

# Scaling Laws for Mesoscopic Damage Evolution<sup>1</sup>

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## Abstract

Scaling laws derived from mesoscopic discrete lattice models are typically used for coupling the mesoscopic damage evolution with the continuum damage response and in determining the size effects on the constitutive response of materials. This study develops the scaling laws based on the Renormalization Group (RG) methodology for the number of bonds broken in a discrete lattice before the macroscopic fracture occurs. The developed scaling laws imply the existence of a finite critical crack cluster threshold, defined as a fraction of number of broken bonds to the total number of bonds, below which macroscopic fracture of an infinite system does not occur. The existence of a finite critical crack cluster threshold can be associated with a critical crack size, below which macroscopic fracture of a specimen does not occur.

Further, in materials with *broadly distributed* microscopic heterogeneities, the fracture strength distribution corresponding to the peak load of the material response does not follow the commonly used Weibull and (modified) Gumbel distributions. Instead, a *lognormal* distribution represents the fracture strength of the macroscopic system more adequately than the conventional Weibull and (modified) Gumbel distributions. Lognormal distribution arises naturally as a consequence of multiplicative nature of large number of random distributions representing the individual conditional probabilities of breaking of bonds leading up to the peak load. Numerical simulations based on two-dimensional triangular and diamond lattice topologies with increasing system sizes substantiate that a *lognormal* distribution represents an excellent fit for the fracture strength distribution at the peak load. The mean fracture strength decreases with increasing lattice system size as an affine transformation of  $1/L$ , and approaches a constant threshold value, as the lattice system size,  $L$  approaches infinity. This critical fracture threshold, below which macroscopic fracture of an infinite system does not occur may be associated with the driving force necessary for macroscopic crack propagation that ultimately leads to the failure of the lattice system.

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