Nilsson-orbit and weak-coupling model: non-axial deformation

A.T. Kruppa* and W. Nazarewicz†

*Institute of Nuclear Research, Bem tér 18/c, 4026 Debrecen, Hungary
†Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996
Physics Division, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, Tennessee 37831
Institute of Theoretical Physics, Warsaw University, ul Hoża 69, PL-00681 Warsaw

Abstract. Models for deformed proton emitters are reviewed. The non-adiabatic coupled-channel framework with rotational coupling and the adiabatic Nilsson-orbit model are discussed and compared. An improvement of the adiabatic approach is obtained by taking care of the diagonal part of the Coriolis coupling. For the description of the proton radioactivity of ^{141}Ho, we investigate the role of γ-vibrational excitations in the daughter nucleus ^{140}Dy. It is shown that the coupling to triaxial degrees of freedom strongly influences theoretical predictions.

INTRODUCTION

Theoretical models applied to the description of non-spherical proton emitters can be divided into two groups. The core-plus-particle models describe the radioactive parent nucleus in terms of a proton interacting with a core (i.e., the daughter nucleus). Usually, the core is modeled by some phenomenological collective model, e.g., the Bohr-Mottelson (geometric) model. Depending on the structure of the daughter nucleus, rotational [1, 2, 3] or vibrational [4, 5] coupling of the proton is assumed. The models belonging to this group employ the coupled-channel reaction theory which have been developed for the description of the elastic or inelastic scattering.

The second group of models uses the framework of the deformed shell model. In the simplest case, the proton resonance corresponds to a singe-particle resonant (Gamow) state of a deformed field [6, 7, 8, 9, 10]. This model can be generalized to include BCS pairing correlations (see these proceedings).

We may refer to the first group of models as weak coupling models or coupled-channel models. For the second group of models, we reserve the term resonance Nilsson-orbit (or adiabatic) description. The term “adiabatic" requires an explanation. It is very difficult, if not impossible, to relate both groups of models to each other, because they operate on different approximation levels. In special situations, however, the relation between the two types of models can be revealed. For instance, if one considers axial-symmetric nuclei, strong rotational coupling [11], and the degenerate ground-state rotational band in the daughter nucleus, one recovers the resonance Nilsson-orbit model. So one may say that *in this case* the adiabatic model is an approximation of the weak-coupling non-adiabatic model. Generally, however, the relation between the models is not that simple. For example, the resonance Nilsson-orbit model with a non-axial symmetric potential (i.e., nonzero γ deformation) cannot be trivially related to a weak coupling model with non-axial deformation.

If the coupled-channel model with the rotational coupling is applied to the nucleus ^{141}Ho, the ground-state decay (half-life time and branching ratio) is poorly described [3, 2]. There are several explanations possible. For example, it may be that the Coriolis mixing is too strong [3]. This can be partly cured if pairing correlation is introduced (see these proceedings). Another possibility, explored in this work, is the coupling to triaxial vibrations. Indeed, in particle-rotor calculations, the best description of the experimentally observed band structures of ^{141}Ho can be explained if γ deformation is introduced [12]. In addition, in the neighboring N=74 isotones there are low-lying 2_2^+ and 3^+ levels. We may interpret these states as members of the γ-vibrational band. There are also other indications that in this mass region the nuclei may have triaxial shapes [13, 14].

The ground-state $K = 0$ rotational band of ^{140}Dy has recently been observed [15]. In this work, we assume that ^{140}Dy also has K=2 γ-vibrational band. This structure can be coupled to the ground-state band if the proton-daughter interaction in the body-fixed system deviates from the axial symmetry. In our weak-coupling model we do not assume, however, that the daughter nucleus has a permanent γ deformation. Our aim is to investigate the possibility of triaxial vibrations around the deformed axial shape. The experimentally observed rotational band of the parent nucleus is assumed to be a $K = 7/2^-$ band [12]. In the strong-coupling picture, this implies the presence of two addtional rotational bands in ^{141}Ho with quantum numbers $7/2 \pm 2$, i.e., $K^\pi = 3/2^-$ and $K^\pi = 11/2^-$.

