Single-photon and Two-photon Wavepackets in Spontaneous Parametric Down-Conversion

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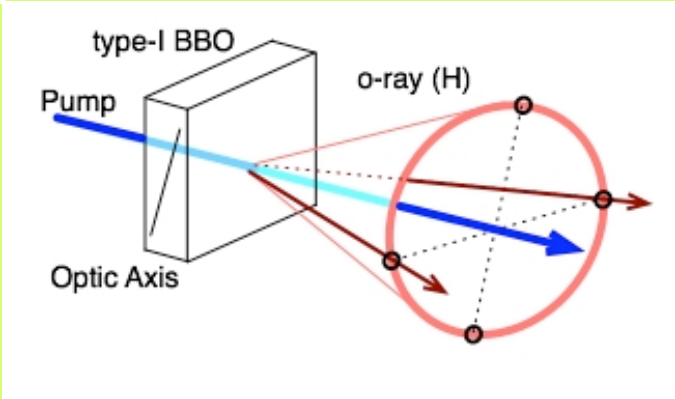
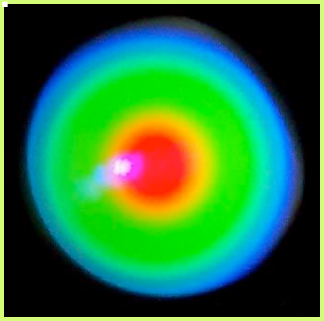
ORNL is managed for the US DOE by UT-Battelle, LLC, under contract no. DE-AC05-00OR22725

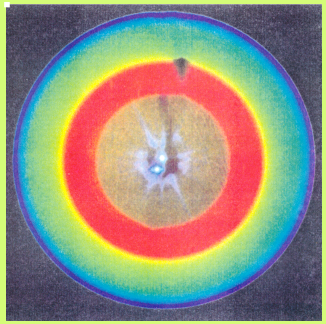
Spontaneous Parametric Down-Conversion (SPDC)

- Spontaneous decay of a pump photon into a pair of lower energy photons in a transparent non-centrosymmetric crystal
- Total energies and momenta are conserved (phase matching condition)
- Good source of non-classical (entangled) light

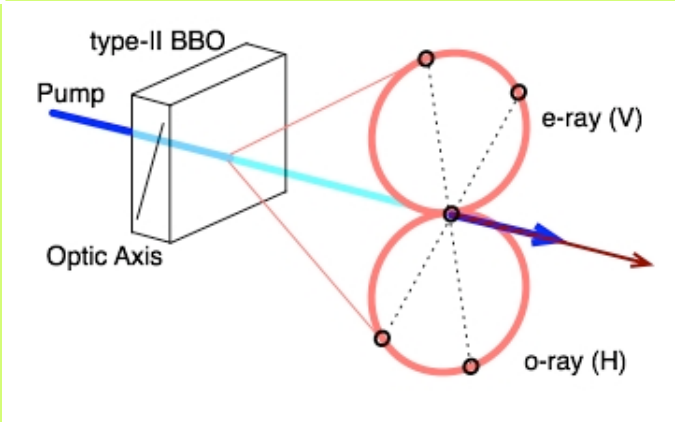
type-I phase matching: the photon pair has the same polarization:

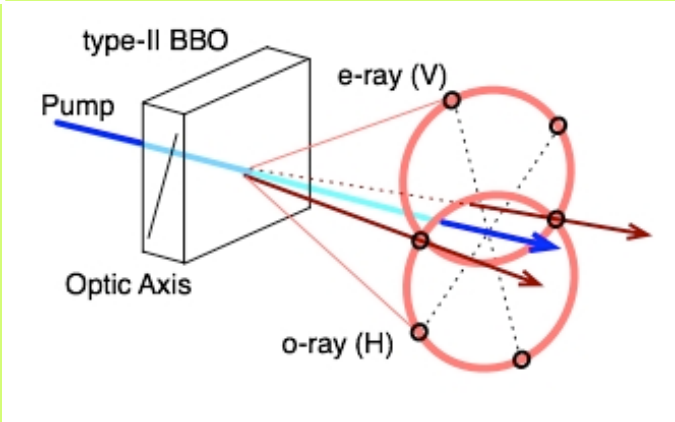






type-II phase matching: the photon pair is orthogonally polarized:





