Scaling Laws for Mesoscopic Damage Evolution

Phani Kumar V.V. Nukala and Srdjan Simunovic

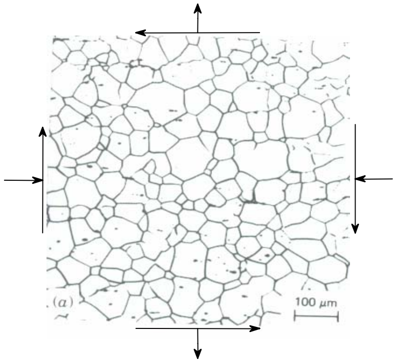
Oak Ridge National Laboratory

Computer Science and Mathematics Division

Motivation: Stress Induced Microcracking Evolution

Macroscopic properties and behavior of quasi-brittle materials are significantly affected by the internal microstructure and damage/microcracking evolution

Stress Field



Microcracking evolution

Controlling of microstructure state and damage evolution leads to improved macroscopic behavior

Modeling at the mesoscale will lead to a fundamental understanding of the effect of microstructural features on the microcracking evolution in brittle materials

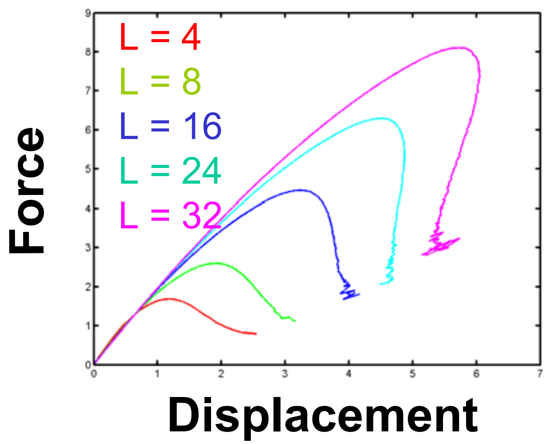
Objective

Mesoscale Damage Evolution:

- depends on the system size L
- computationally intractable $\approx O(L^4)$

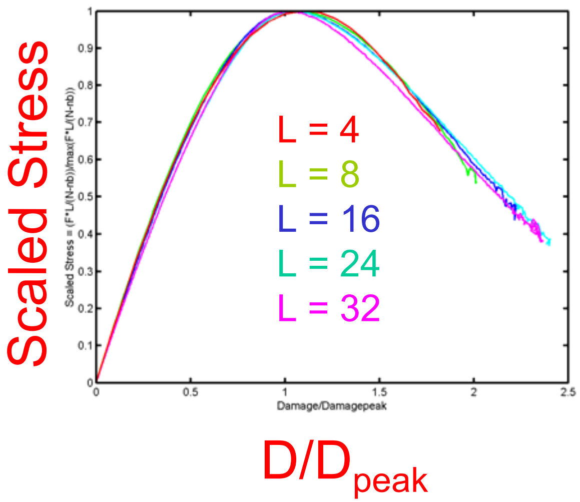
Relevant Questions:

- size effect on failure
- intensive measure of damage comparing the extent of damage between two specimens?



Material response is size dependent

Scaling laws are required to obtain a "normalized" response that couple mesoscopic and continuum length scales



Explicit modeling of material microstructure combined with the scaling theory accounts for size effects and local stress field interactions during damage/microcrack evolution

Mesoscopic Simulation: Discrete Lattice Models

Focus of the study is not on any particular material

But, in capturing the generic features of damage evolution

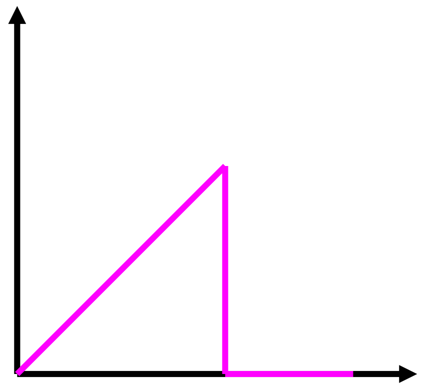
Essential ingredients of breakage process:

- Initial material disorder (inhomogeneities)
- redistribution of stresses due to damage evolution

Discrete Lattice Models:

- disorder in bond strength and stiffness
- elastic response characteristics of the bonds
- bond breaking rule (failure criteria)

Perfectly brittle bond

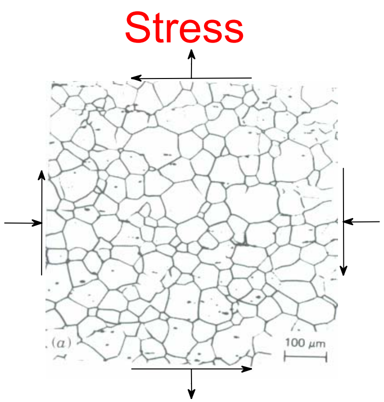


Any realistic damage evolution description must be capable of reproducing the behavior of these idealized discrete lattice models

Mesoscopic Modeling Approach

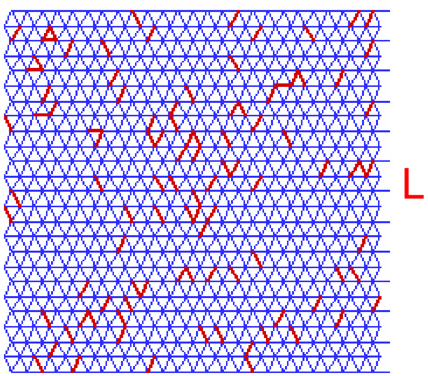
Failure of a bond is governed by

- weakest bond of the disordered medium
- stress concentration around material inhomogenities