This paper is organized as follows. We will begin with the overview of the weak coupling model in the case of rotational coupling. We will then discuss the Nilsson-orbit model and its relation to the weak coupling approach. In particular, we consider the diagonal part of the Coriolis coupling, which was neglected in earlier calculations. Finally, we present numerical results. Here we show how the position of excited states of the daughter nucleus can influence predictions of the weak-coupling model. We also present preliminary results for the proton emission from ^{141}Ho assuming the presence of the γ-vibrational band in the daughter nucleus. The final conclusion is that the coupling to γ-vibration improves the agreement between the weak-coupling model and experiment.

WEAK COUPLING MODEL: NON-ADIABATIC APPROACH

The model of the parent nucleus describes a single proton interacting with a deformed core. The model Hamiltonian can be written as

$$H_{\rm rot} = H_{\rm d} - \frac{\hbar^2}{2m} \triangle_{\mathbf{r}} + V(\mathbf{r}, \omega), \tag{1}$$

where $H_{\rm d}$ is the collective Hamiltonian of the daughter nucleus, the second term represents the relative kinetic energy, and V is the proton-core interaction, which depends on the position of the proton $\mathbf{r}$ and the orientation ω of the core. In the laboratory system, the daughter proton interaction is given by

$$\begin{aligned} V(\mathbf{r},\omega) &= V^{(1)}(\mathbf{r},\omega)+a_2V^{(2)}(\mathbf{r},\omega) \\ &= \Sigma_{\lambda\mu}V_\lambda^{(1)}(r)D^\lambda_{\mu 0}Y_{\lambda,\mu}(\hat{r})+a_2\Sigma_{\lambda\mu}V_\lambda^{(2)}(r)\left(D^\lambda_{\mu 2}(\hat{r})+D^\lambda_{\mu-2}\right)Y_{\lambda,\mu}(\hat{r}), \end{aligned} \tag{2}$$

where the deformation parameters are a_0 and a_2 [$V_\lambda^{(1)}(r)$ depends on a_0]. For the core we take the rotational-vibrational collective model. The daughter states $\phi_{I\mu K}$ are given by the standard ansatz

$$\phi_{I\mu K}=\sqrt{\frac{2I+1}{16\pi^2(\delta_{K,0}+1)}}\left[D^{I*}_{\mu K}+(-1)^I D^{I*}_{\mu -K}\right]\chi_{Kn_2}(a_2)|\text{g.s.}\rangle. \tag{3}$$

The wave function of the parent nucleus can be written in the weak-coupling form

$$\Psi^{JM}=\sum_{IKlj}\frac{f^J_{IKlj}(r)}{r}\Phi_{JMIKlj}, \tag{4}$$

where the channel function is given by

$$\Phi_{JMIKlj}=\sum_{\Omega\mu}\langle j\Omega I\mu|JM\rangle\mathscr{Y}_{lj\Omega}\phi_{I\mu K}, \tag{5}$$

and

$$\mathscr{Y}_{lj\Omega}=\sum_{ms}\langle lm\frac{1}{2}s|j\Omega\rangle i^l Y_{lm}(\hat{r})\chi_{1/2}(s) \tag{6}$$

arises from the coupling of the proton spin with the orbital angular momentum. The unknown radial functions $f^J_{IKlj}(r)$ can be then obtained from the set of coupled-channel equations:

$$\begin{aligned} &\frac{\hbar^2}{2m}\left(-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right)f^J_{IKlj}+\Sigma_{\lambda I'l'j'}A_\lambda(Ilj,I'l'j',J)B_\lambda(II'K)V^{(1)}_\lambda f^J_{I'Kl'j'}+ \\ &\Sigma_{\lambda I'K'l'j'}A_\lambda(Ilj,I'l'j',J)C_\lambda(IKI'K',a_2)V^{(2)}_\lambda f^J_{I'K'l'j'}=(E-E_{IK})f^J_{IKlj}. \end{aligned} \tag{7}$$

Here, the r-independent coupling coefficients can be written in terms of the reduced nuclear matrix elements

$$B_\lambda(II'K)=\langle\Phi_{IK}||D^\lambda_{;0}||\Phi_{I'K}\rangle \tag{8}$$

and

$$C_\lambda(IKI'K',a_2)=\langle\Phi_{IK}||a_2(D^\lambda_{;2}+D^\lambda_{;-2})||\Phi_{I'K'}.\rangle \tag{9}$$

The explicit expressions for geometric coefficients A_λ are given, e.g., in Ref. [16]. The nuclear structure model of the daughter nucleus enters the formalism through the reduced matrix elements B_λ and C_λ.