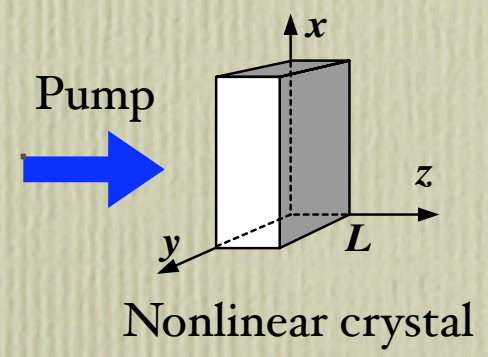
Quantum state of SPDC

Ideal case: assumes perfect phase matching and a monochromatic pump

$$|\psi\rangle = \sum_{s,i} \delta(\omega_s + \omega_i - \omega_p)\delta(\vec{k}_s + \vec{k}_i - \vec{k}_p) a_s^\dagger(\omega(\vec{k}_s)) a_i^\dagger(\omega(\vec{k}_i))|0\rangle$$

- photon pair is perfectly entangled in energy and momentum
- infinite two-photon coherence time

In reality, phase matching is not perfect even with a monochromatic pump



$$|\psi\rangle = -\frac{i}{\hbar}\int_{\infty}^{\infty} dt\, \mathcal{H}|0\rangle$$

$$\mathcal{H} = \epsilon_0 \chi^{(2)} \int_V d^3\vec{r}\, E_p^{(+)} E_s^{(-)} E_i^{(-)}$$

$$|\psi\rangle = \int_{-\infty}^{\infty} \mathcal{S}(\nu)\mathcal{P}(\nu) a_s^\dagger(\Omega_p/2 + \nu) a_i^\dagger(\Omega_p/2 - \nu)|0\rangle$$

$\mathcal{P}(\nu)$: phase term

$\omega_s + \omega_i = \Omega_p$

$\nu = \omega_s - \Omega_p/2$

Ω_p : pump frequency

$$D = \frac{1}{u_i(\Omega_p/2)} - \frac{1}{u_s(\Omega_p/2)}$$

$$D'' = \left.\frac{d^2k}{d\omega^2}\right|_{\omega=\Omega_p/2}$$

type-I SPDC: $\mathcal{S}_1(\nu) = \mathrm{sinc}\left(\frac{\nu^2 D'' L}{2}\right)$

type-II SPDC: $\mathcal{S}_2(\nu) = \mathrm{sinc}\left(\frac{\nu D L}{2}\right)$

Finite coherence time

Unique wavepacket

SPDC Correlation Function (Wavepacket)

1st order correlation function:

$$\hat{\rho}_s = tr_i[|\psi\rangle\langle\psi|]$$
$$= \int_{-\infty}^{\infty} d\nu\, |S(\nu)|^2\, a_s^\dagger(\Omega+\nu)|0\rangle\langle 0|a_s(\Omega+\nu)$$

using

$$G^{(1)}(\tau) = tr[\hat{\rho}_s E_s^{(-)}(t) E_s^{(+)}(t+\tau)]$$

$$E_s^{(-)}(t) = \int_0^{\infty} d\omega\, a_s^\dagger(\omega)\exp[i\omega t]$$

$$G^{(1)}(\tau) = \int_0^{\infty} d\omega |S(\omega-\Omega)|^2 e^{-i\omega\tau}$$

Wiener-Khintchine theorem: the autocorrelation function of a stationary random process and the power spectrum of the process form a Fourier transform pair.

2nd order correlation function:

$$|\psi\rangle = \int_{-\infty}^{\infty} \mathcal{S}(\nu)\mathcal{P}(\nu) a_s^\dagger(\Omega_p/2+\nu) a_i^\dagger(\Omega_p/2-\nu)|0\rangle$$

using

$$G^{(2)}(\tau) = |\langle 0|E_2^{(+)}(t+\tau)E_1^{(+)}(t)|\psi\rangle|^2$$

$$E_1^{(+)}(t+\tau) = \int_0^{\infty} d\omega_s\, a_s(\omega_s)\, \exp[-i\omega_s(t+\tau)]$$