Discretization with random disorder distributions

Lattice system with disorder



Disorder Type

+

Lattice Topology

+

Lattice Bond Model

Applied Stress

Failure Criteria

Mesoscopic Damage Evolution

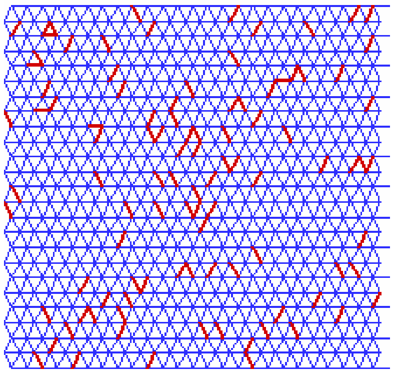
Analysis Procedure

Procedure:

Step 0: For each bond in the lattice system, assign unit stiffness and random force threshold f_i^{th}

Step 1: Impose a unit macroscopic displacement

Step 2: Calculate the force f_i in each bond through lattice equilibrium



Lattice system with disorder

$$\mathbf{K} = \sum_i f_i^2$$

K Global stiffness

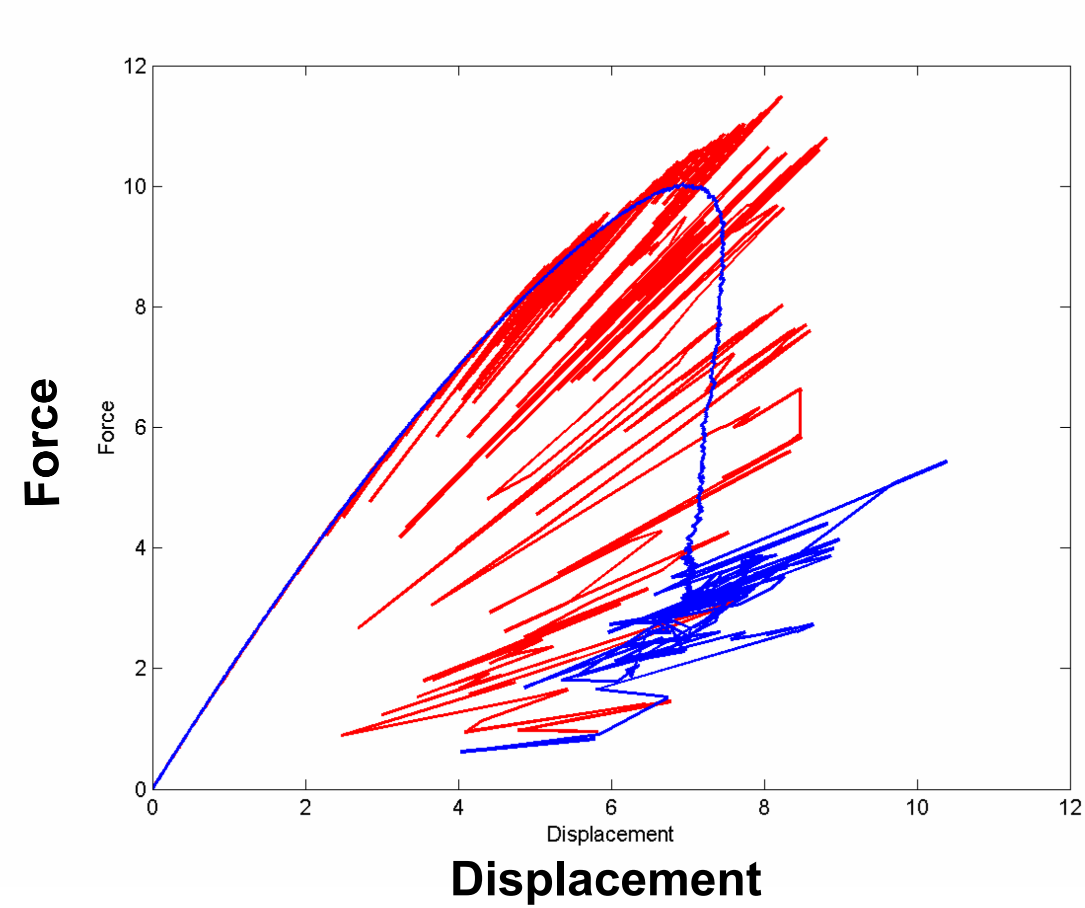
Step 3: Determine the bond i_c for which

$$\frac{1}{\lambda} = \max_i \left(\frac{f_i}{f_i^{th}} \right)$$

Step 4: Record the lattice displacement and force $(\lambda, K\lambda)$

Step 5: Remove the bond i_c and repeat steps 1-4, until the entire lattice system breaks apart

