Scaling Laws for Damage Evolution in Quasi-brittle Materials
- An Application of Field Induced Percolation

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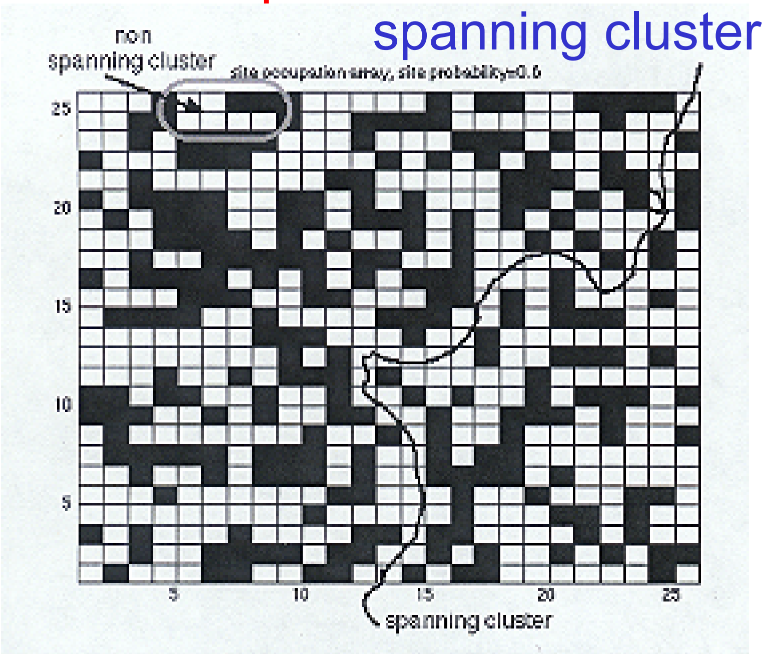
Percolation Basics

What is Percolation?

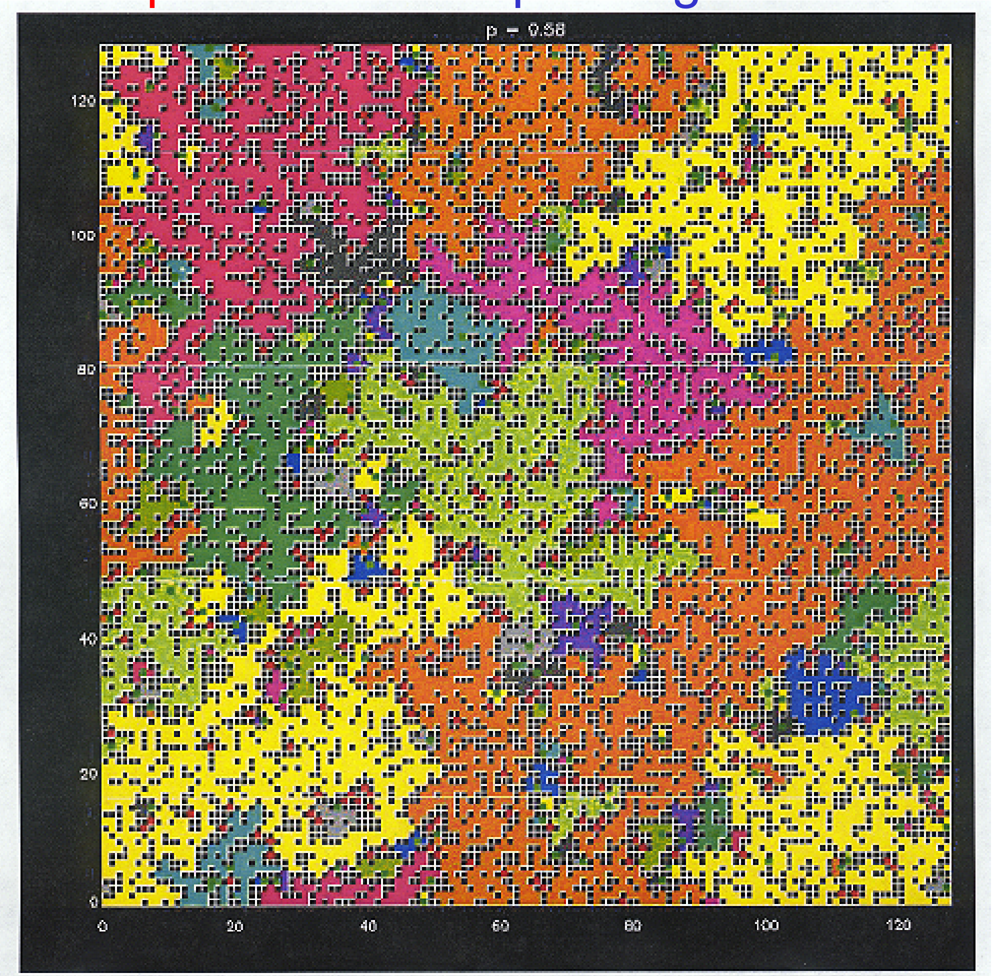
p = 0.60

spanning cluster



25 x 25 square lattice

p = 0.58 No spanning cluster



128 x 128 square lattice

Phase transition:

System changes from connected phase to disconnected phase as the critical point $p_c = 0.592746$ is approached from above

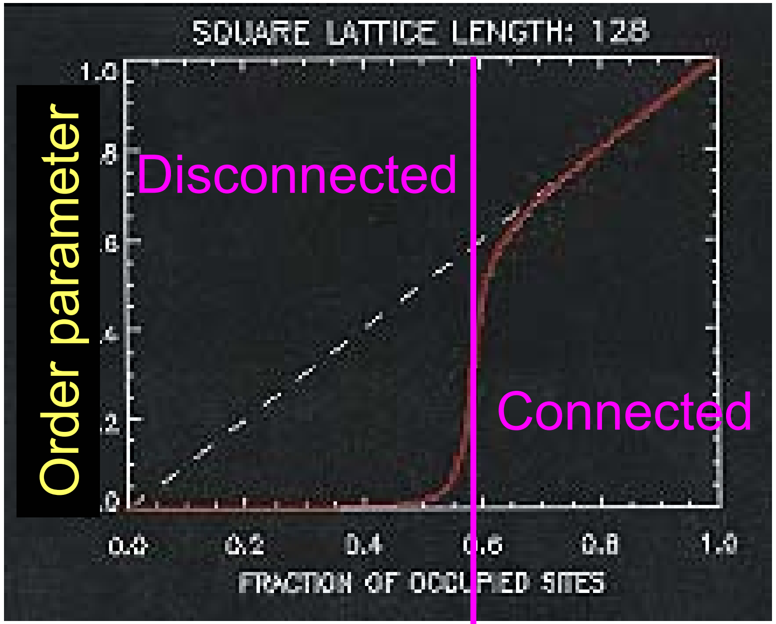
Why should we use Percolation Theory?

Percolation and Fracture Similarity

Fracture is a phase transition:
From a connected system to
a disconnected system

Stiffness of the system is the
Order parameter:
- non-zero in the connected phase
- zero in the disconnected phase

128 x 128 square lattice



critical percolation
threshold $p_c = 0.592746$

Percolation theory describes how
system approaches criticality

Scaling Laws

System behavior with change in length scale
- couples meso- and macro- phenomena

System size effect

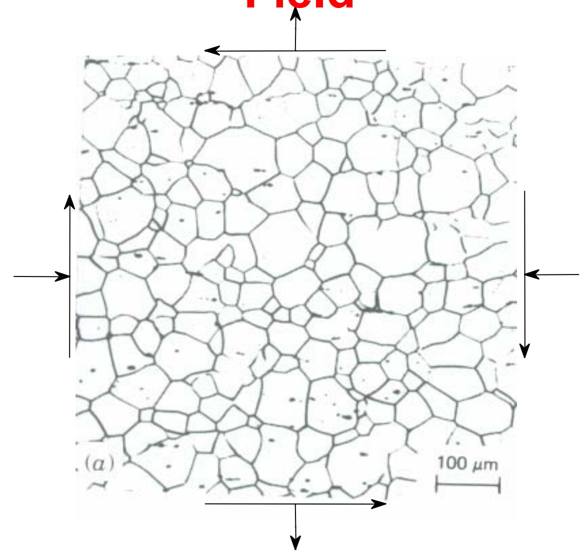
Field Induced Percolation

But, really, fracture is not a random percolation!
Stress distribution plays a significant role.