Weak-coupling model in the body-fixed frame

The ansatz (4) is given in the laboratory frame but the total wave function can be easily transformed to the body-fixed system. Following Ref. [17], where the α-decay was described in the adiabatic limit of the weak-coupling approach, one obtains:

$$\Psi^{JM} = \sum_{K\Omega lj} \frac{g^J_{K,\Omega lj}(r)}{r}\sqrt{\frac{2J+1}{16\pi^2}}$$
$$\left(\mathscr{Y}'_{lj\Omega} D^{J*}_{M,\Omega+K} + (-1)^{J-j}\mathscr{Y}'_{lj-\Omega} D^{J*}_{M,-\Omega-K}\right)\chi_{Kn_2}(a_2)|\text{g.s.}\rangle. \tag{10}$$

The new radial wave functions $g^J_{K,\Omega lj}(r)$ are related to the laboratory-system wave functions through the equation

$$g^J_{K,\Omega lj}(r) = \sqrt{\frac{1}{\delta_{K,0}+1}}\sum_I \frac{\hat{I}}{\hat{J}}\langle j\Omega IK|J\Omega+K\rangle f^J_{IKlj}(r). \tag{11}$$

In the body-fixed frame the proton daughter interaction is

$$V_{\text{def}}(r,\theta'\phi') = V^{(1)}_{\text{def}}(r,\theta') + a_2 V^{(2)}_{\text{def}}(r,\theta'\phi')$$
$$= \sum_\lambda V^{(1)}_\lambda(r)Y'_{\lambda,0}(\theta') + a_2\sum_\lambda V^{(2)}_\lambda(r)\left[Y'_{\lambda,2}(\theta',\phi') + Y'_{\lambda,-2}(\theta',\phi')\right], \tag{12}$$

i.e., it is given by the triaxial average potential. The action of the Hamiltonian of the daughter nucleus on the laboratory channel function is very simple: $H_d\Phi_{JMIKlj} = E_{IK}\Phi_{JMIKlj}$. On the other hand, the action of the daughter's Hamiltonian on the wave function (10) is more complicated:

$$H_d\Psi^{JM} = \sum_{Klj}\sum_{\Omega\Omega'} A^J_{Klj}(\Omega,\Omega')\frac{g^J_{K,\Omega lj}(r)}{r}\Phi^{\text{body}}_{JMK\Omega' lj}, \tag{13}$$

where

$$A^J_{Klj}(\Omega,\Omega') = \sum_I \frac{2I+1}{2J+1}E_{IK}\langle j\Omega IK|J\Omega+K\rangle\langle j\Omega' IK|J\Omega'+K\rangle \tag{14}$$

and

$$\Phi^{\text{body}}_{JMK\Omega lj} = \sqrt{\frac{2J+1}{16\pi^2}}\left(\mathscr{Y}'_{lj\Omega} D^{J*}_{M,\Omega+K} + (-1)^{J-j}\mathscr{Y}'_{lj-\Omega} D^{J*}_{M,-\Omega-K}\right)\chi_{Kn_2}(a_2)|\text{g.s.}\rangle. \tag{15}$$

For simplicity, we give the coupled-channel equations in the body-fixed system in the case of axial symmetry

$$\begin{aligned} -\frac{\hbar^2}{2m}\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right] g^J_{0,\Omega lj} \quad &+ \quad \sum_{l'j'}\langle \mathscr{Y}_{lj\Omega}|V(r,\theta')|\mathscr{Y}_{l'j'\Omega}\rangle\, g^J_{0,\Omega l'j'} \\ &+ \quad \sum_{\Omega'} A^J_{0lj}(\Omega\Omega')g^J_{0,\Omega' lj} = E\, g^J_{0,\Omega lj}. \end{aligned} \tag{16}$$

We must emphasize that the coupled-channel equations (7) and (16) are completely equivalent. We note that under special circumstances (E_{IK} are given by the pure rotational formula) the third term of the left hand side of Eq. (16) is the rotational energy in the strong coupling limit [3].