Typical Loading Response

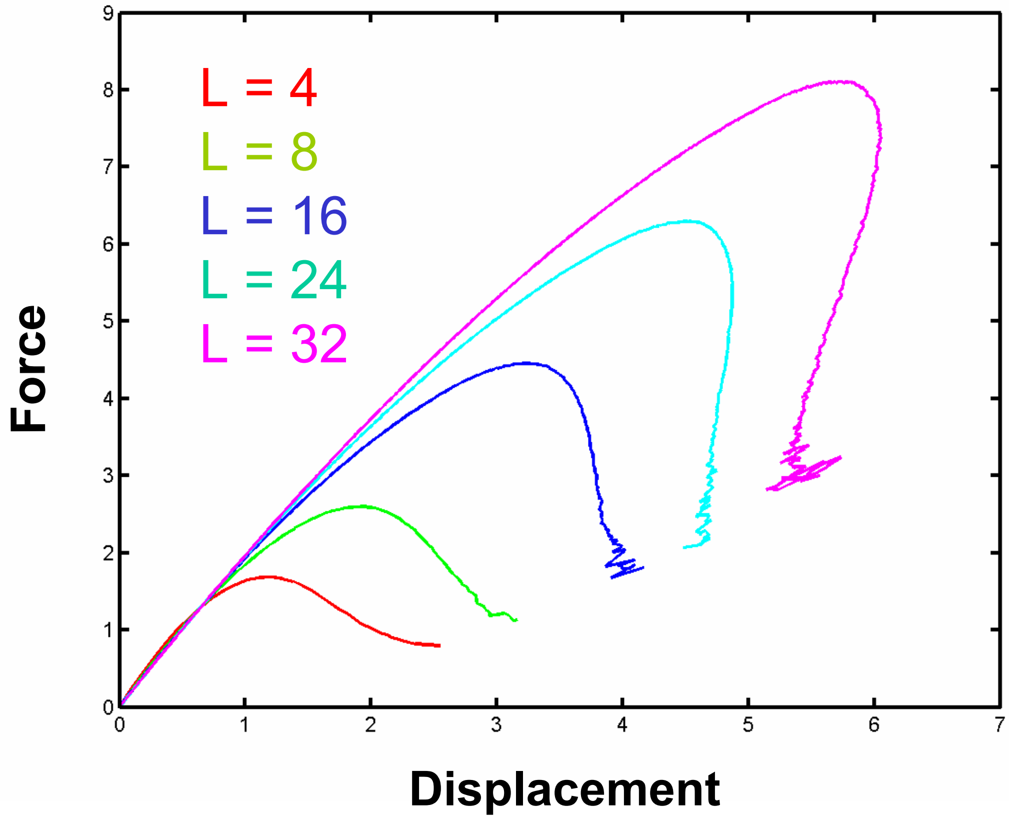


Single lattice response

Lattice response averaged over 5000 samples

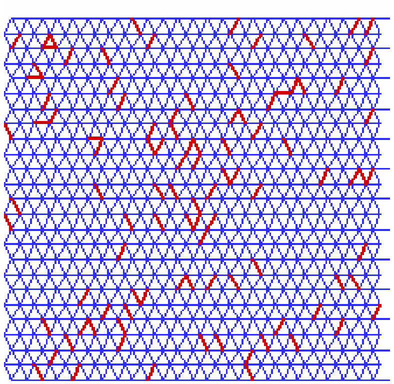
In the hardening regime, average material response is obtained with fewer number of samples, whereas in the softening regime, averaging over many number of samples is required to obtain a representative material response

Lattice Response versus System Size

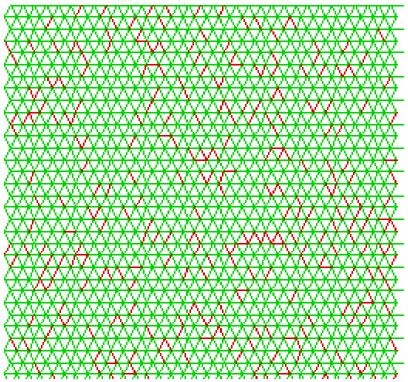


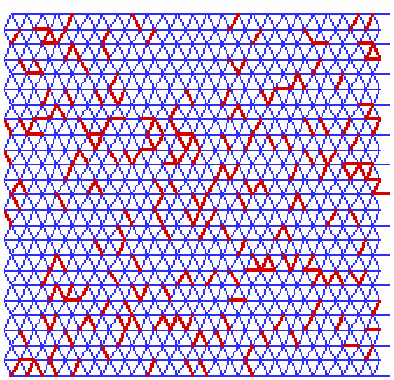
- Lattice response depends on the system size
- Scaling laws are required to obtain a “normalized” response that couples the mesoscopic scale response to the continuum scale response

What is the Intensive Measure of Damage?

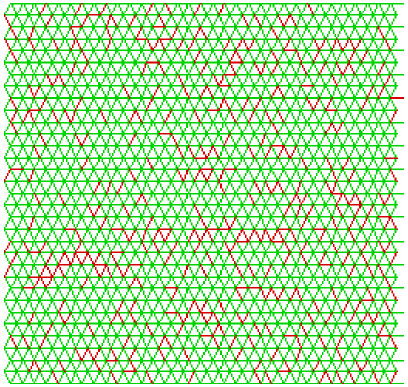


Nucleation Phase:
Diffusive Damage

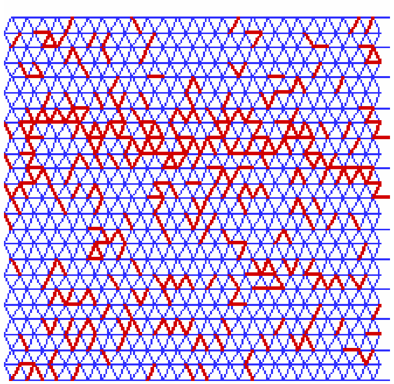




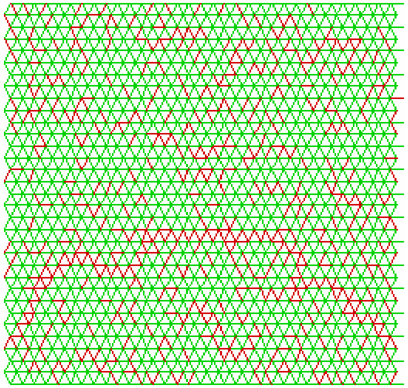
Growth Phase:
Stress Concentration effects are dominant