Fields:

- mechanical: stress, strain
- thermal
- electrical
- magnetic

Field



Microstructure (cluster) evolution:

- microcracks
- second-phases

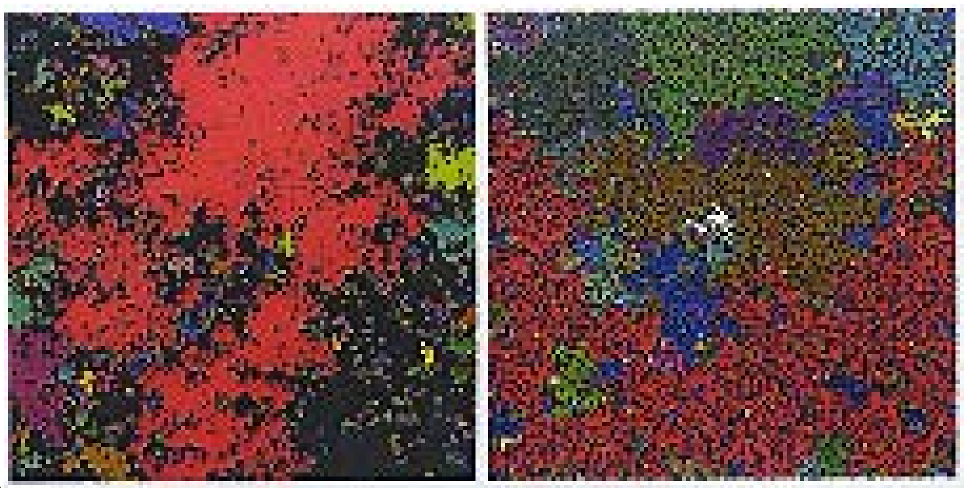
Geometric percolation:

- clusters formed with randomly occurring events

Field induced percolation:

- current event is dependent on the past history of events
- applied field introduces a bias in the occurrence of events

Microstructure evolution



Correlated Uncorrelated

General Applications

Materials Science Applications

Thermo-Mechanical Fields:

- Nucleation and growth of damage
  - Intergranular and transgranular cracking
- Dynamic recrystallization
- Stress induced boundary migration
  - Migration of both low and high angle flat boundaries
  - Twinning in ferromagnetic shape memory alloys

Electrical and Magnetic Fields:

- Evolution of magnetic domains
- Magnetic Fields in Solidification
  - Electromagnetic stirring (Dendrite morphology)
- Electro-migration of interfaces
  - Boundary migration related to grain boundary potential

Chemical Bond Fields:

- Polymer gelation, vulcanization; Glass transition

General Applications

Thermal Fields:

- Boiling of water: liquid-gas phase transition
- Paramagnetic to ferromagnetic phase transition

Electrical Fields:

- Fuse problem: conducting to non-conducting transition
- Dielectric breakdown: non-conducting to conducting transition

Fluid Flow: Geological Applications

- Flow through fractured rocks and porous media
- Earthquakes, fracture and fault patterns

Traffic Flow: Transportation Applications

- Traffic flow on a network

Information Flow on www: Computer Science Applications

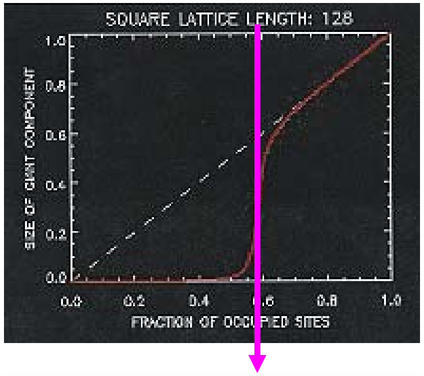
- Information flow on www network
- Overloading of computer network within a massively parallel system

Graph Theory:

- Structural failure of a highly redundant system

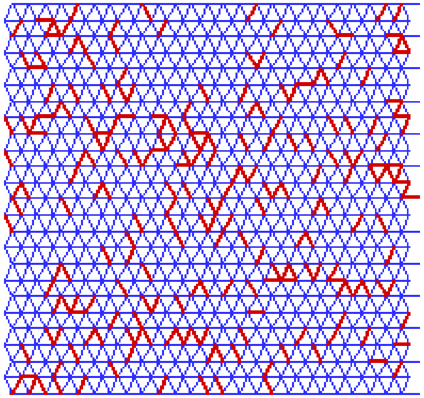
Common Theme

System Behavior: undergoes phase transition at a critical point



System Model: random graph with vertices, mutual interactions as bonds

System Evolution: approaches criticality through a change in the interaction strength

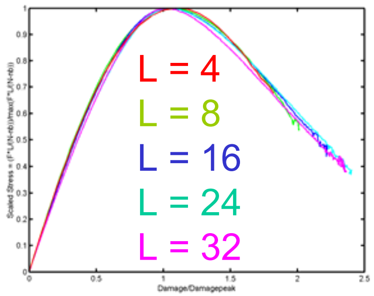


Conservation laws;
Kirchhoff equations

Redistribution: underlying physics governs the redistribution of the applied field

Scaling Laws: comparison of system behavior at different length scales is possible

System size effect on system behavior

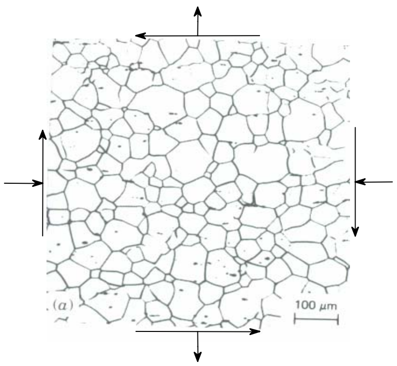


Damage Evolution in Brittle Materials

Motivation: Stress Induced Microcracking Evolution

Macroscopic properties and behavior of quasi-brittle materials are significantly effected by the internal microstructure and damage/microcracking evolution

Stress Field



Microcracking evolution

Controlling of microstructure state and damage evolution leads to improved macroscopic behavior

Modeling at the mesoscale will lead to a fundamental understanding of the effect of microstructural features on the microcracking evolution in brittle materials

Objective

Mesoscale System Response:

– depends on the system size
– computationally intractable

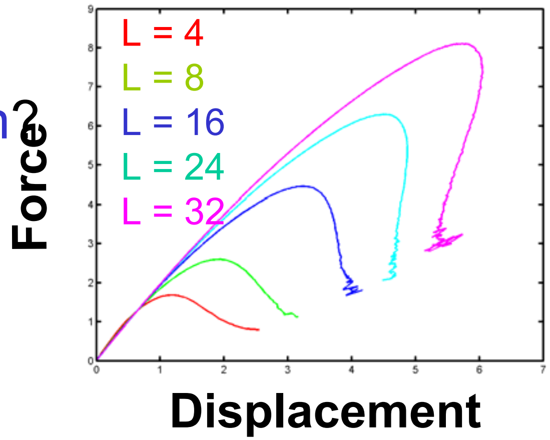
→ Scaling Laws

Open Questions?:

- How does a disordered solid breakdown?

- What is the size effect on failure?

