NON-LIPSHITZIAN CONTROL OF FRICTION

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Abstract

Recently, we proposed a new algorithm to control frictional dynamics of an array of particles towards pre-assigned values of the average sliding velocity [1]. The algorithm is based on the concepts of terminal attractor and global targeting, which endow the control with robust efficiency. In this paper, we focus on the transient times needed to reach the prescribed behavior and their dependence on the control parameters.

Despite great progress made during the past half century, many issues in fundamental tribology, such as the origin of friction and failure of lubrication, have remained unsolved. Moreover, the current reliable knowledge related to friction and lubrication is mainly applicable to the macroscopic systems and machinery and, most likely, will be only of limited use for micro- and nano-systems. Indeed, when the thickness of the lubricant film is comparable to the molecular or atomic size, its behavior becomes significantly different from the behavior of macroscopic (bulk) lubricant [2]. Better understanding of the intimate mechanisms of friction, lubrication, and other interfacial phenomena at the atomic and molecular scales is expected to provide designers and engineers with the required tools and capabilities to monitor and control friction, reduce unnecessary wear, and predict mechanical faults and failure of lubrication in MEMS and nano-devices [3].

The ability to control and manipulate friction during sliding is extremely important for a large variety of technological applications. The outstanding difficulties in realizing efficient friction control are related to the complexity of the task, namely dealing with systems with many degrees of freedom, under strict size confinement, and only very limited control access. Moreover, a nonlinear system driven far from equilibrium can exhibit a variety of complex spatial and temporal behaviors, each resulting in different patterns of motion and corresponding to different friction coefficient [4].

Friction can be manipulated by applying small perturbations to accessible elements and parameters of the sliding system. This operation requires a-priori knowledge of the strength and timing of the perturbations. Recently, the groups of J. Israelachvili [5] (experimental) and U. Landman [6] (full-scale molecular dynamics computer simulation) showed that friction in thin-film boundary lubricated junctions can be reduced by coupling the small amplitude (of the order of 1Å) directional mechanical oscillations of the confining boundaries to the molecular degree of freedom of the sheared interfacial lubricating fluid. Using a surface force apparatus, modified for measuring friction forces while simultaneously inducing normal (out-of-plane) vibrations between two boundary-lubricated sliding surfaces, load- and frequency-dependent transitions between a number of "dynamical friction" states have been observed [5]. In particular, regimes of vanishingly small friction at interfacial oscillations were found. Extensive grand-canonical molecular dynamics simulations [6] revealed the nature of the dynamical states of confined sheared molecular films, their structural mechanisms, and the molecular scale mechanisms underlying transitions between them. Significant changes in frictional responses were observed in the two-plate model [7] by modulating the normal response to lateral motion [8]. In addition, surface roughness and thermal noise are expected to play a significant role in deciding upon control strategies at the micro and the nano-scale [9,10]. These results point to a completely new direction for realizing ultra-low friction in mechanical devices.

In a previous paper [1], we proposed a global feedback control scheme, based on the properties of terminal attractors [11, 12]. The main advantage of terminal attractor algorithms consists in their robustness and efficiency [1]. In this paper, we continue the study of the non-Lipschitzian control algorithms for friction, by focusing on the dependence of the transient times on the parameters of the control.

We illustrate the proposed control strategy on a phenomenological model of friction [7,13-16]. Despite their relative simplicity, phenomenological models [10,13-16] show a fair agreement with many experimental results using the friction force apparatus [7,18,19] and quartz microbalance experiments [9,17,20]. The basic equations for the driven dynamics of a one dimensional particle array of N identical particles moving on a surface are given by a set of coupled nonlinear equations of the form [16]:

$$m\ddot{x}_n + \gamma \dot{x}_n = -\partial U / \partial x_n - \partial V / \partial x_n + f_n + \eta(t)\,,\ n=1,...N \tag{1}$$