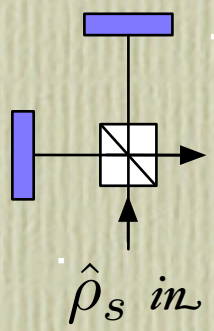
$$E_2^{(+)}(t) = \int_0^{\infty} d\omega_i\, a_i(\omega_i)\, \exp[-i\omega_i t]$$

$$G^{(2)}(\tau) = \left|\int_{-\infty}^{\infty} S(\nu) e^{-i\nu\tau}\right|^2$$

- 1st and 2nd order corelation functions (or wavepackets) may have quite different shapes. For example, $G^{(1)}(\tau)$ does not get affected by the introduction of a medium with a group velocity dispersion, but $G^{(2)}(\tau)$ gets broadened by it.

Single-photon and Two-photon wavepacket calculations

***Single-photon wavepacket*: consider a Michelson interferometer.**



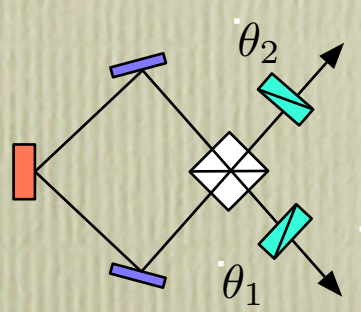
$$E^{(+)}(t) = \int \{a(\omega)e^{-i\omega t} + a(\omega)e^{-i\omega(t+\tau)}\}d\omega$$

Count Rate:

$$R_s = tr[\hat{\rho}_s E^{(-)}(t)E^{(+)}(t)]$$
$$= \frac{1}{2}\left\{1 + g^{(1)}(\tau)\cos(\Omega\tau)\right\}$$

$$g^{(1)}(\tau) = \frac{|G^{(1)}(\tau)|}{|G^{(1)}(0)|}$$

***Two-photon wavepacket*: consider a SA/HOM interferometer.**



$$E_2^{(+)}(t_2) = -i\sin\theta_2 \int d\nu\, a_s(\Omega+\nu)e^{-i(\Omega+\nu)t_2} + \cos\theta_2 \int d\nu\, a_i(\Omega-\nu)e^{-i(\Omega-\nu)(t_2+\tau)}$$

$$E_1^{(+)}(t_1) = -\sin\theta_1 \int d\nu\, a_s(\Omega+\nu)e^{-i(\Omega+\nu)t_1} + i\cos\theta_1 \int d\nu\, a_i(\Omega-\nu)e^{-i(\Omega-\nu)(t_1+\tau)}$$

Coincidence Rate:

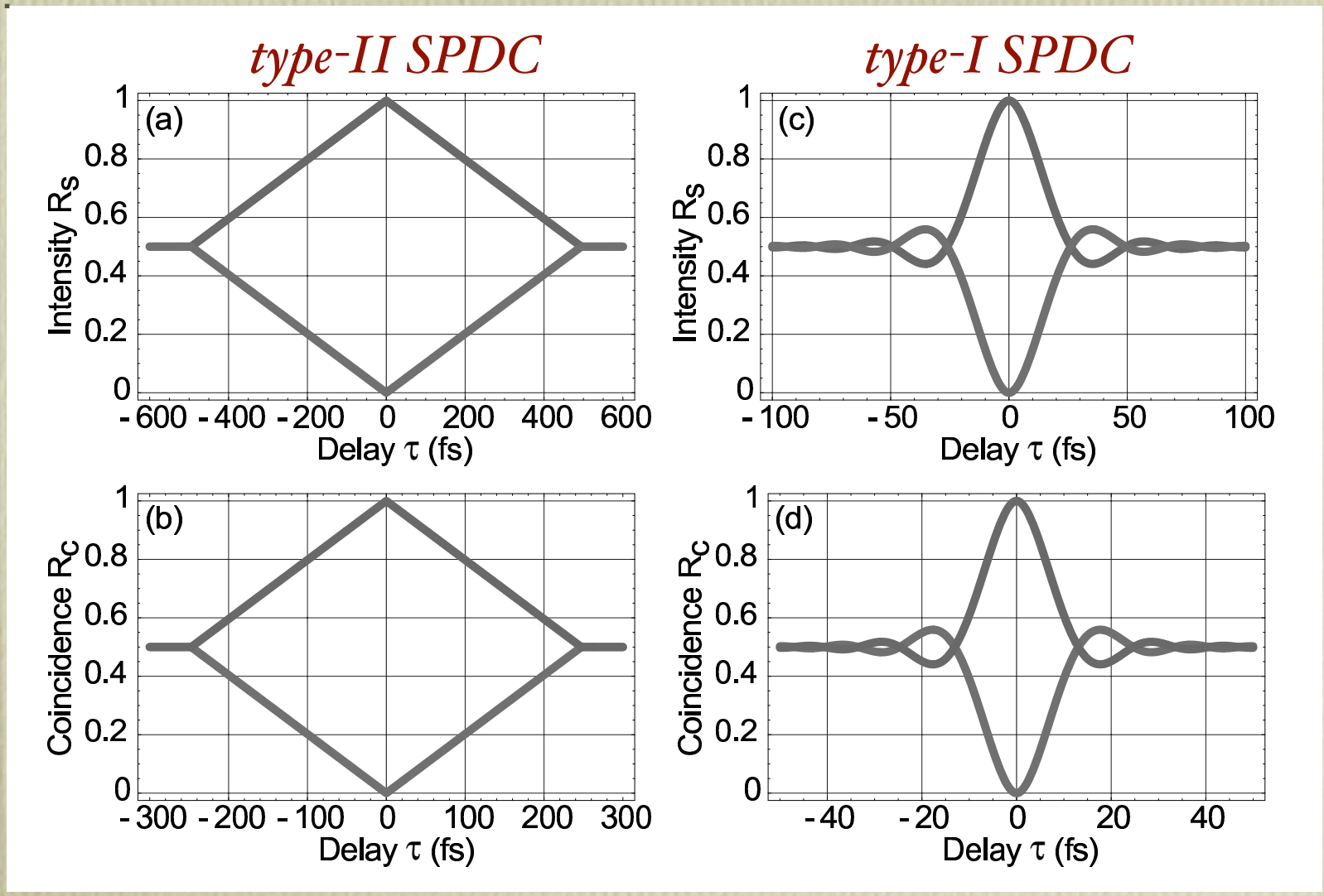
$$R_c = \int |\langle 0|E_2^{(+)}(t_2)E_1^{(+)}(t_1)|\psi\rangle|^2 dt_1 dt_2$$
$$= \frac{1}{2}\left\{1 \pm g^{(1)}(2\tau)\right\} \quad \begin{cases} - \text{ for } \theta_1 = \theta_2 = 45^\circ \\ + \text{ for } \theta_1 = -\theta_2 = 45^\circ \end{cases}$$