- What are the scaling laws of failure?



- How does one quantify damage? and how do we compare the extent of damage between two specimens?

- What is the connection between mesoscopic damage and the phenomenological continuum damage evolution?

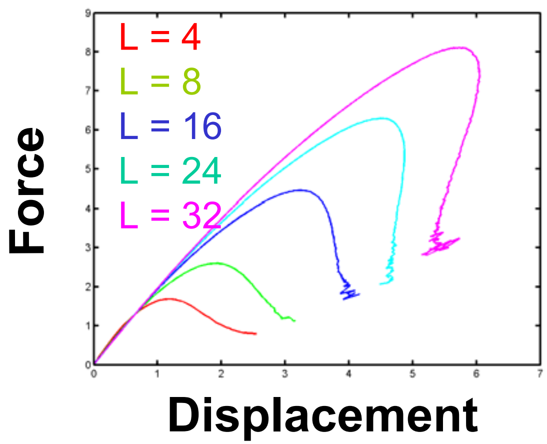
Objective:

– Describe continuum damage evolution based on mesoscopic modeling using scaling laws

Current Status of Material Models

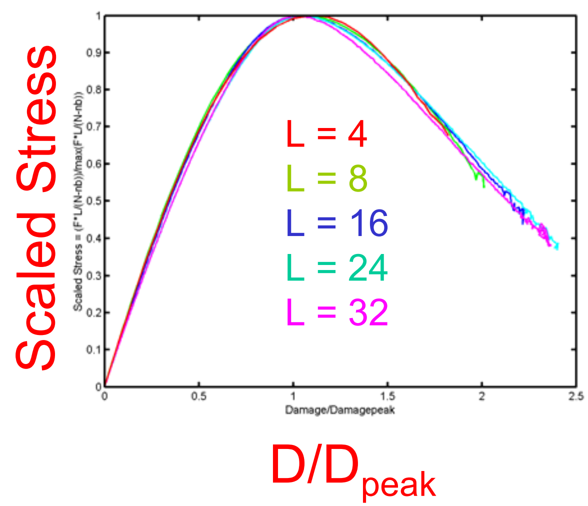
Phenomenological Material Models:

- progressive damage and cracking are microstructure-insensitive
- based on simplified assumptions for the evolution of damage
    - valid only for moderate damage levels
    - local stress field fluctuations and interactions are not considered



Material response is size dependent

Scaling laws are required to obtain a "normalized" response that couple mesoscopic and continuum length scales



Solution:

Explicit modeling of material microstructure combined with the scaling theory accounts for size effects and local stress field interactions during damage/microcrack evolution

Numerical Methodology

Mesoscopic Simulation: Discrete Lattice Models

Focus of the study is not on any particular material

But, in capturing the generic features of damage evolution

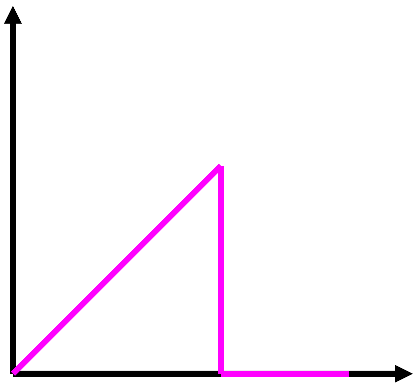
Essential ingredients of breakage process:

- Initial material disorder (inhomogeneities)
- redistribution of stresses due to damage evolution

Discrete Lattice Models:

- disorder in bond strength and stiffness
- elastic response characteristics of the bonds
- bond breaking rule (failure criteria)

Perfectly brittle bond



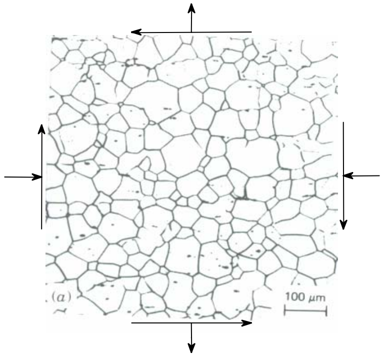
Any realistic damage evolution description must be capable of reproducing the behavior of these idealized discrete lattice models

Mesoscopic Modeling Approach

Failure of a bond is governed by

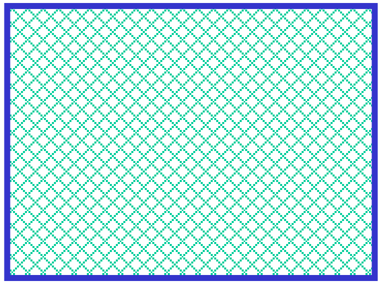
- weakest bond of the disordered medium
- stress concentration around material inhomogenities

Stress



Discretization with random disorder distributions

Lattice system with disorder



L

Disorder Type

+

Lattice Topology

+

Lattice Bond Model

Applied Stress

Failure Criteria

Mesoscopic Damage Evolution

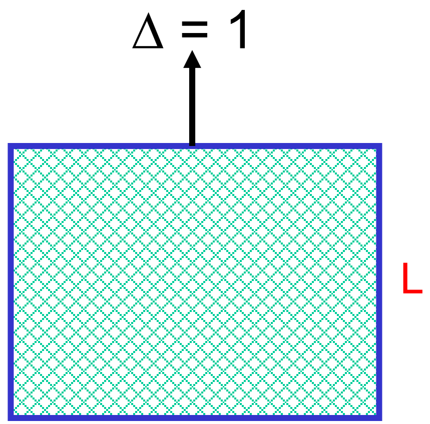
Analysis Procedure

Procedure:

Step 0: For each bond in the lattice system, assign unit stiffness and random force threshold f_i^{th}

Step 1: Impose a unit macroscopic displacement

Step 2: Calculate the force f_i in each bond through lattice equilibrium



Lattice system with disorder

$$K = \sum_i f_i^2$$

K Global stiffness

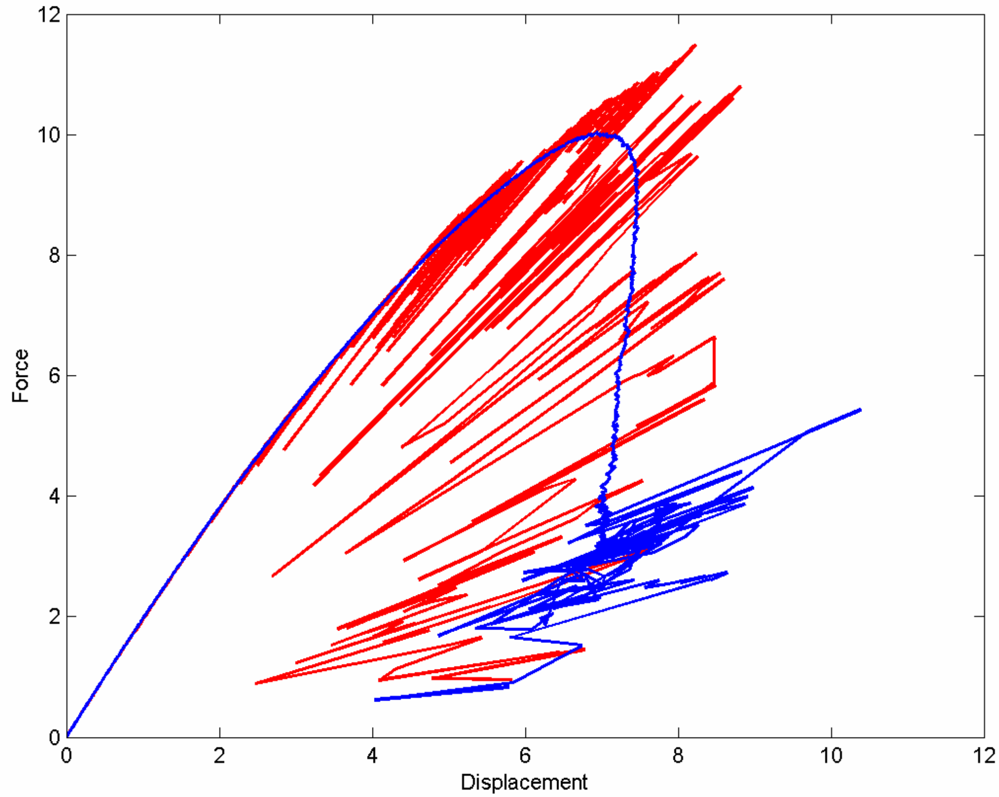
Step 3: Determine the bond i_c for which

$$\frac{1}{\lambda} = \max_i \left(\frac{f_i}{f_i^{th}} \right)$$

Step 4: Record the lattice displacement and force $(\lambda, K\lambda)$

Step 5: Remove the bond i_c and repeat steps 1-4, until the entire lattice system breaks apart