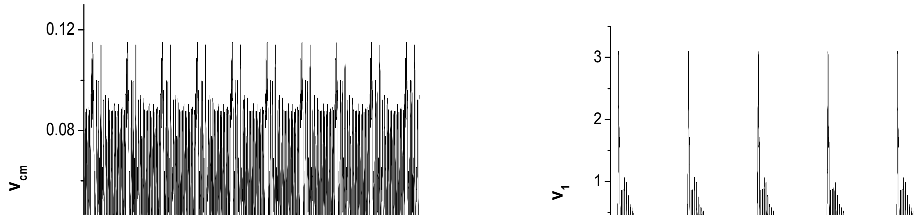
where x_n is the coordinate of the *nth* particle, m is its mass, γ is the linear friction coefficient representing the single particle energy exchange with the substrate, f_n is the applied external force, and $\eta(t)$ is Gaussian noise. The particles in the array are subjected to a periodic potential, $U(x_n + a) = U(x_n)$, and interact with each other via a pair-wise potential $V(x_n - x_j),\ n,\ j = 1, 2, ...N.$ The system (1) provides a general framework of modeling friction although the amount of details and complexity varies in different studies from simplified 1D models [15,16,21,22] through 2D and 3D models [17,23-25] to a full set of molecular dynamics simulations [25].

To better present our ideas, we make the following simplifications, namely: (i) the substrate potential has a simple periodic form, (ii) there is a zero misfit length between the array and the substrate, (iii) the same force f is applied to each particle, and (iv) the interparticle coupling is linear. The coupling with the substrate is, however, strongly nonlinear. For this case, using the dimensionless phase variables $\phi_n = 2\pi x_n / a,$ the equations of motion reduce to the dynamic Frenkel-Kontorova model [16]

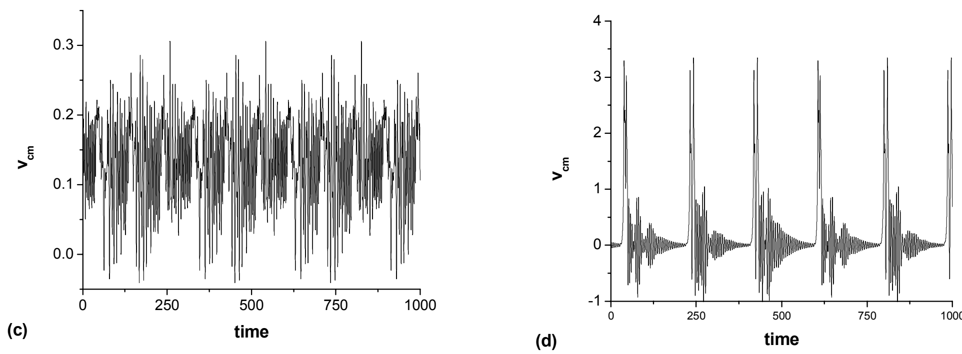
$$\ddot{\phi}_n + \gamma\dot{\phi}_n + \sin(\phi_n) = f + \kappa(\phi_{n+1} - 2\phi_n + \phi_{n-1}), \quad n = 1, 2, ...N \tag{2}$$

Throughout this paper, we shall use an array with $N=25$ particles. We performed extensive numerical simulations for other arrays sizes ($3 < N < 40$) to verify that we indeed present a typical example. Without control, we observed four co-existing different regimes: periodic sliding, periodic stick-slip, chaotic stick-slip, and rest (no motion). All motion types are obtained by only changing the initial conditions of the particle's positions and velocities, but not the system's parameters. The average (center of mass) velocity for the "natural" (i.e., uncontrolled) motion may take only a limited range of values, namely: (i) $v = f / \gamma$ for periodic sliding motion, (ii) $v = nv_0$, where n is an integer, and $v_0 = \dfrac{2\pi}{nN\gamma}\sqrt{\dfrac{\pi - \cos^{-1} f}{\pi}(\kappa - \kappa_c)^{1/2}}$, for periodic stick-slip motion, (iii) $v = 0$ for rest (no sliding). [16]. In the range of parameters under consideration, we observed only one single value of the average velocity for chaotic stick-slip.

