APPLICATION OF GROUND-STATE FACTORIZATION TO NUCLEAR STRUCTURE PROBLEMS

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We compute accurate approximations to the low-lying states of ^{44}Ti by ground-state factorization. Energies converge exponentially fast as the number of retained factors is increased, and quantum numbers are reproduced accurately.

The nuclear shell model is difficult to solve for more than a few valence nucleons due to the large dimensions of the underlying Hilbert space. Furthermore, the complexity of the interaction makes it challenging to devise approximations that significantly reduce the size of the problem while still being sufficiently accurate. In recent years, various approximation schemes have been proposed[1,2,3,4,5,6,7,8]. We particularly mention the mixed-mode shell-model theory[7], the approach based on a quasi-SU(3) truncation scheme[5], and the very recently proposed ground-state factorization[8]. The first two of these approaches use a basis truncation scheme that is based on a small number of SU(3) coupled irreps and thereby includes important collective configurations. The third approach approximates the ground state in terms of a small number of products of optimally chosen proton and neutron states. Particularly promising results of the mixed-mode shell-model[7] and the ground-state factorization[8] have been reported for the sd-shell nucleus ^{24}Mg. This is interesting since this nucleus exhibits competing single-particle and collective degrees of freedom. In this work we apply the ground-state factorization to the pf-shell nucleus ^{44}Ti. This nucleus is an interesting test case as its SU(3) symmetry-breaking has recently been explored in detail[9].

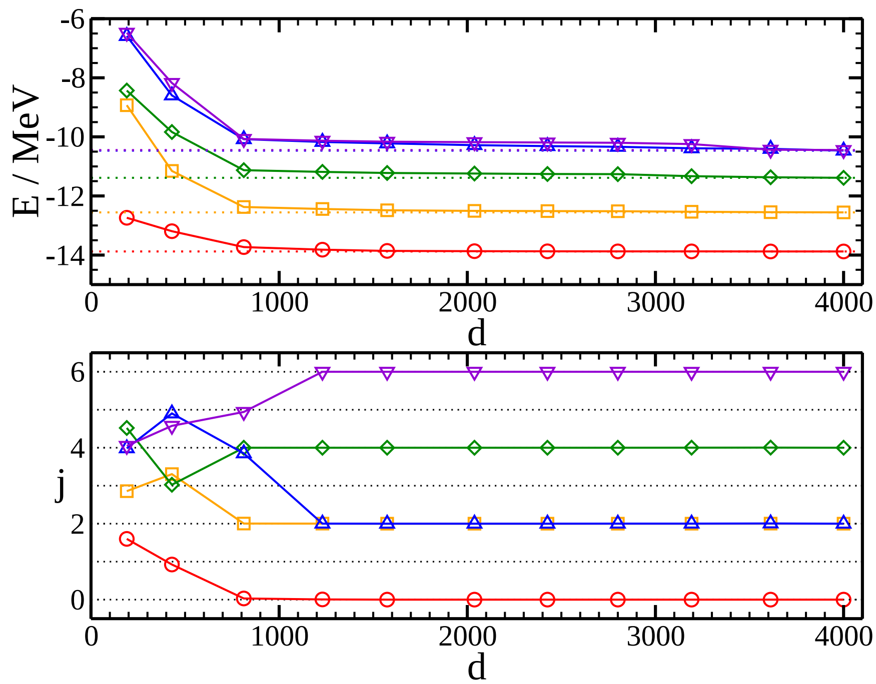

Figure 1. Low-lying states of ^{44}Ti (KB3 interaction) computed from ground-state factorization. Top: Energy spectrum (data points connected by full lines) and exact results (dotted lines) versus the dimension d of the eigenvalue problem. Bottom: Angular momentum quantum number j from $j(j+1) = \langle J^2 \rangle$ (data points connected by full lines) versus d. An exact diagonalization has dimension $d_{\max} = 4000$.

The ground-state factorization is based on the ansatz

$$|\psi\rangle = \sum_{j=1}^{\Omega} |p_j\rangle |n_j\rangle \tag{1}$$

for the ground state $|\psi\rangle$. Here, the unknown factors are the proton states $|p_j\rangle$ and the neutron states $|n_j\rangle$ which are of dimension d_P and d_N, respectively. The truncation is controlled by the fixed input parameter Ω which counts the number of retained factors. Variation of the energy $E = \langle\psi|\hat{H}|\psi\rangle / \langle\psi|\psi\rangle$ yields eigenvalue problems of dimension Ωd_P (Ωd_N) for the proton states (neutron states). Note that these dimensions are usually much smaller than the dimension $d_P d_N$ of the full problem. For details, we refer the reader to Ref.[8].

We apply the ground-state factorization pf-shell nucleus ^{44}Ti and use the KB3 interaction[10]. In m-scheme, the Hilbert space has dimension $d_{\max} = 4000$, and the eigenvalue problem for the factorization has an Ω-dependent dimension $d \leq d_{\max}$. Figure 1 shows the energies and angular momentum quantum numbers of the low-lying states versus the dimension d of the eigenvalue problem. Note the exponential convergence with respect to increasing dimension d of the eigenvalue problem. Very good energies are obtained once the dimension d exceeds $d \approx 0.2 d_{\max}$, while the angular momenta stabilize around $d \approx 0.3 d_{\max}$. Note also that the angular momenta of the quasi-degenerate third and fourth excited states are accurately reproduced. The rapid convergence of the excited states suggests that they can be approximated by factors that are similar to those of the ground state. This similarity of the structure of low-lying states is also reflected in the strenght distribution of the SU(3) Casimir operator C_2 [9].

The results of this work and the results of Ref.[8] demonstrate the accuracy and efficiency of the ground-state factorization for a variety of nuclei. This opens a promising avenue for large-scale nuclear structure calculations.

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