Typical Loading Response



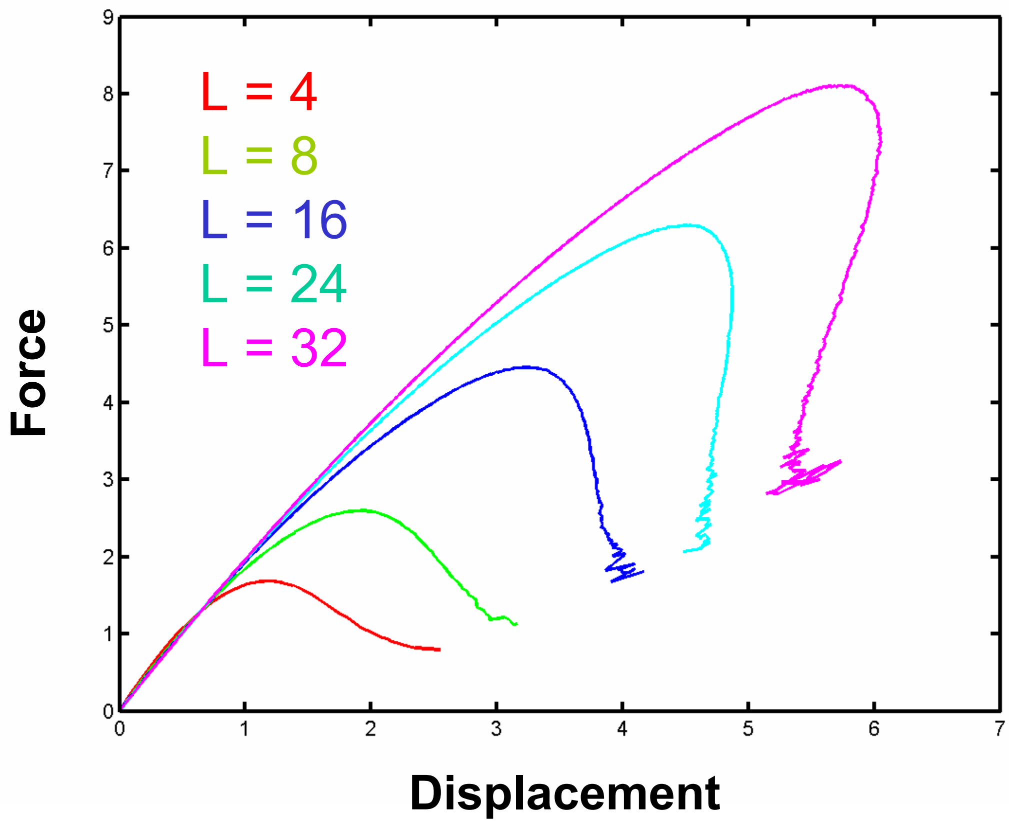
Force

Displacement

Single lattice response
Lattice response averaged over 5000 samples

In the hardening regime, average material response is obtained with fewer number of samples, whereas in the softening regime, averaging over many number of samples is required to obtain a representative material response

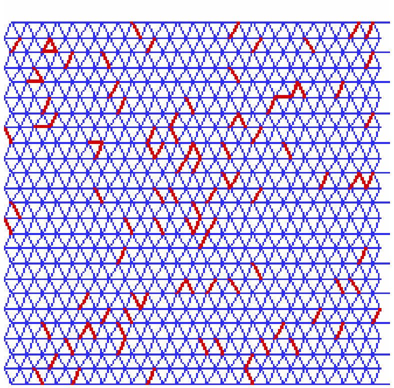
Lattice Response versus System Size



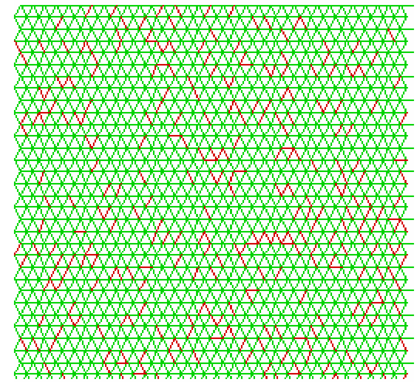
- Lattice response depends on the system size
- Scaling laws are required to obtain a "normalized" response that couples the mesoscopic scale response to the continuum scale response

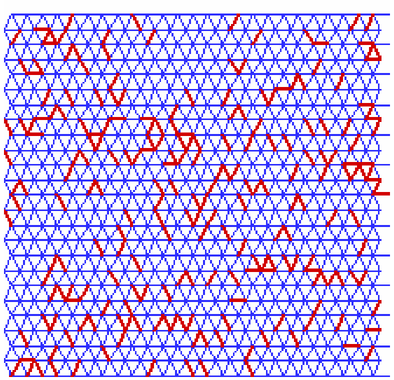
Numerical Results

What is the Intensive Measure of Damage?

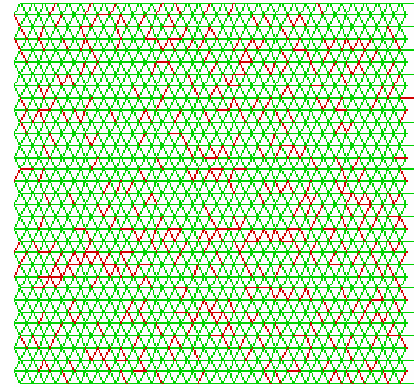


Nucleation Phase:
Diffusive Damage

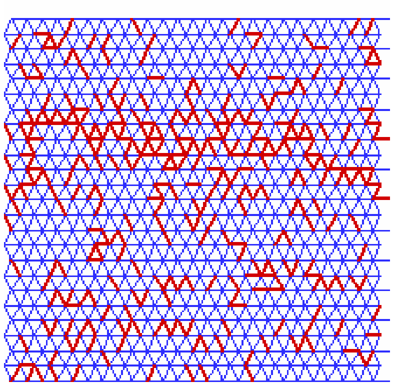




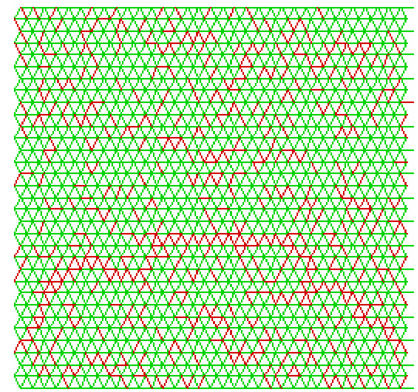
Growth Phase:
Stress Concentration effects are dominant