NILSSON-ORBIT MODEL: ADIABATIC APPROACH

In the spirit of the deformed shell model, we assume that the state of the emitted proton is the lowest resonance single-particle orbit of a deformed potential. In the body-fixed coordinate system the deformed potential is given by Eq. (12). The Hamiltonian of the resonance Nilsson-orbit model contains only the deformed potential (12), and it can be written as

$$H_{\mathrm{def}} = -\frac{\hbar^2}{2m} \triangle_{\mathbf{r}'} + V_{\mathrm{def}}(r,\theta'\phi'). \tag{17}$$

The wave function of a Nilsson-orbit can be expanded

$$\Psi_{\mathrm{def}} = \sum_{\Omega l j} \frac{\bar{g}_{\Omega l j}}{r} \mathscr{Y}'_{l j\Omega}. \tag{18}$$

For the radial functions, one obtains a set of coupled-channel equations:

$$\begin{aligned}\frac{\hbar^2}{2m}\left(-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right)\bar{g}_{\Omega l j}(r) + \textstyle\sum_{l' j'}\langle \mathscr{Y}'_{l j\Omega}|V^{(1)}_{\mathrm{def}}|\mathscr{Y}'_{l' j'\Omega}\rangle \bar{g}_{\Omega l' j'}(r) \\ + a_2 \textstyle\sum_{\Omega' l' j'}\langle \mathscr{Y}'_{l j\Omega}|V^{(2)}_{\mathrm{def}}|\mathscr{Y}'_{l' j'\Omega'}\rangle \bar{g}_{\Omega' l' j'}(r) = E\bar{g}_{\Omega l j}(r).\end{aligned} \tag{19}$$

For simplicity, let us assume that the daughter nucleus is axially deformed ($a_2 = 0$). The comparison of Eqs. (19) and (16) reveals that if the last term on the left hand side of (16) is neglected, then the Nilsson-orbit model and the weak-coupling model are equivalent. This approximation is severe. The term in question vanishes only if all the energies of the excited states of the daughter nucleus are set to zero. Only in the case of extreme degeneracy (or infinite collective moment of inertia) is the resonance Nilsson-orbit model related to the weak-coupling model.

Corrected Nilsson-orbit model: inclusion of the diagonal part of the Coriolis coupling

In the above section we have demonstrated that the Nilsson-orbit model (an extension of the classical Nilsson-orbit picture to narrow resonances) is an approximation to the weak-coupling model. Nevertheless, it has proved to be a fairly useful approach for the description of proton-emitting nuclei. There is also a very practical reason why it is useful to study connections between the non-adiabatic approach and the Nilsson-orbit description. The number of coupled-channel equations in the weak-coupling model quickly increases with the number of active states of the daughter nucleus.

One of the drawbacks of the Nilsson-orbit model is that the excitation energies of the daughter nucleus are neglected. This is clearly an unphysical assumption. The excitation energies come into play through the action of the operator H_d, which is given by Eqs. (13) and (14). Since our aim is to avoid the increase of the number of coupled-channel equations, we neglect the non-diagonal part of the Ω coupling and approximate (13) by

$$H_d \Psi^{JM} \approx \sum_{Klj} \sum_{\Omega} A^J_{Klj}(\Omega,\Omega) \frac{g^J_{K,\Omega lj}(r)}{r} \Phi^{\text{body}}_{JMK\Omega lj}. \tag{20}$$

Taking into account the above expression, the coupled-channel equation (16) turns into

$$\begin{aligned} \frac{\hbar^2}{2m}\left(-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right) h^J_{0,\Omega lj}(r) \quad &+ \quad \sum_{l'j'} \langle \mathscr{Y}'_{lj\Omega} | V^{(1)}_{\text{def}} | \mathscr{Y}'_{l'j'\Omega} \rangle h^J_{0,\Omega l'j'}(r) \\ &= \quad \left[E - A^J_{0lj}(\Omega,\Omega)\right] h^J_{0,\Omega lj}(r). \end{aligned} \tag{21}$$