L = 24



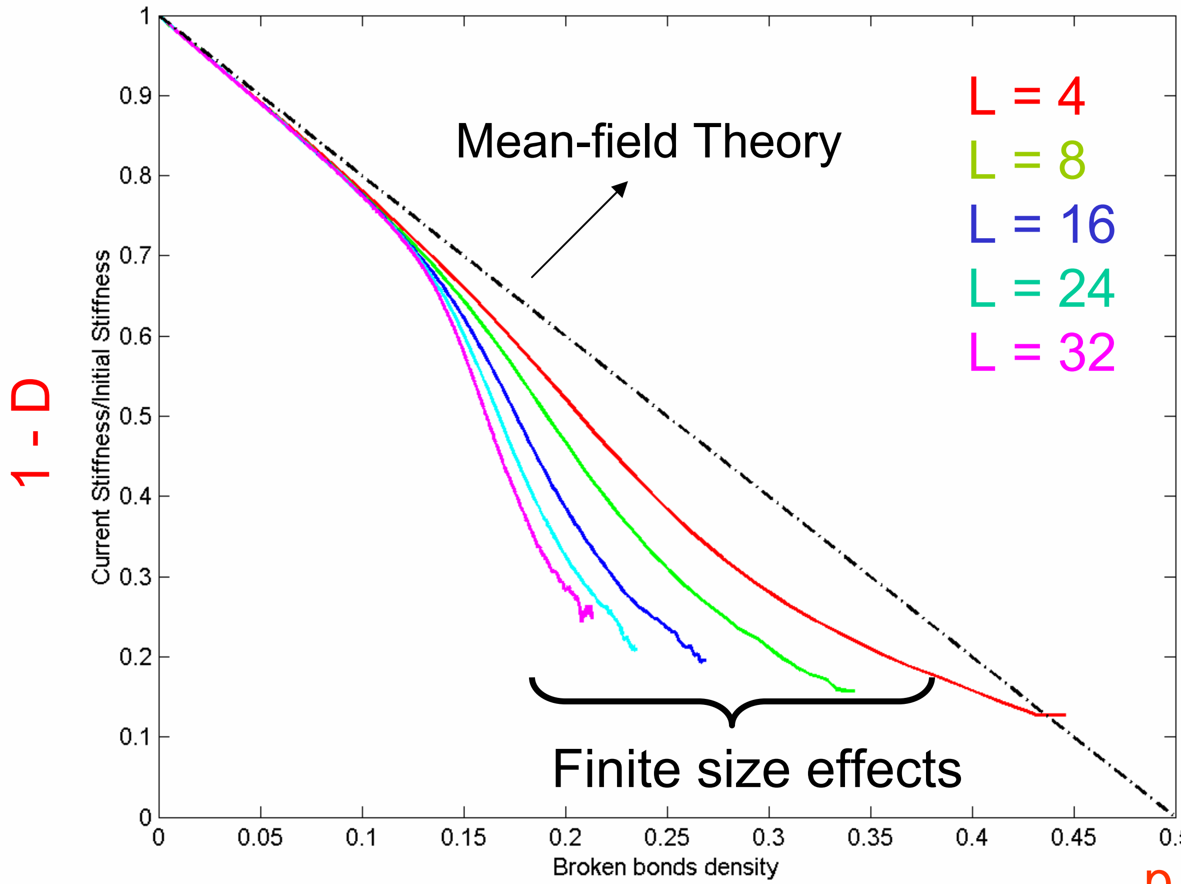
Coalescence:
Localization of damage to form a percolating crack



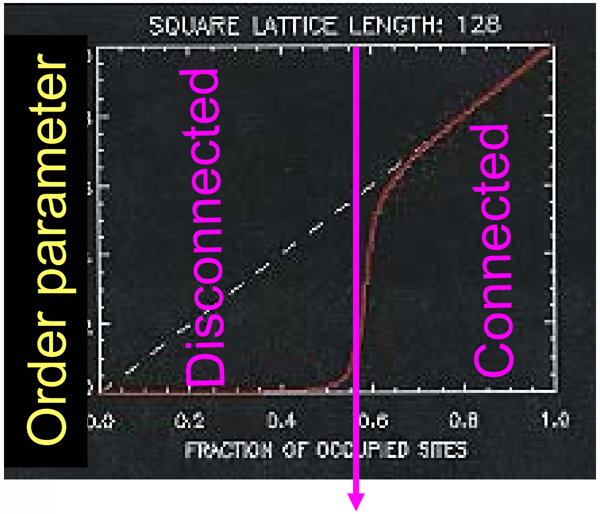
L = 32

Damage Definition Based on Stiffness Degradation

$$\text{Damage Variable} = \left\{ 1 - \frac{\text{Current Stiffness}}{\text{Initial Stiffness}} \right\}$$ → Close to being intensive!