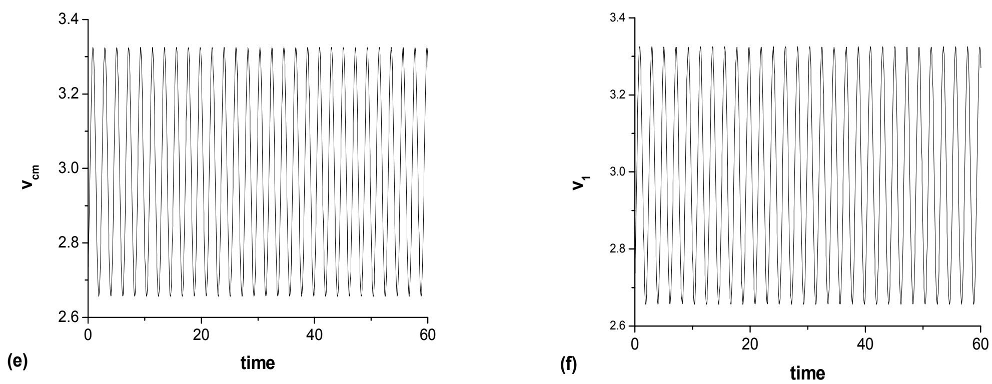
To better illustrate the dynamics of the uncontrolled system, we display the time series of the average velocity and the velocity of the first particle in the array for different sets of initial positions and initial velocities.



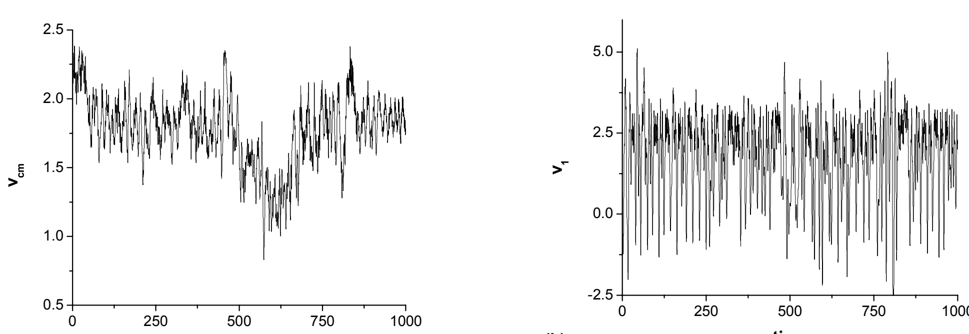
Figures 1a-b: (a) Time series of the average velocity and (b) time series of the first particle in the array with no control. The average velocity is 0.0625. The other parameters are: $N = 25; f = 0.3; \gamma = 0.1$; $\kappa = 0.26$.



Figures 1c-d: (c) Time series of the average velocity and (d) the time series of the first particle in the array with no control. The average velocity is 0.13. The other parameters are: $N = 25; f = 0.3; \gamma = 0.1$; $\kappa = 0.26$.



Figures 1e-f: (e) Time series of the average velocity and (f) time series of the first particle in the array with no control. The average velocity is 3.0. The other parameters are: $N = 25; f = 0.3; \gamma = 0.1$; $\kappa = 0.26$.



Figures 1g-h: (g) Time series of the average velocity and (h) time series of the first particle in the array with no control. The average velocity is 1.81. The other parameters are: *N = 25; f = 0.3; γ = 0.1*; κ = *0.26.*

Our objectives are to: (i) achieve any targeted value of the average sliding velocity using only small values of the control and (ii) significantly reduce the transient time needed to reach the desired behavior. To that effect, we propose the following control algorithm:

$$C(t) = \alpha(v_{target} - v_{cm})^{\beta} \tag{3}$$

where $v_{cm} = (1/N)\sum_{n=1}^{N}\dot{\phi}_n$ is the average (center of mass) velocity, v_{target} is a constant targeted velocity for the center of mass, $\beta = 1/(2n+1)$, and $n = 1,2,3\dots$. Note that the control term is identical for all the particles in the array and requires only the knowledge of the average velocity of the array. This control utilizes the concept of "terminal attractors" [11,12] whose relevance for the dynamics is explained below.

