dawson, *Control of Robot Manipulators*, MacMillan Publishing Co.: New York, 1993.

[13] A. Loria, R. Kelly, and R. Ortega, "On Global Output Feedback Regulation of Euler-Lagrange Systems with Bounded Inputs," *Proc. of the IFAC World Congress*, San Francisco, California, July 1996, pp. 161-166.

[14] A. Loria and H. Nijmeijer, "Bounded Output Feedback Tracking Control of Fully-Actuated Euler-Lagrange Systems," *Systems & Control Letters*, Vol. 33, No. 3, pp. 151-161 (1998).

[15] R. Lozano, and B. Brogliato, "Adaptive Control of Robot Manipulators with Flexible Joints," *IEEE Trans. on Automatic Control*, Vol. 37, pp. 174-181 (1992).

[16] V. Santibáñez and R. Kelly, "Global Regulation for Robot Manipulators Under SP-SD Feedback," *Proc. IEEE Int. Conf. Robotics and Automation*, April 1996, pp. 927-932.

[17] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*, Prentice Hall Co.: Englewood Cliffs, N.J., 1989.

[18] J.J.E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, Inc.: Englewood Cliff, NJ, 1991.

[19] E. Zergeroglu, W. E. Dixon, A. Behal, and D. M. Dawson, "Adaptive Set-Point Control of Robotic Manipulators with Amplitude-Limited Control Inputs," *Robotica*, Vol. 18, No. 2, pp. 171-181 (2000).