L = 24



Coalescence:
Localization of damage to form a percolating crack

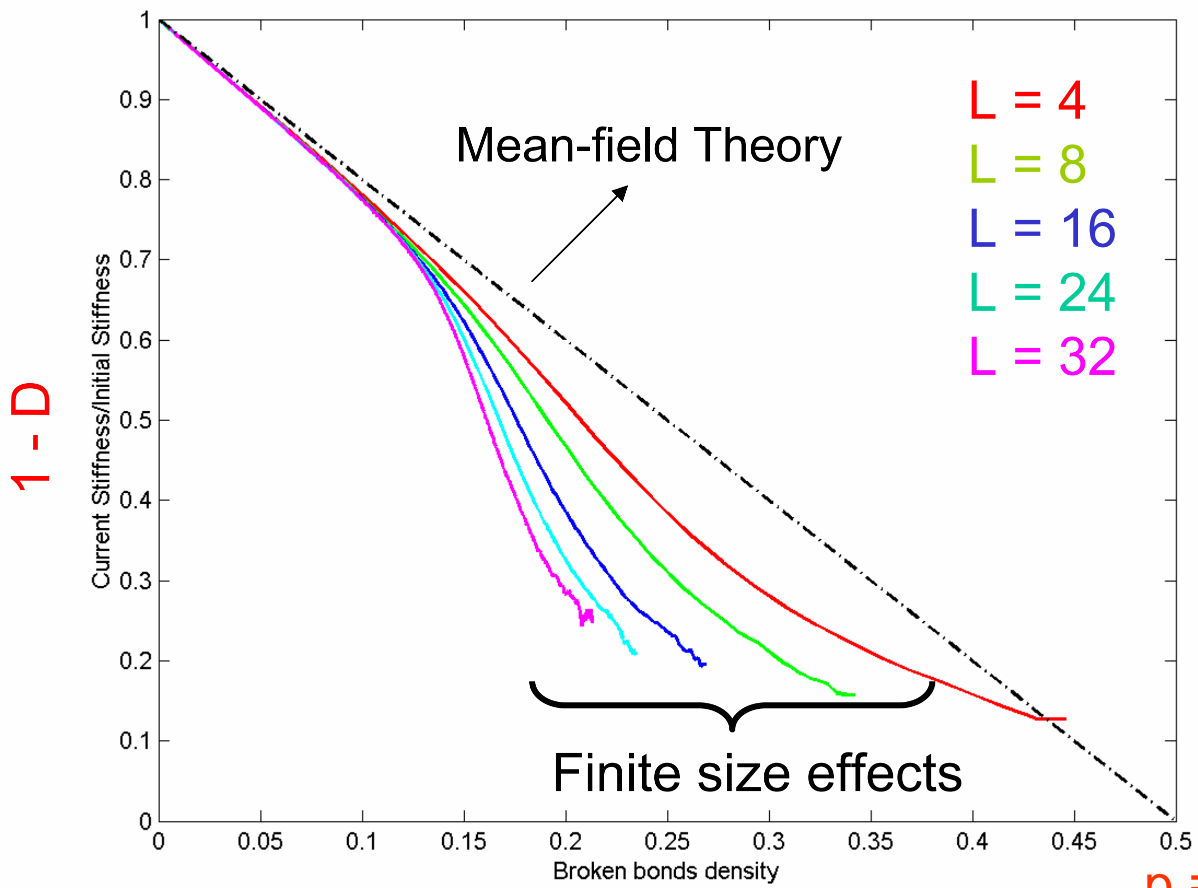


L = 32

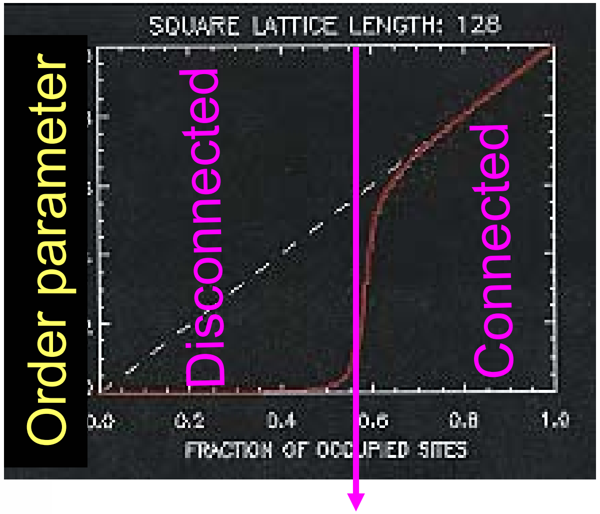
What is the intensive damage variable in the problem?

$$\text{Damage Variable} = \left\{ 1 - \frac{\text{Current Stiffness}}{\text{Initial Stiffness}} \right\} \longrightarrow$$

Close to being intensive!



$$p = \frac{n_b}{L^d}$$



Scaling Law

$$\mathbf{p_{cL} - p_c = cL^{-\alpha}}$$

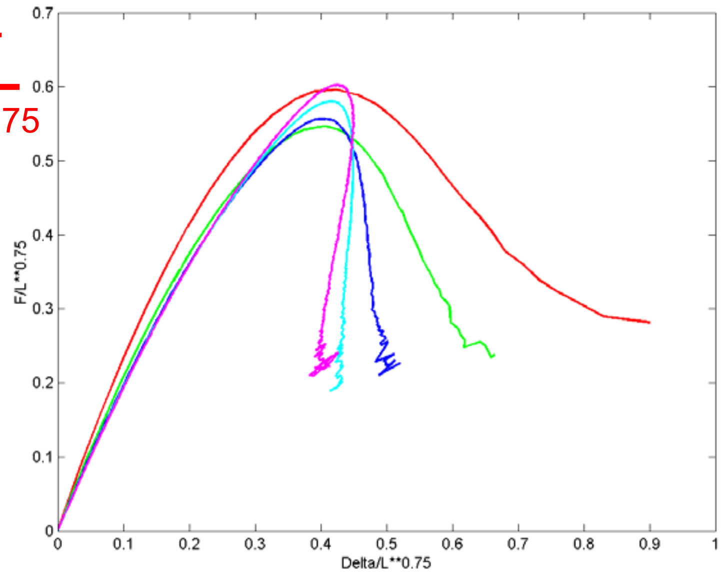
non-zero p_c indicates critical crack size needed for macroscopic fracture

Scaling Laws for Lattice Response

Scaling proposed in this study

Scaling proposed in the literature

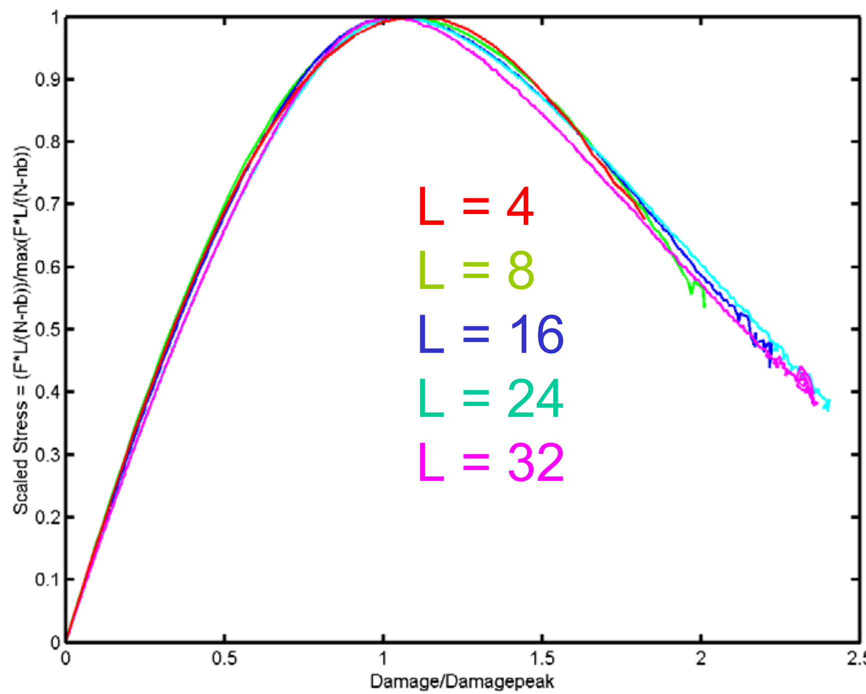
$\frac{F}{L^{0.75}}$



$\frac{\Delta}{L^{0.75}}$

Valid in the hardening regime only

Scaled Stress



D/D_{peak}

Scaling is valid until fracture!