The equations of motion (Equation 2) in the presence of the control term *C(t)* (Equation 3) reads:

$$\ddot{\phi}_n + \gamma\dot{\phi}_n + \sin(\phi_n) = f + \kappa(\phi_{n+1} - 2\phi_n + \phi_{n-1}) + C(t) \tag{4}$$

System (4) can be written as a *2N*-dimensional first order system:

$$\dot{\phi}_n - F_n(\phi_1, \phi_2, \dots \phi_{2N}) = 0, \quad n = 1,2,3,\dots,2N \tag{5}$$

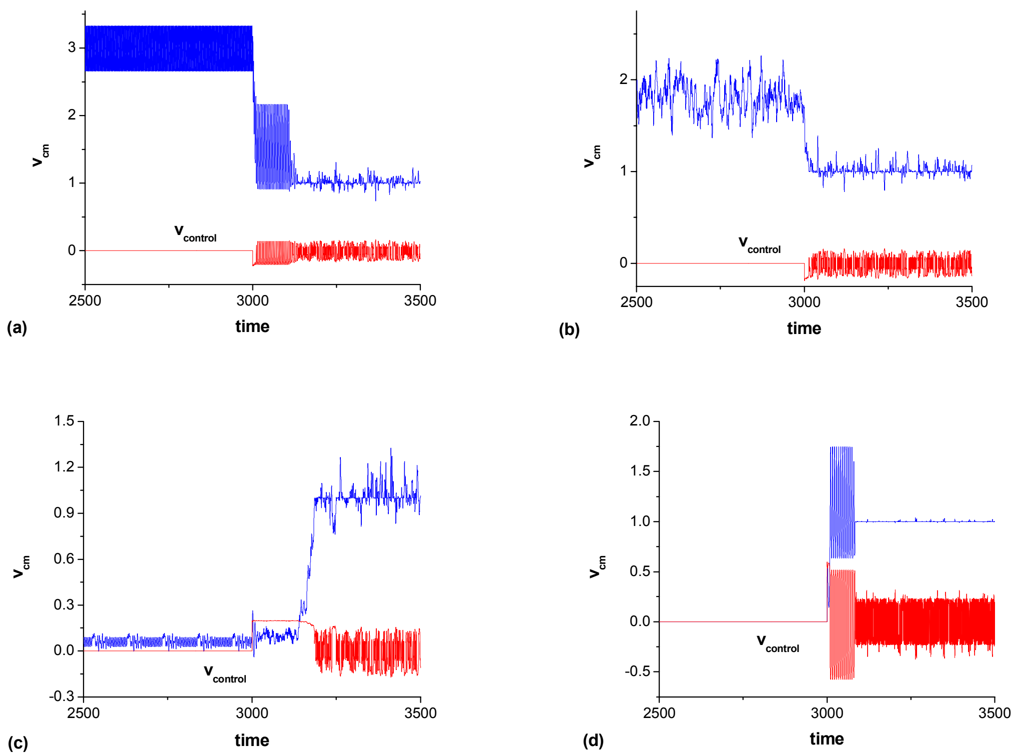
where, for simplicity, we maintain the same notation for the (now different) unknown functions. The fixed points of this *2N*-dimensional, dissipative dynamical system are obtained by solving the stationary version of Eq. (5). If the real parts of the eigenvalues μ_e of the Jacobian matrix, ***M***, $M_{nm} = \partial F_n / \partial\phi_m$, at a fixed point are all negative (that is $Re\,\mu_e < 0$) then this point is locally asymptotically stable and constitutes a local attractor of the dynamics.

When a nonlinear dynamical system satisfies the Lipschitz condition, namely $|\partial F_n / \partial\phi_m| \le K < \infty$, there is a unique solution for each initial configuration. Moreover, the time spent by each trajectory to reach an attractor is, in principle, infinite. Therefore, the time needed to reach the desired target within the needed precision may become unacceptably large.