In this way we have introduced an effective excitation energy dependence in each (lj) channel. Consequently, Eq. (21) introduces the J-dependence into the Nilsson-orbit model. We have achieved an interesting result. Namely, using the coupled equations (21) for a fixed Ω, we are able to calculate not only the band head ($J=\Omega$) but also the excited states of the parent nucleus by putting $J=\Omega+1, \Omega+2, \ldots$ in (21). We will refer to the calculations based on (21) as “dynamically corrected Nilsson-orbit model" or “dynamically corrected adiabatic description" (ADI-D). The standard Nilsson-orbit description will be referred to as the adiabatic approach (ADI).

NUMERICAL RESULTS

The numerical tests have been performed for the nucleus ^{141}Ho, viewed as the composite system of a proton and the daughter nucleus ^{140}Dy (collective core). For the rotational bands in the daughter nucleus, we have fixed the maximum spin to $I=12$. In the resonance Nilsson-orbit model, the maximum of the proton j value was taken to be $27/2$. As for the parameterization of the Woods-Saxon (WS) potential, we have used the Chepurnov set employed in Ref. [2].

Dynamically corrected adiabatic approach

For each value of $E_2 = E_{20}$, we determine the WS potential strength in the weak-coupling model so as to get the $J = \frac{7}{2}^-$ state at the experimental value of 1.19 MeV. Having established the single-particle potential, we carried out the ADI and ADI-D calculations. The position of the resulting resonance is displayed in Fig. 1. It is seen that the real part of the energy calculated in ADI-D is dramatically improved as compared with ADI. The difference between the ADI-D and the weak-coupling treatment is due to the off-diagonal Coriolis coupling. Since the Coriolis coupling is completely neglected in

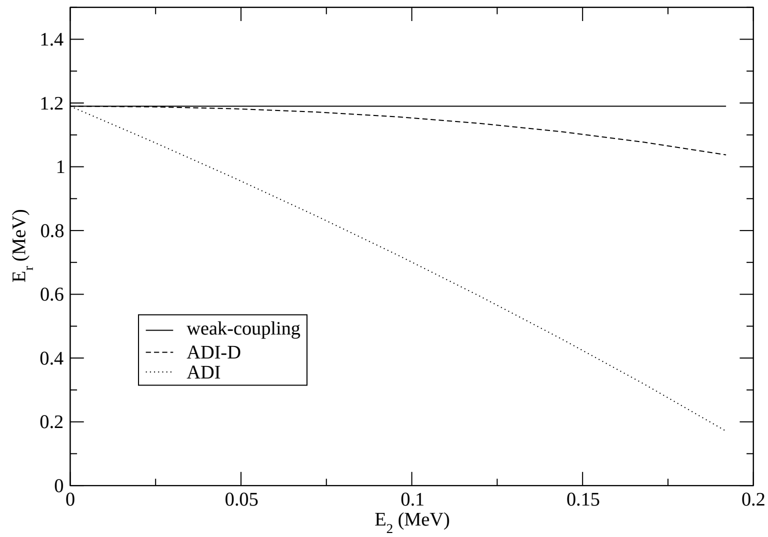

FIGURE 1. The real part of the energy of the $J = \frac{7}{2}^-$ resonance in ^{141}Ho as a function of the excitation energy E_2 of the 2^+ state of the daughter nucleus. The WS strength has been adjusted at each E_2 in the weak-coupling model to reproduce the experimental Q_p value.

the adiabatic model, the ADI description significantly deviates from the weak-coupling result.

Effect of the structure of the daughter nucleus

The excitation energies of the daughter nucleus are calculated using the pure rotational expression $E_{I0} = \frac{\hbar^2}{2\Theta} I(I+1)$. Instead of the moment of inertia parameter, we use the excitation energy E_2 of the 2^+ state in ^{140}Dy as a parameter of the calculations. Figure 2 shows the proton half-life $T_{1/2}$ as a function of E_2. Not surprisingly, $T_{1/2}$ is very sensitive to the position of the 2^+ state. Taking the experimental 2^+ excitation energy 0.202 MeV [15], one obtains $T_{1/2}$=45 msec. If we repeat the weak-coupling calculation in such a way that the energies of the remaining rotational states are taken from the VMI fit to the experimental data (not from the pure rotational formula), then $T_{1/2}$ is reduced to 21 msec (filled circle in the Fig. 2). This demonstrates that $T_{1/2}$ is sensitive not only to the correct position of 2^+ level but also to the placement of other rotational states of the daughter nucleus.