Single-photon and Two-photon wavepacket calculations

$$R_s = \frac{1}{2}\left\{1 + g^{(1)}(\tau)\cos(\Omega\tau)\right\}$$

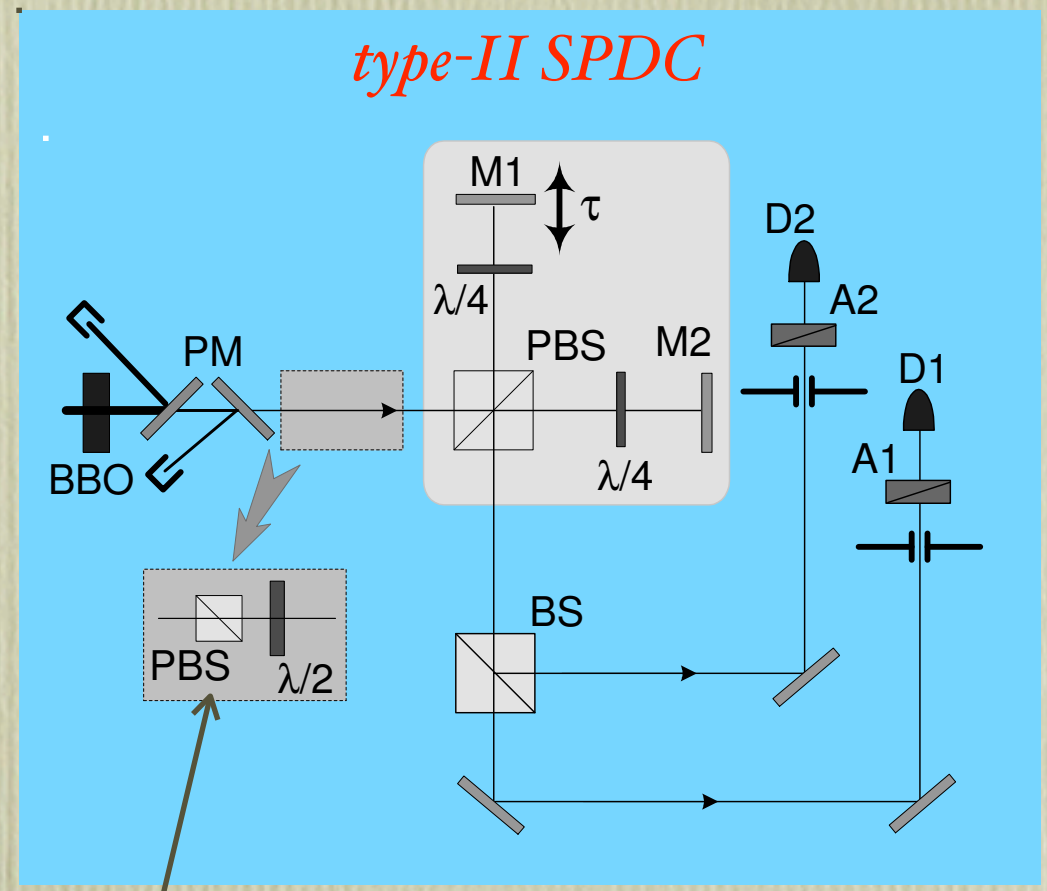
$$R_c = \frac{1}{2}\left\{1 \pm g^{(1)}(2\tau)\right\}$$

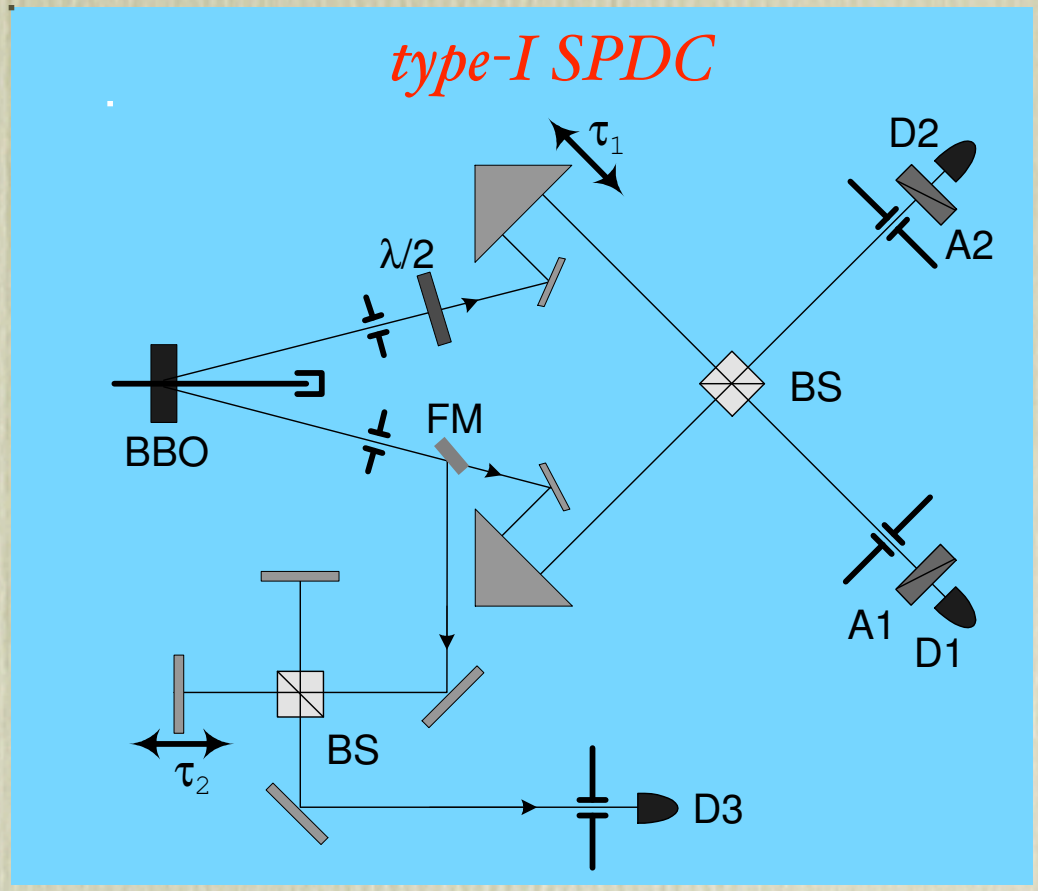
Both R_s and R_c do not depend on $G^{(2)}(\tau)$.