$p = \frac{n_b}{L^d}$

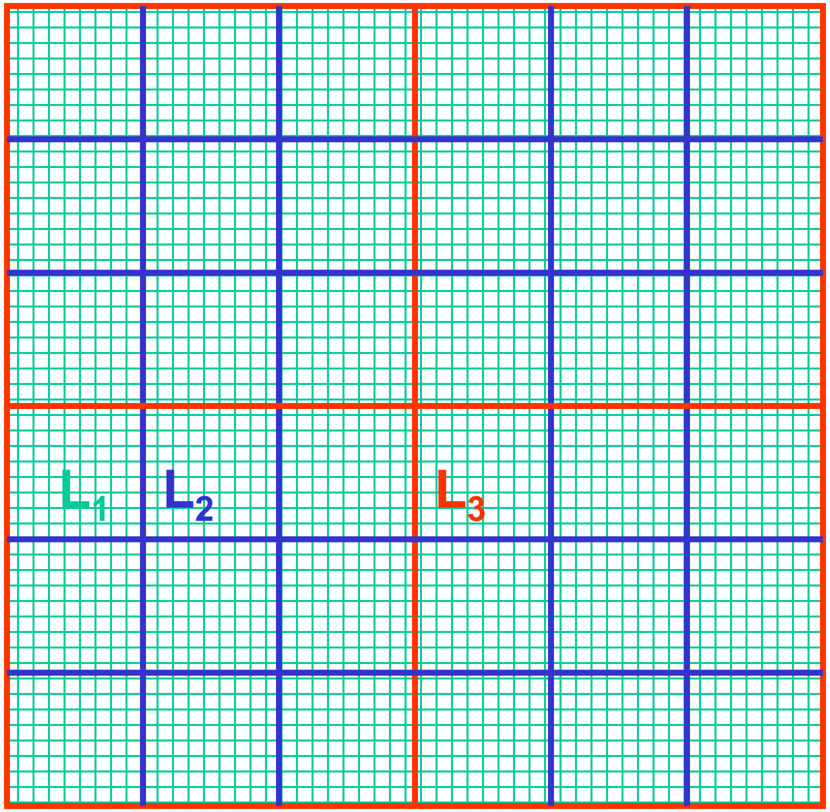


Renormalization Approach for Scaling

Let $L_1 < L_2 < L_3$

p fraction of broken bonds at scale L_1

p' fraction of broken bonds at scale L_2



Coarse Graining*

$$p' = R_l(p) \quad \text{where} \quad l = \frac{L_2}{L_1}$$

$$p' - p_\infty = R_l(p) - R_l(p_\infty)$$

$$= \left.\frac{\partial R_l}{\partial p}\right|_{p=p_\infty} (p - p_\infty)$$

$$= \Lambda_l (p - p_\infty)$$

where $\Lambda_l = l^{-\alpha}$ ⟶ $p_L - p_\infty = c_\infty L^{-\alpha}$

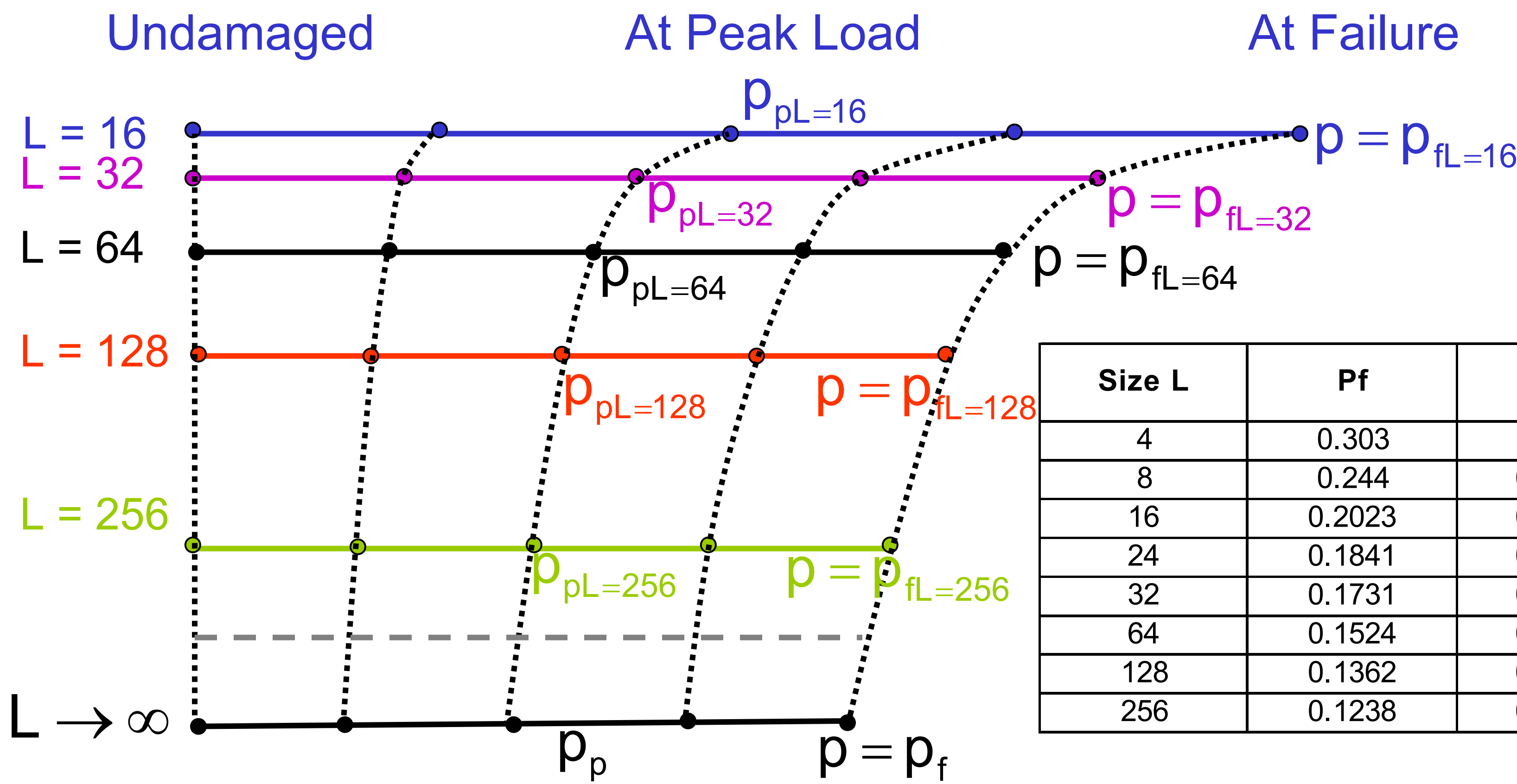
(* such that the probability of failure under the influence of external load remains the same at both scales)

non-zero p_∞ indicates critical crack size needed for macroscopic fracture

Intensive Definition of Damage

$$p_L - p_\infty = c_\infty L^{-\alpha} \quad \Longrightarrow \quad n_L = N_{el}\left(p_\infty + c_\infty L^{-\alpha}\right)$$