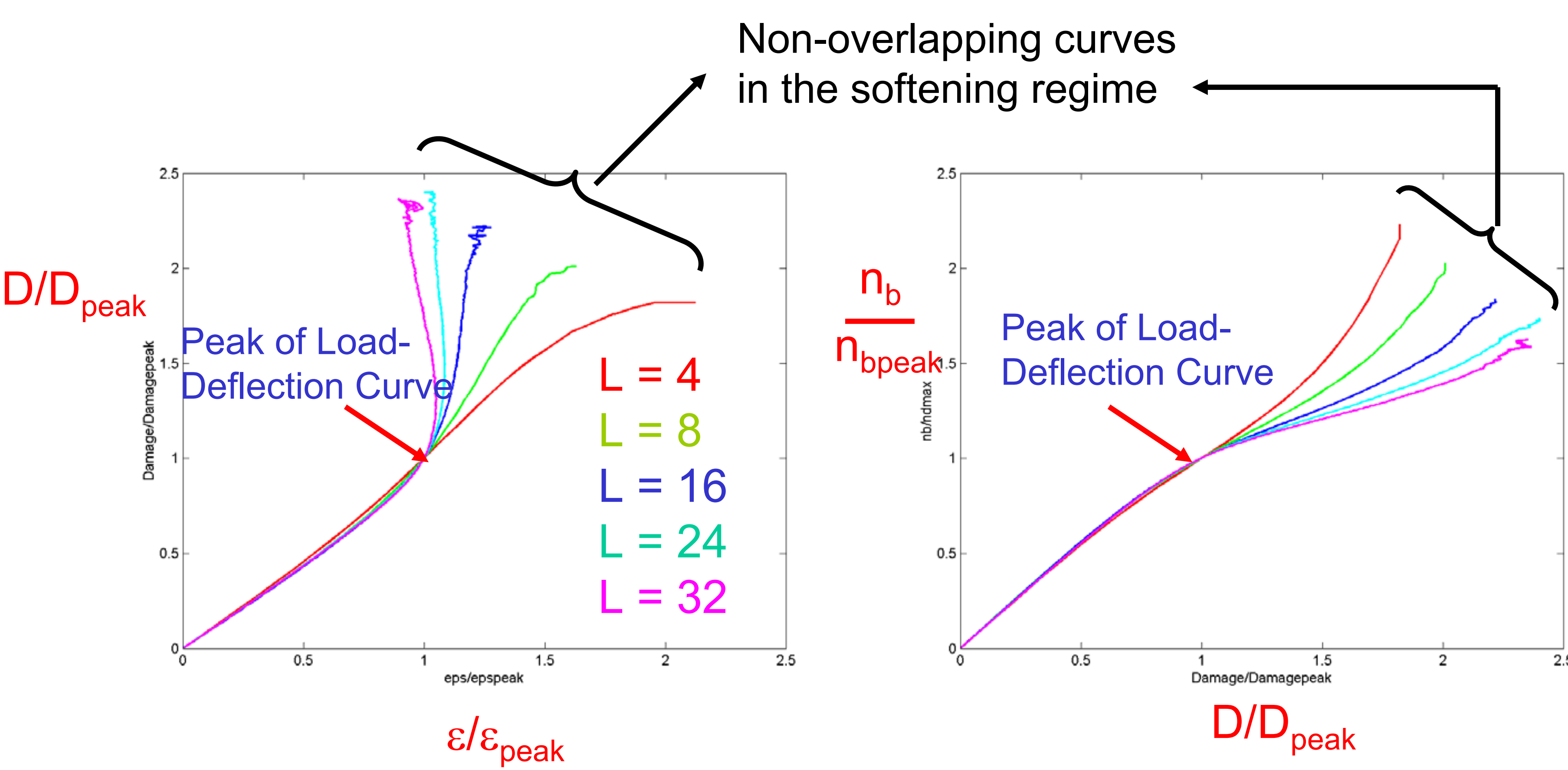
Proposed Scaling Law:

$$\frac{D}{D_{peak}} = \varphi\left(\frac{\sigma}{\sigma_{peak}}\right)$$

$D_{peak} = 0.633\ L^{-0.2}$

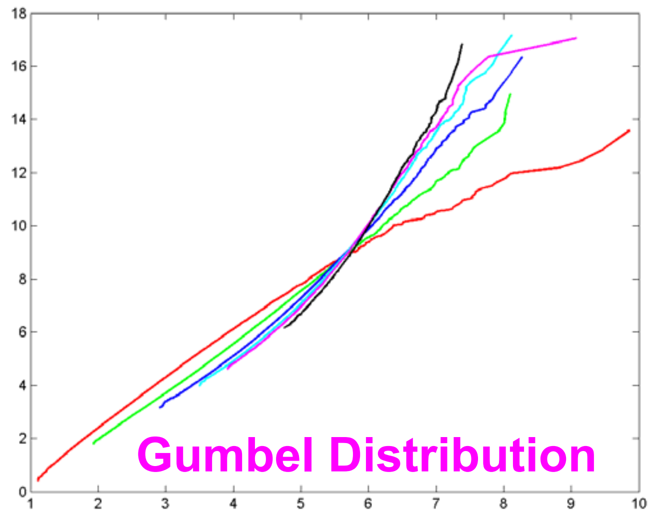
$\sigma_{peak} = 0.2605 + 1.0649 / L$

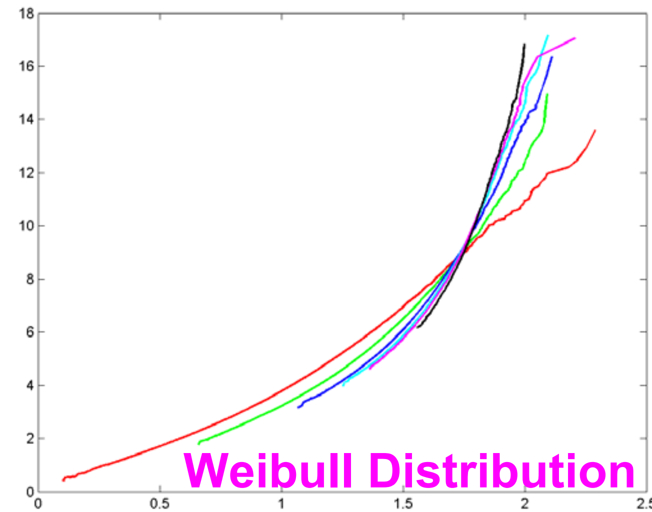
Scaling Laws for Lattice Strain

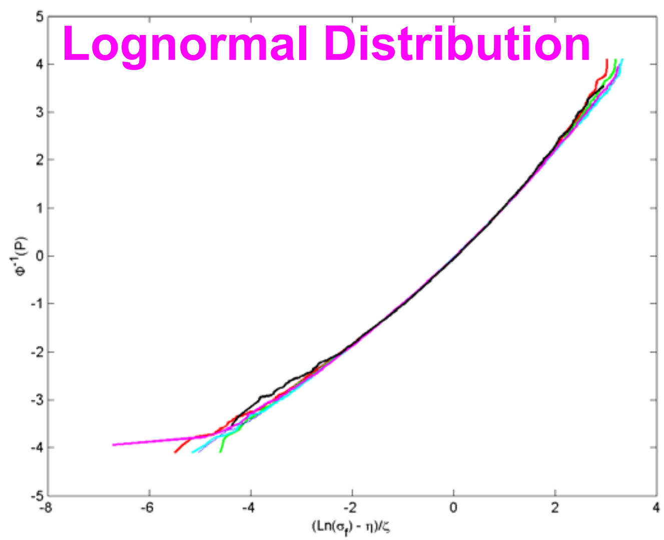


- Strain and broken bond density are not good measures in the softening regime
- However, lattice force is a good measure of damage over the entire range