(BBO = 2mm thick; 351.1 nm pump, 702.2 nm SPDC)

Experimental Setup





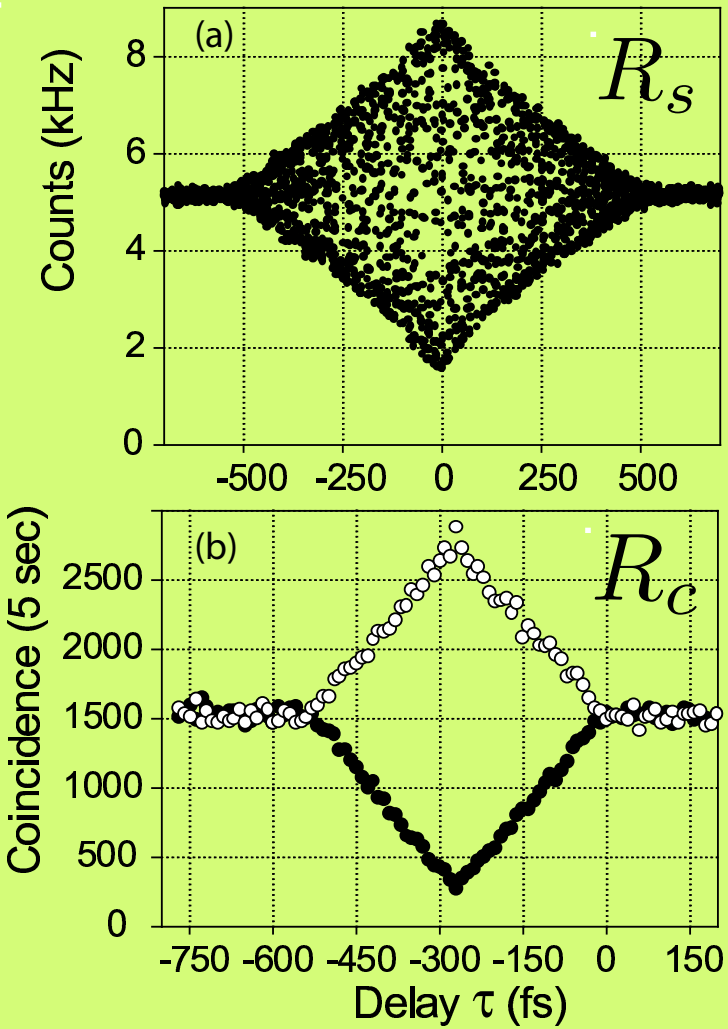
- Used only when measuring $G^{(1)}(\tau)$.
  Removed when measuring $G^{(2)}(\tau)$.

- D3: for measuring $G^{(1)}(\tau)$.
  D1-D2: for measuring $G^{(2)}(\tau)$.

BBO = 2mm thick; 351.1 nm pump; 702.2 nm SPDC.
All quarter-wave plates oriented at 45 deg.
All half-wave plates oriented at 22.5 deg.