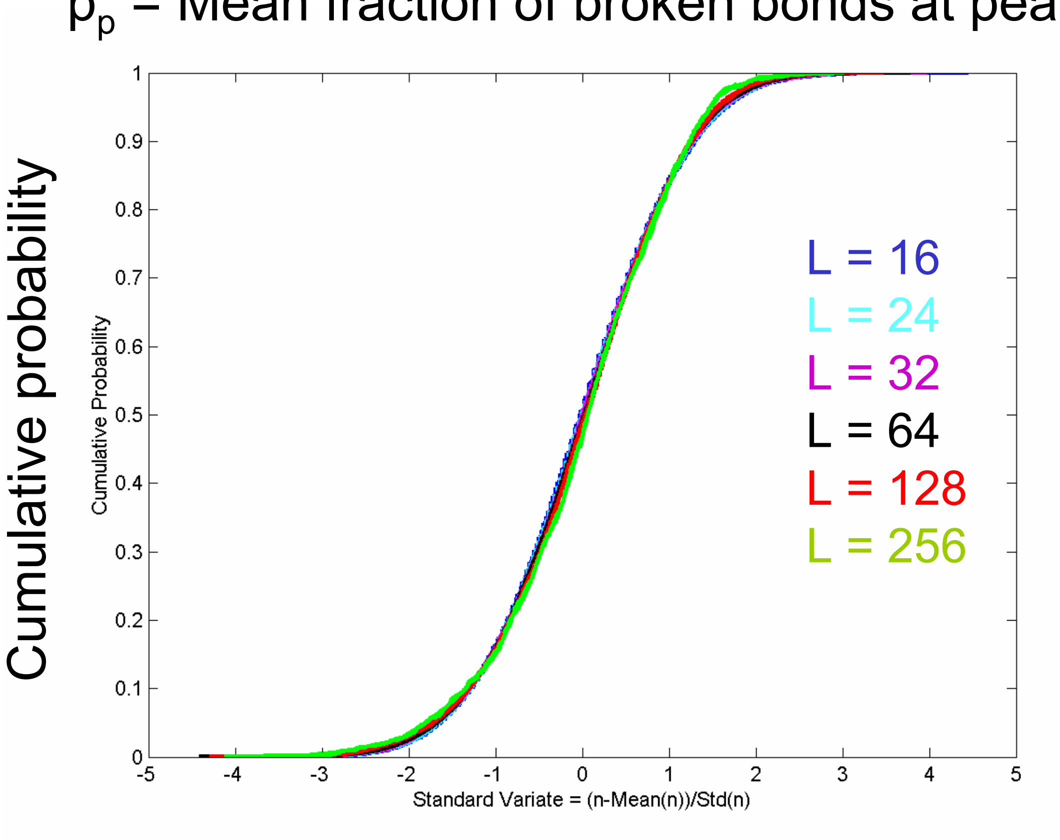
$$c_\infty = c(p_\infty)$$



Size L	Pf	Pp
4	0.303	0.207
8	0.244	0.1813
16	0.2023	0.1612
24	0.1841	0.1513
32	0.1731	0.1451
64	0.1524	0.1325
128	0.1362	0.1222
256	0.1238	0.1142

Probability Distributions for Fraction of Broken Bonds at Failure and at Peak Load

p_f = Mean fraction of broken bonds at failure
p_p = Mean fraction of broken bonds at peak load



Standard variate = (p – Mean(p)) / Std(p)

Probability distributions for fraction of broken bonds at failure as well as at peak load are identical

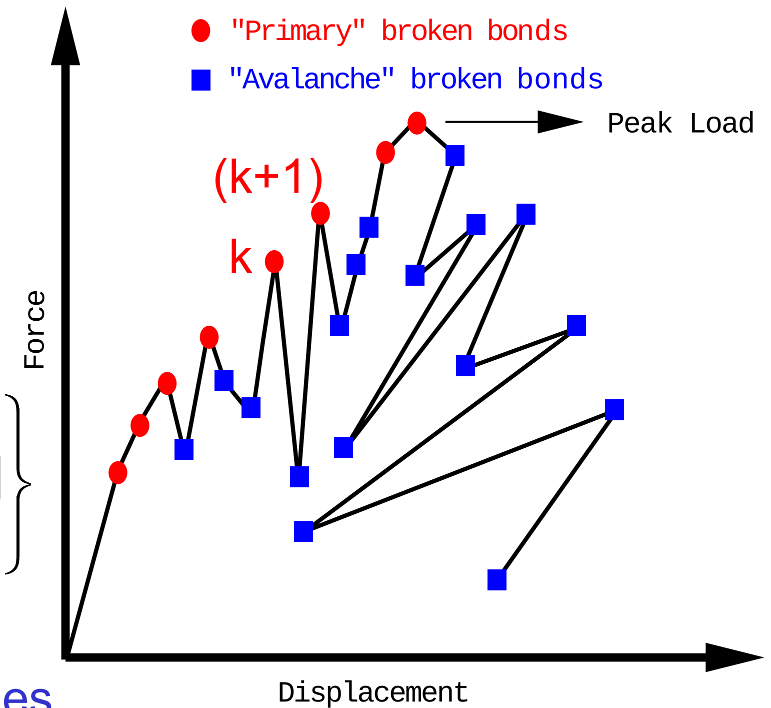
Failure Load Distribution

Let B_k = {set of k "primary" broken bonds}

Probability $f_{(k+1)}$ that $(k+1)^{th}$ "primary" bond fails

$$f_{(k+1)} = \prod_{j \in B_k} f_j$$

Define $\mathcal{G} = \left\{ g_j = \frac{\sigma_j}{\sigma_{(j-1)}} \quad \forall j \in B_k \text{ and } g_1 = 1 \right\}$