The excited states of the daughter nucleus correspond to open or closed channels. To check the importance of the closed channels, we carried out the following weak-coupling calculation. For the open channels (the 0^+, 2^+, 4^+, and 6^+ states of the daughter nucleus) we have used the experimental excitation energies, and for the closed channels we have taken the energies calculated by the pure rotational formula. This yields $T_{1/2}$=31 msec (shown by a filled square in Fig. 2). From this exercise we can conclude that the effect of the closed channels is as important as the open ones.

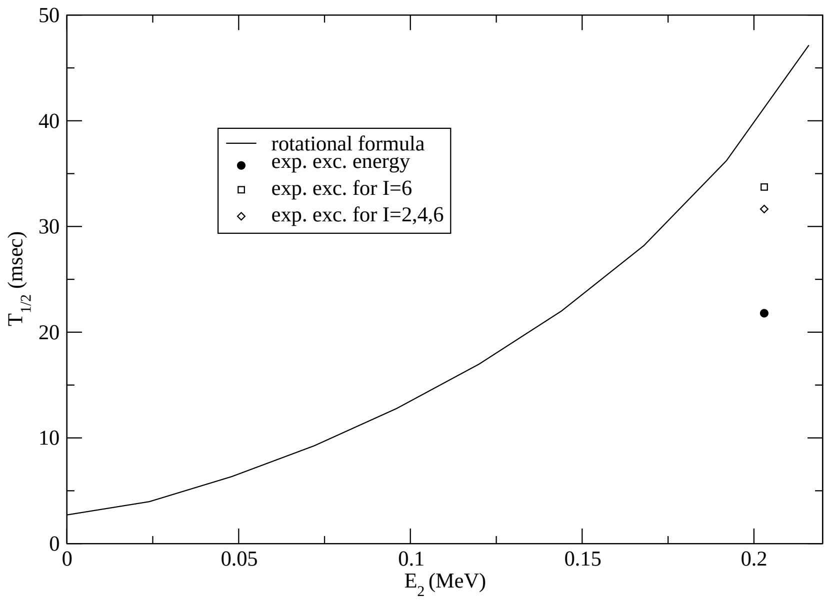

FIGURE 2. Half-life of the $J = \frac{7}{2}^-$ resonance in ^{141}Ho as a function of the excitation energy E_2 of the 2^+ state in ^{140}Dy. The solid curve shows the result when the excitation energies are calculated by the pure rotational formula. The result obtained by using the experimental energies of the ground-state band in ^{140}Dy is marked by a dot. The filled square marks the result obtained by assuming that the energies of closed-channel states are taken from the pure rotational formula.

Effect of γ-vibrations

We have seen that the structure of the daughter nucleus (e.g., the exact placement of rotational members of the ground-state band) has a large influence on $T_{1/2}$. If we assume that the daughter nucleus ^{140}Dy has other excited states, such as a low-lying γ-vibrational band, this will also have a large effect on the calculated proton-emission observables. The coupling of the γ-vibrational $K = 2$ rotational band to the ground-state band is possible if the proton-daughter interaction has a non-axial component in the body-fixed system.

Figure 3 shows results of coupling to the γ-vibrational band of the daughter nucleus. Guided by experimental systematics, we assumed the energy of the K=2 bandhead to be 0.789 MeV. The positions of higher-lying states of the γ-band were calculated using the pure rotational formula. For the ground-state band, we have used the experimental values of the excitation energies. The deformation parameter a_0 was set to the value of 0.244, which is consistent with earlier investigations [15, 12]. Different curves in Fig. 3 display results of calculations carried out using different number of states in the γ-band. It is seen that in our preliminary calculations we have not yet reached full convergence with