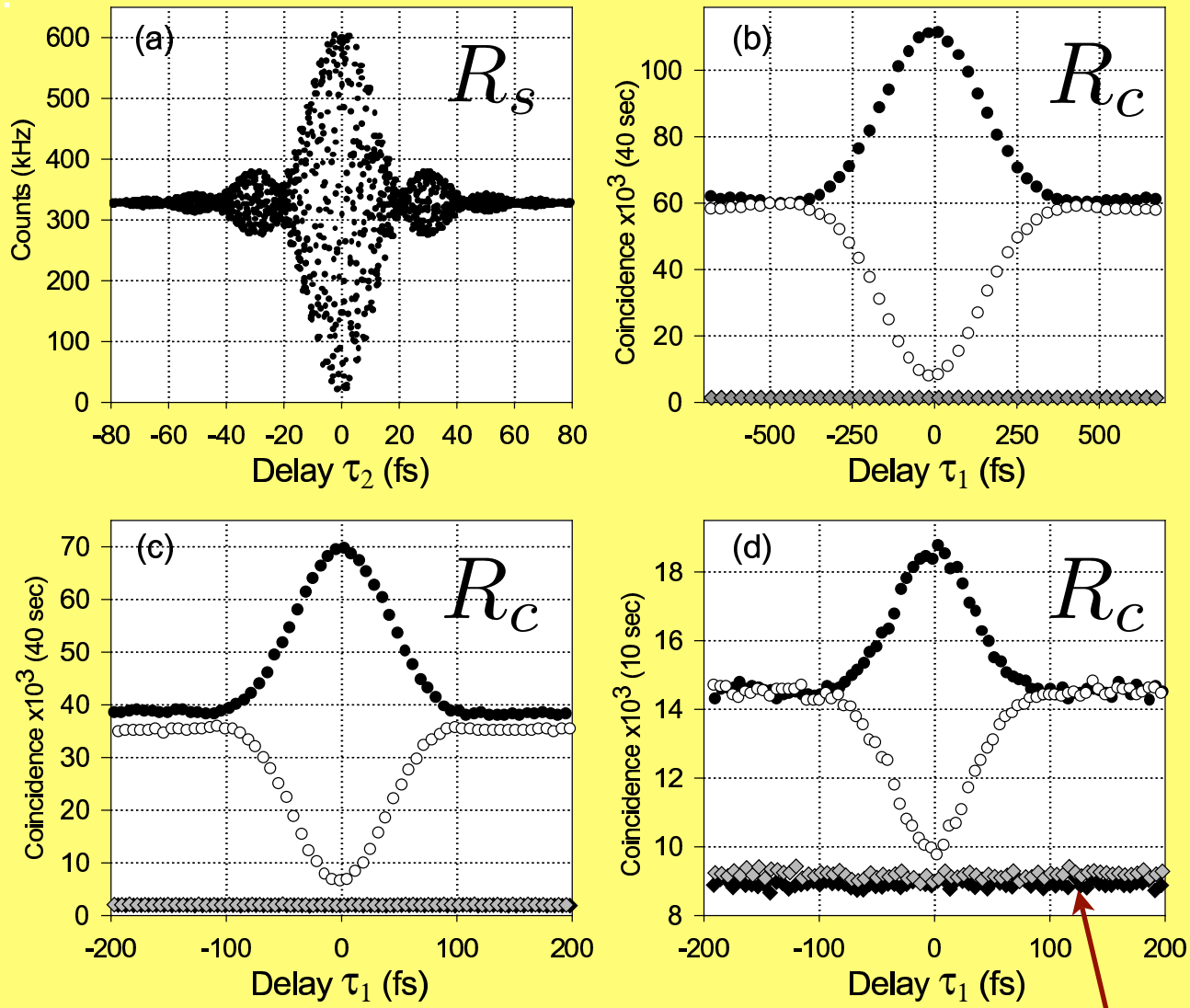
Experimental Data

type-II SPDC



* Data agree well with theory.

type-I SPDC



Accidental Coincidence

* Fig. (a) agrees well with theory.
* (b) 3nm filters; (c) 20 nm filters; (d) 80 nm filters.
* Fig. (d) does not agree with theory.