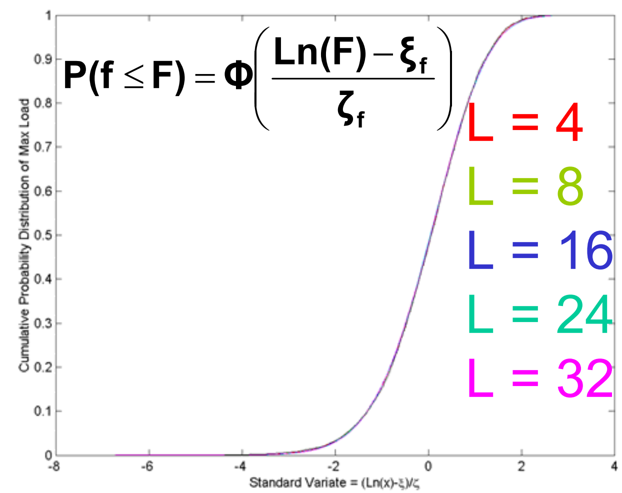
Scaling of Failure Load Distribution







- **Cumulative Probability Scaling Law**

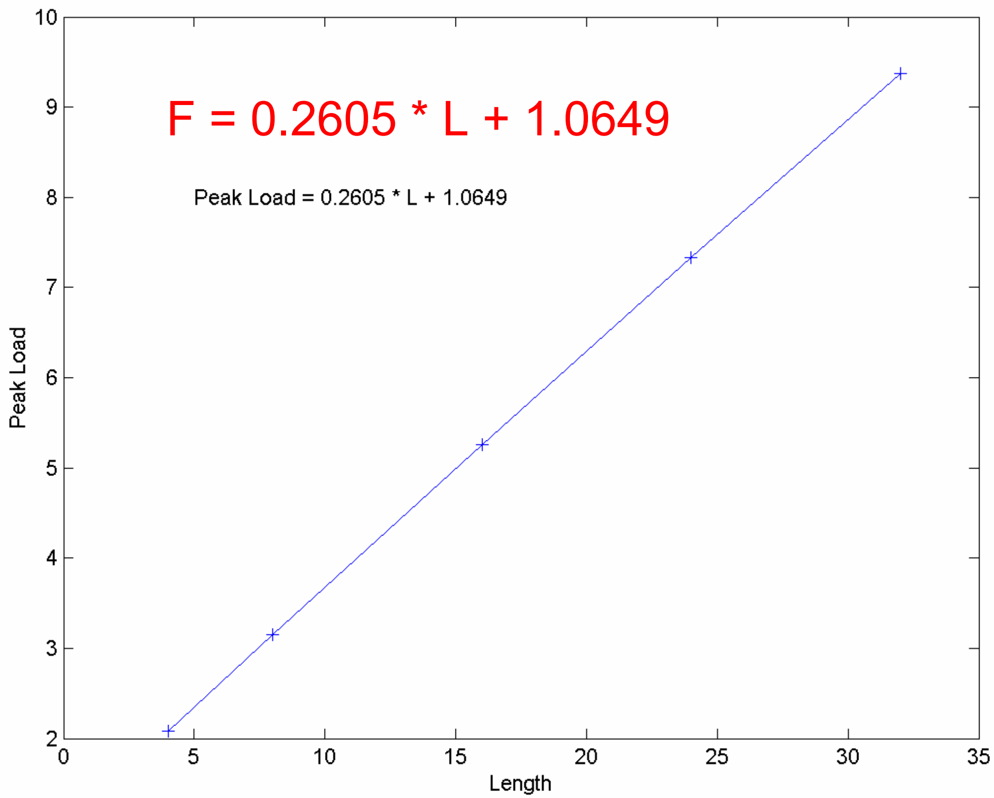


Standard Variate = $(Ln(F)-\xi)/\zeta$

Size Effect on the Mean Failure Load

- **Mean failure stress is inversely dependent on size**

Peak Load

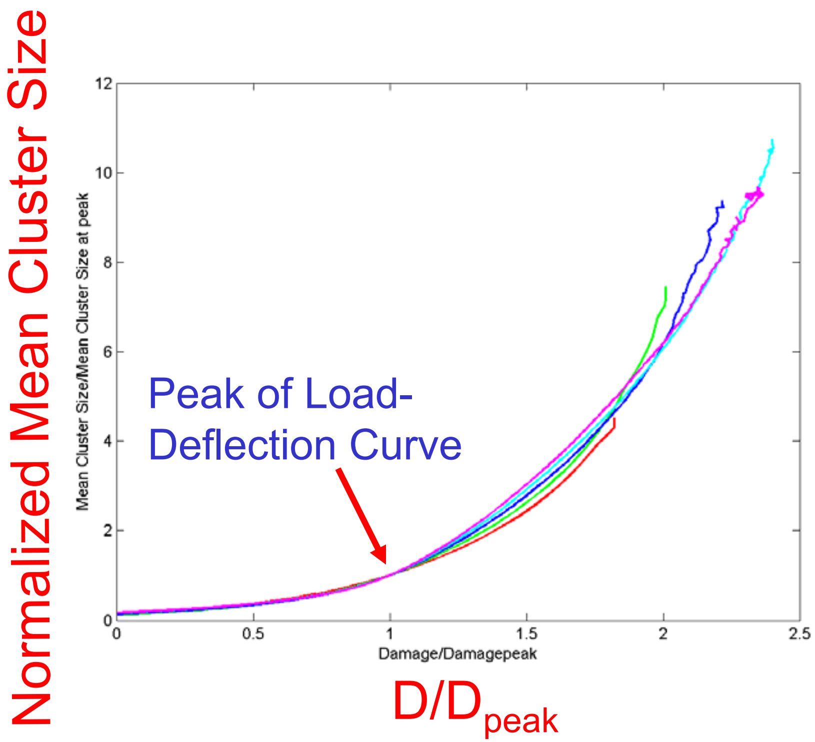


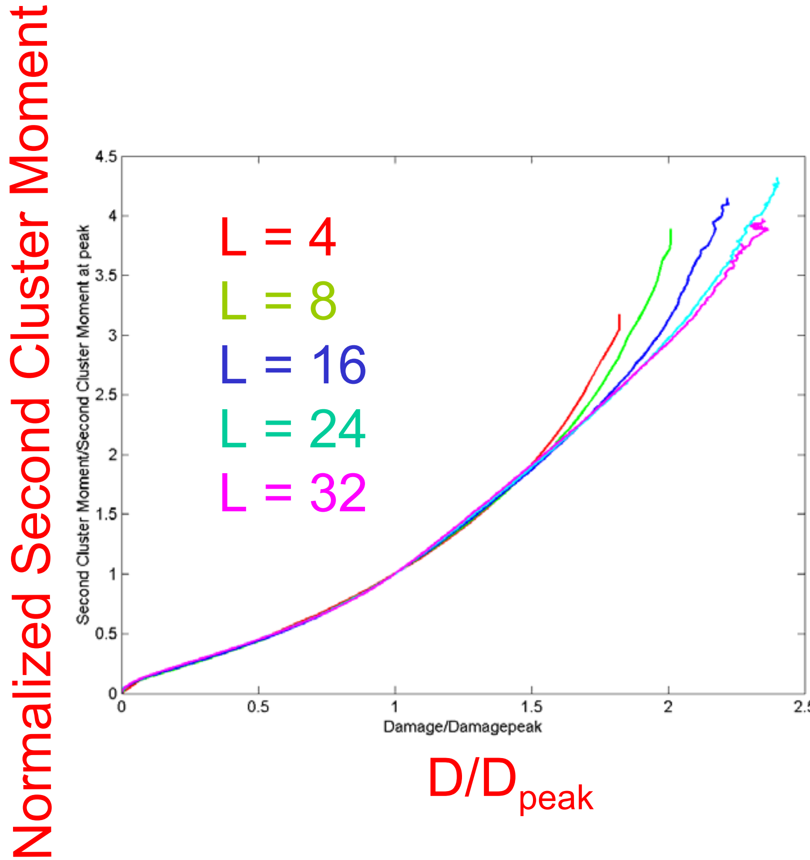
System Size

$$\sigma_{peak} = 0.2605 + \frac{1.0649}{L}$$

Griffith's crack driving force necessary for macrocrack propagation

Geometric Significance of Damage Variable





Second Cluster Moment

Damage

Mean Cluster Size

Correlation Length
-Measures correlation of statistical fluctuations in stress

Computing Requirements

Lattice Size (L)	CPU Time (minutes) Standard Algorithm	CPU Time (minutes) New Algorithm
4	0.008	0.003
8	0.012	0.005
16	0.032	0.014
24	0.12	0.05
32	0.3	0.15
64	5.1	2.5
128	185	51

CPU Time (minutes) = $9.036 * 10^{-8} \; L^{4.14}$

For L = 1000 ⟹ Time ~ 166 days!

Mesoscopic simulations require $O(L^4)$ cpu time for 2D and $O(L^6)$ for 3D, where "La" is the specimen size and "a" is the average grain size

⟹ Computationally intractable using serial versions

⇓

Parallel implementation on multiple processors using domain decomposition techniques and parallel solvers is **essential**

Meso to Macro: Preliminary Results

Mesoscopic to Continuum Damage Evolution

Uniaxial Case:

$$F = K_0\left(1-D\right)\Delta$$

Scaling Law:

$$\frac{D}{D_{peak}} = \varphi\left(\frac{\sigma}{\sigma_{peak}}\right)$$

$$D_{peak} = 0.633\ L^{-0.2}$$

$$\boldsymbol{\sigma_{peak} = 0.2605 + \frac{1.0649}{L}}$$

Damage Evolution:

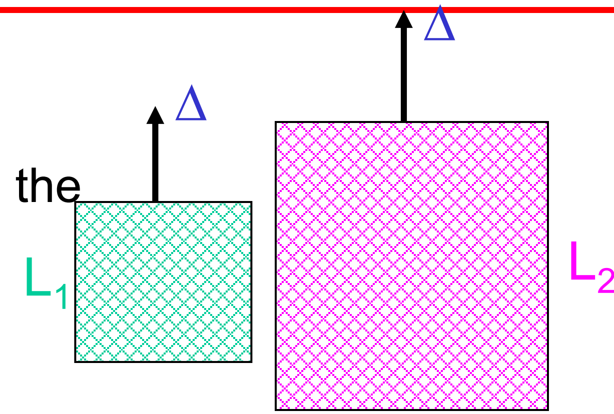
$$\dot{D} = \frac{D_{peak}}{\sigma_{peak}}\varphi'\left(\frac{\sigma}{\sigma_{peak}}\right)\dot{\sigma}$$

→ Similar to plastic strain evolution in ductile materials

Includes scaling and size effects

Given: F and L → **Compute:** σ, σ_{peak}, D_{peak} → **Estimate:** D from scaling law → **Compute:** Δ

Comparison of Damage within Different Specimens



Given:

Two different specimens of size L_1 and L_2 and the lattice forces F_1 and F_2 respectively

Compute:

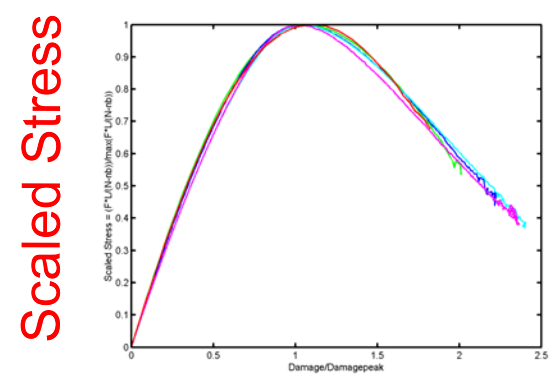
- Scaled stresses σ_1 and σ_2 corresponding to F_1 and F_2
- D_{1peak} and D_{2peak} based on L_1 and L_2

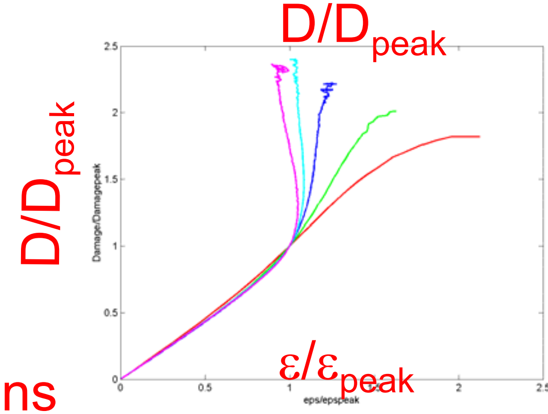
Estimate:

Damage D_1 and D_2 within the specimens based on σ_1 and σ_2 using the scaling law for lattice forces

Compute:

Strains ε_1 and ε_2 within the specimens based on D_1 and D_2 using the scaling law for lattice strains





Damage estimates based on stresses exhibit excellent scaling compared to those based on strains

Summary: Scaling Laws and Size Effect in Brittle Materials

Scaling Laws

- Allow for comparison of damage between specimens of different sizes
- Couple mesoscopic and continuum damage evolution

Size Effect on Mean Failure Stress

- Mean peak stress is inversely dependent on lattice size
- CDF of peak load follows lognormal and exhibits excellent scaling with system size

Field induced percolation provides the necessary framework for developing damage evolution scaling laws in brittle materials

Mesoscopic Simulation: Discrete versus Continuum Models

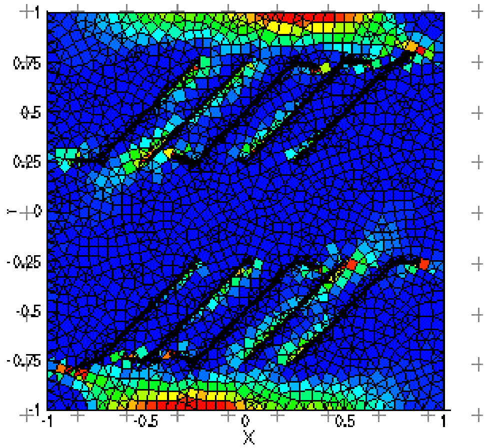
Discrete Lattice Models

- Suitable for studying the behavior of complex microstructures with heterogeneities
- Captures crack propagation and microstructure evolution with relative ease
- Ideal for studying statistical behavior including scaling and size effects
- Not readily applicable for capturing plasticity dominated phenomena

Continuum Models

- Suitable for studying the behavior of homogeneous solids
- Mesh size should be much smaller than typical inhomogeneity (crack, grain) size
- Captures inter-granular cracks using cohesive laws
- Not readily applicable for large number of heterogeneities
- Recent investigations on extended FE methods show promise in capturing inter- and trans-granular cracks and their interaction

Extended FE Models:
Multiple cracks, growth, interaction and coalescence



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