In contrast, the terminal attractor dynamics that we are utilizing violates *by construction* the Lipschitz condition. As a result, trajectories reach the terminal attractor in finite time. To illustrate this point, consider one of the simplest systems with terminal attractor [11], i.e. the equation $\dot{\phi} = -\phi^{1/3}$. At the equilibrium point, ϕ *=0*, the Lipschitz condition is violated, since

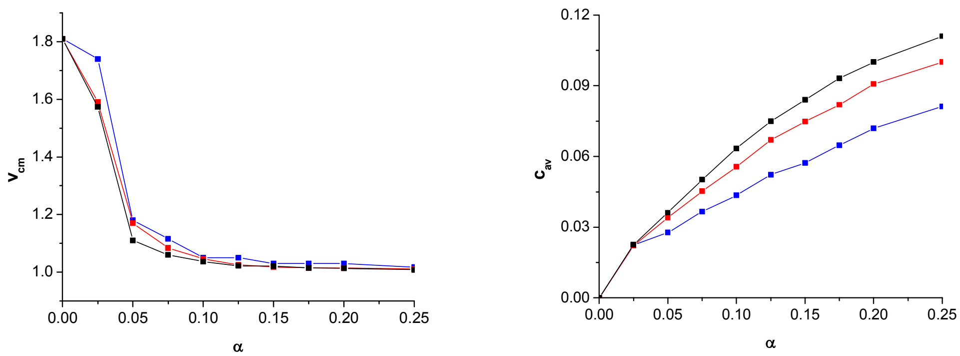
$\partial\dot{\phi}/\partial\phi = -(1/3)\phi^{-2/3}$ tends to minus infinity as ϕ tends to zero. Also, one can easily check that the trajectory started at the initial point ϕ_0 reaches the terminal attractor in a finite time, $\tau = \frac{3}{2}\phi_0^{2/3}$.

This is precisely the effect realized by the non-Lipschitzian control term, $C(t)$. The "infinite attraction power" of the "terminal" (non-Lipschitzian) attractor endows the proposed algorithm with excellent efficiency and robustness , as illustrated in Figure 2 for the target velocity of: v_{target} = 1. Red color lines indicate the time series of the control (Equation 3), while the blue lines show the time series of the velocity of the center of mass. In all cases, we reached and sustained the (arbitrarily chosen) target value for rather small values of the control.



Figures 2a-d. Time series of the average velocities (blue lines) and of the control (red lines). In all cases, the targeted velocity is equal to 1.0, while the initial uncontrolled velocities are: 3, 1.81, 0.0625, and 0. The other parameters are the same as in Figure 1, and the control parameters are: α = 0.15, and β = 1/7. The control is applied at time t = 3000.

We performed extensive testing of the proposed algorithm (Eqs.3-4) by choosing numerous values of the target velocity [1]. For some values of v_{target}, the dependence of the average velocity, v_{cm} on α appears to be irregular. In all cases, however, we succeeded to identify a specific value of control α to reach the desired value of the average velocity. We also studied the convergence process, i.e. the dependence of the average velocity of the center of mass as a function of the control amplitude α. Figures 3a-b illustrate this dependence.



Figures 3a-b. (a) Dependence of the average velocity on the control amplitude α; (b) dependence of the time averaged control on the control amplitude. The exponent values are: $\beta = 1/7$ (black line), $\beta = 1/5$ (red line), and $\beta = 1/3$) blue line), and all the other parameters are as in Figure 1.

Figure 3a illustrates that the convergence to the targeted value is faster for smaller values of the exponent. The reason for this behavior is the fact that smaller exponents correspond to larger average controls in the vicinity of the target (Fig. 3b).

In summary, we proposed a new type of algorithm to control friction of sliding nano-arrays. This algorithm is based on the concepts of "terminal attractor" and global targeting and requires only the knowledge of the velocity of the center of mass of the array. We demonstrated the ability to control the array towards the desired sliding velocity and this control was achieved in a short transient time. Finally, we studied the dependence of the transient time on the parameters of the control.

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