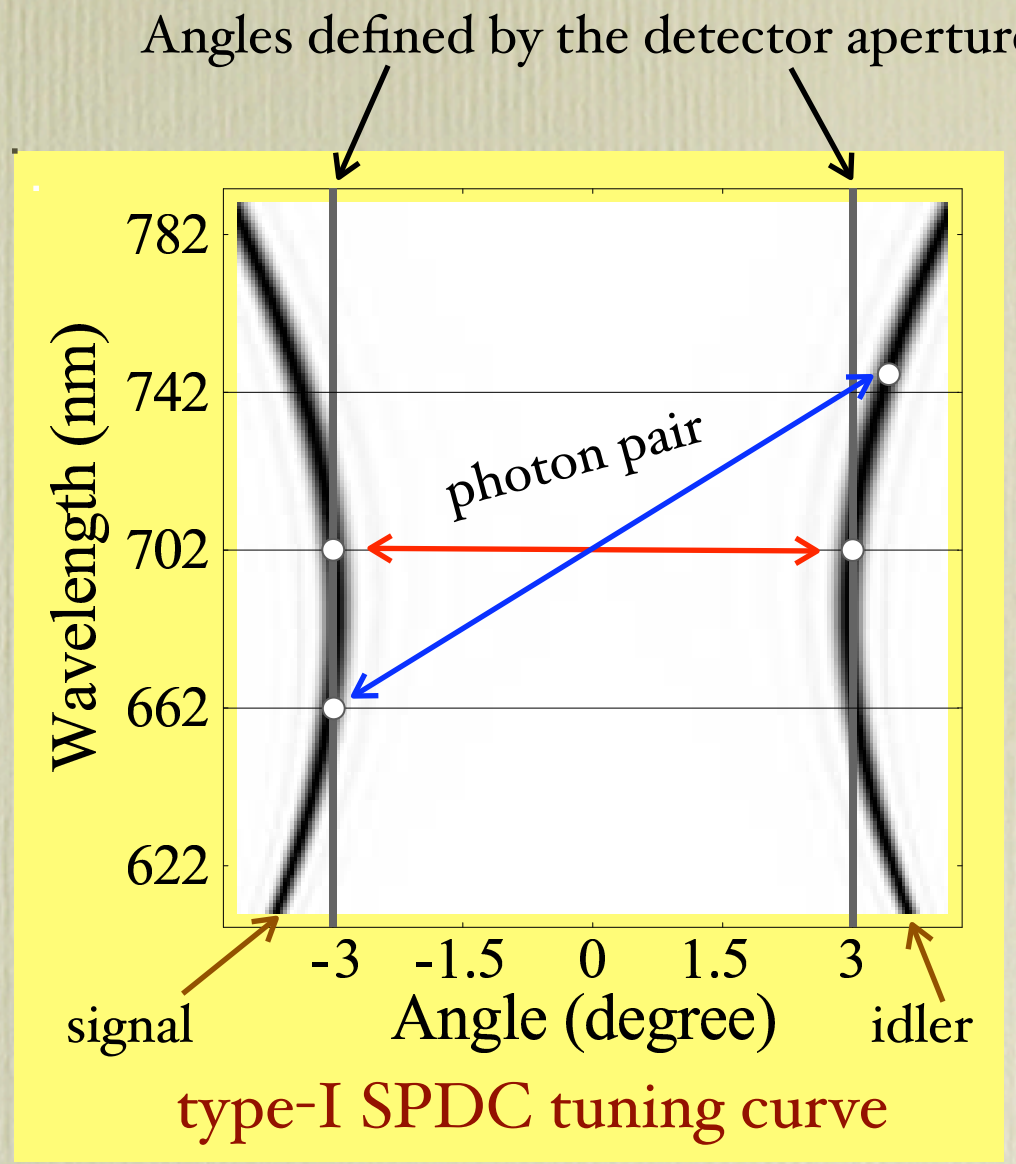
Discussion: type-I SPDC

* Fig.(a) suggests that the power spectrum is not limited by the detector apertures.
* Fig.(d) however suggests that the two-photon bandwidth is limited by the detector apertures.
* Type-I SPDC tuning curve indeed shows that the two-photon bandwidth is limited by the detector apertures:

351.1 nm → 702.2 nm + 702.2 nm
pair detection event!

351.1 nm → 662 nm + 748 nm
click! no-detection

No coincidence count → Reduced two-photon bandwidth!



Summary

- Calculated and Measured one-photon and two-photon wavepackets of type-I and type-II SPDC.
- Two-photon wavepacket of type-I SPDC was broader than the theoretically expected value: although the power spectrum is not limited by the apertures, the two-photon bandwidth may still be limited as joint-detection events has to be considered.
- For broadband type-I SPDC, one-dimensional theory does not work well: the tuning curve characteristics should be considered.
- When utilizing energy and momentum entanglement of type-I SPDC photon pairs (e.g. quantum metrology), care must be taken not to over-estimate the two-photon bandwidth.
- Y.-H. Kim, to appear in J.O.S.A. B (2003).