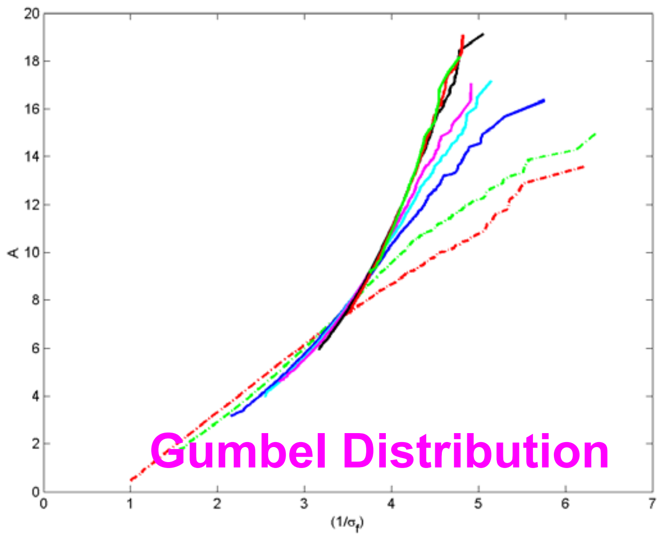


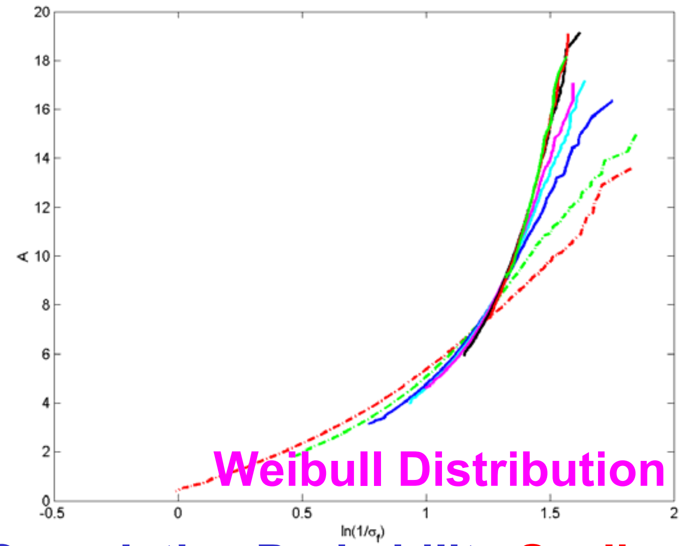
In the case of broadly distributed heterogeneities, the scale factors g_j become independent distributed random variables. (Since they depend not only on the stress concentration factors but also on the initial randomly distributed bond threshold values)

Stress $\sigma_{(k+1)}$ required to break $(k+1)^{th}$ "primary" bond $\sigma_{(k+1)} = \left(\prod_{j \in B_k} g_j \right) \sigma_1$

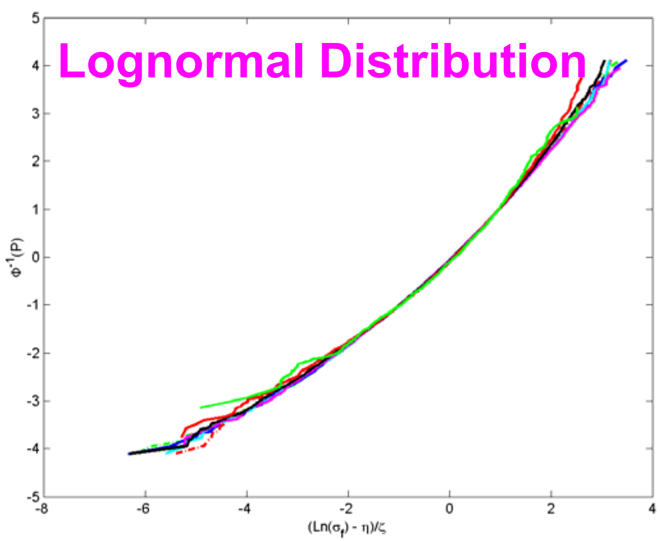
$$\Longrightarrow \quad \mathrm{Prob}\left[\sigma_{(k+1)} \le \sigma\right] \approx LN$$