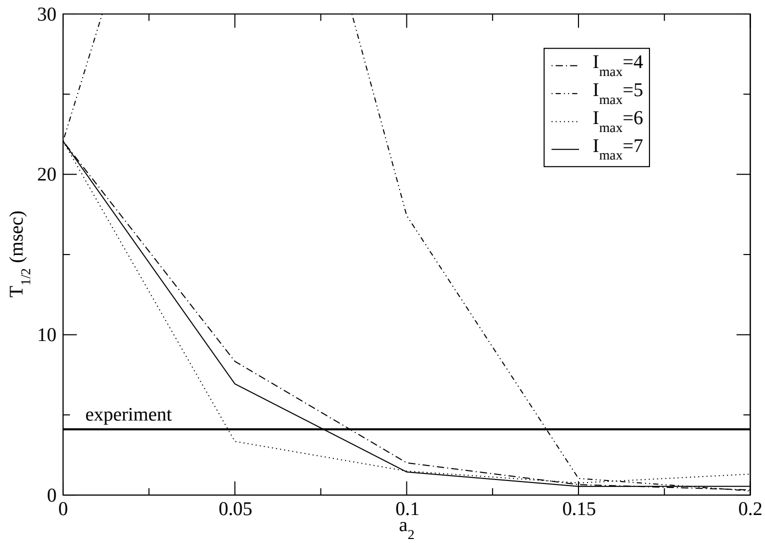

FIGURE 3. Half-life of the $J = \frac{7}{2}^-$ resonance in ^{141}Ho as a function of the deformation parameter a_2. Results of calculations carried out assuming a different number of active states in the γ-band of ^{140}Dy are shown.

respect to the number of states in the γ-band. The trend, however, is such that triaxiality certainly helps to improve the agreement with the experimental lifetime (4.1 msec).

Our calculations have clearly demonstrated that the structure of the daughter nucleus strongly influences the results of the non-adiabatic coupled-channel model. The description of the proton radioactivity of ^{141}Ho is a very challenging task for theory. Different models have been proposed in order to describe simultaneously the half-life time and branching ratio. Our model shows that the assumption of dynamical coupling to low-lying γ-vibrational states should be treated very seriously.

ACKNOWLEDGMENTS

This research was supported by Hungarian OTKA Grant Nos. T37991 and T29003 and by the U.S. Department of Energy under Contract Nos. DE-FG02-96ER40963 (University of Tennessee) and DE-AC05-00OR22725 with UT-Battelle, LLC (Oak Ridge National Laboratory).

REFERENCES

1. Kruppa A.T., Barmore B., Nazarewicz W. and Vertse T., Phys. Rev. Lett., 2000, 84, 4549
2. Barmore B., Kruppa A.T., Nazarewicz W. and Vertse T., Phys. Rev. C, 2000, 62, 054315
3. Esbensen H., and Davids C.N., Phys. Rev. C, 2001, 63, 014315
4. Davids C.N. and Esbensen H., Phys. Rev. C, 2001, 64, 034317
5. Hagino K., Phys. Rev. C, 2001, 64, 041304
6. Maglione E., Ferreira L.S. and Liotta R.J., Phys. Rev. Lett., 1998, 81, 538
7. Davids C.N., et. al., Phys. Rev. Lett., 1998, 80, 1849

8. Maglione E., Ferreira L.S. and Liotta R.J., Phys. Rev. C, 1999, 59, R589
9. Rykaczewski K., et. al., Phys. Rev. C, 1999, 60, 011301
10. Sonzogni A.A, et. al., Phys. Rev. Lett., 1999, 83, 1116
11. Bohr A. and Mottelson B.R., *Nuclear Structure* (Benjamin, New York, 1975), Vol. II
12. Seweryniak D., et. al., Phys. Rev. Lett., 2001, 86, 1458
13. Yan J., Brentano von V.P. and Gelberg A., Phys. Rev. C, 1993, 48, 1046
14. Kortelahti M.O., et. al., Phys. Rev. C, 1990, 42, 1267
15. Królas W., et.al. , Phys. Rev. C, 2002, 65, 031303(R)
16. Tamura T., Rev. Mod. Phys., 1965, 37, 679
17. Fröman P.O., Mat. Fys. Skr. Dan. Vid. Selsk., 1957, 1, No 3.