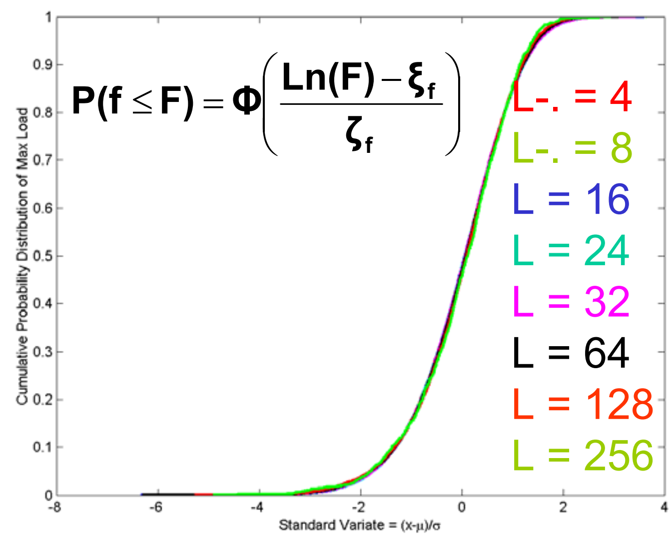
Scaling of Failure Load Distribution





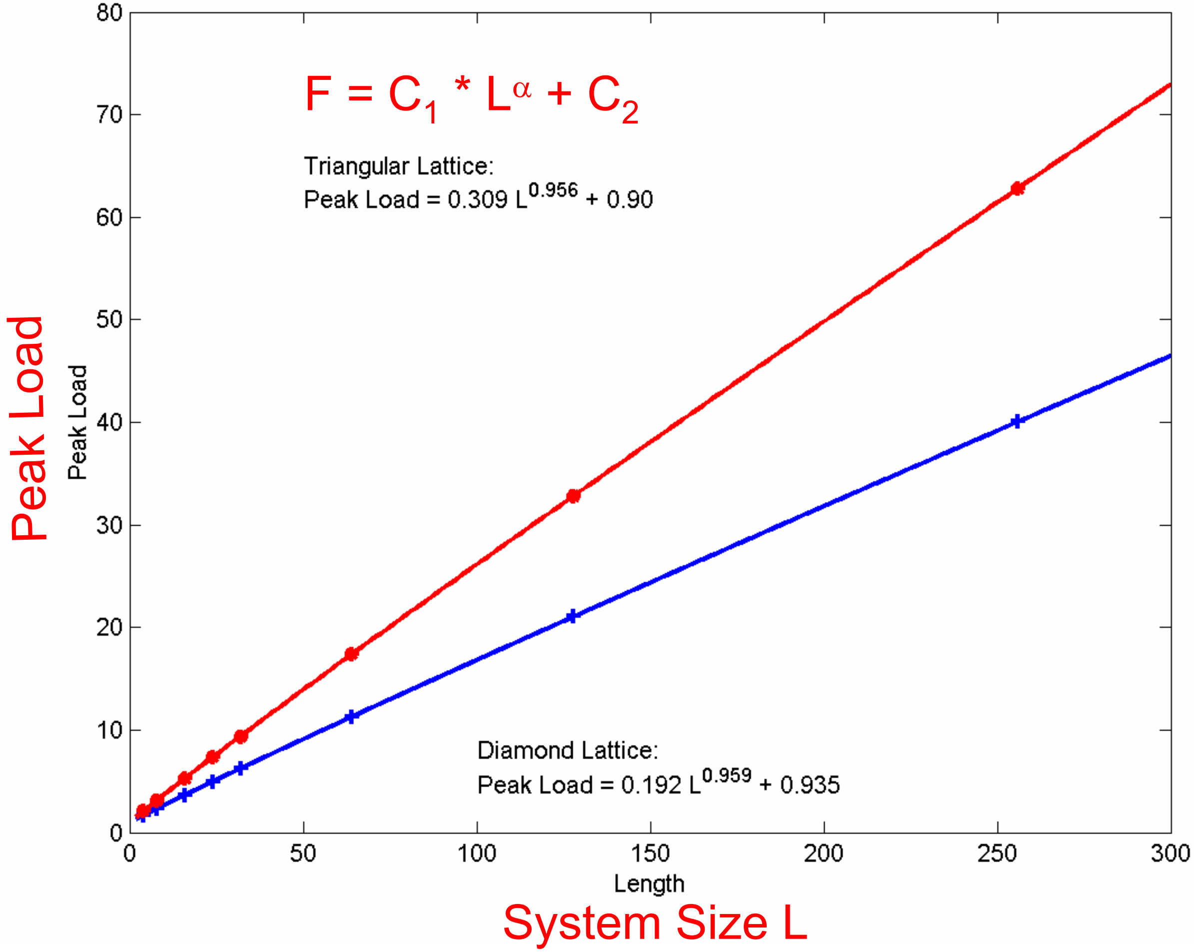
- **Cumulative Probability Scaling Law**





Standard Variate = (Ln(F)-ξ)/ζ

Size Effect on the Mean Failure Load



$$\sigma_{peak} = C_1 L^{(\alpha-1)} + \frac{C_2}{L}$$

Since

$$(1-\alpha) << 1 \Rightarrow L^{(1-\alpha)} \approx (\log(L))^{-\psi}$$

$$\sigma_{peak} = \frac{C_1}{(\log(L))^{\psi}} + \frac{C_2}{L}$$

Mean fracture strength decreases very slowly with increasing system size L, and scales as $\sigma_{peak} \approx \frac{C_1}{(\log(L))^{\psi}}$ for very large L

Computing Requirements

Lattice Size (L)	CPU Time (sec) PCG Algorithm	CPU Time (sec) New Algorithm
32	11.66	0.592
64	173.6	10.72
128	7473	212.2
256		5647
512		93779

CPU Time (sec) = $1.53 * 10^{-7} L^{4.36}$

For L = 1000 ⟹ Time ~ 21 days!

Mesoscopic simulations require $O(L^4)$ cpu time for 2D and $O(L^6)$ for 3D, where "La" is the specimen size and "a" is the average grain size

Summary

- For materials with broadly distributed heterogeneities, the fraction of the broken bonds is finite, even as the sample size becomes large, which is in contrast with the weakest-link case

- The scaling law for the fraction of broken bonds $p_L - p_\infty = c_\infty L^{-\alpha}$ avoids some of the inconsistencies associated with the conventional power law type expressions

- For materials with broadly distributed heterogeneities, a lognormal distribution represents the fracture strength more adequately than the Weibull and modified Gumbel distributions

- Mean fracture strength behaves as $\sigma_{peak} = \frac{C_1}{(\log(L))^{\psi}} + \frac{C_2}{L}$, and scales as $\sigma_{peak} \approx \frac{C_1}{(\log(L))^{\psi}}$ for very large L

Mesoscopic Simulation: Discrete versus Continuum Models

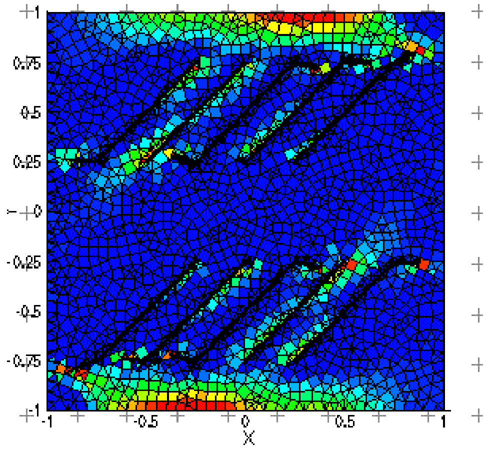
Discrete Lattice Models

- Suitable for studying the behavior of complex microstructures with heterogeneities
- Captures crack propagation and microstructure evolution with relative ease
- Ideal for studying statistical behavior including scaling and size effects
- Not readily applicable for capturing plasticity dominated phenomena

Continuum Models

- Suitable for studying the behavior of homogeneous solids
- Mesh size should be much smaller than typical inhomogeneity (crack, grain) size
- Captures inter-granular cracks using cohesive laws
- Not readily applicable for large number of heterogeneities
- Recent investigations on extended FE methods show promise in capturing inter- and trans-granular cracks and their interaction

Extended FE Models:
Multiple cracks, growth, interaction and coalescence



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