

# Application of Fractal Encoding Techniques for Image Segmentation

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## ABSTRACT

Fractal encoding is the first step in fractal based image compression techniques, but this technique can also be useful outside the image compression field. This paper discusses a fractal encoding technique and some of its variations adapted to the concept of segmenting anomalous regions within an image. The primary goal of this paper is to provide background information on fractal encoding and show application examples to equip the researcher with enough knowledge to apply this technique to other image segmentation applications. After a brief overview of the algorithm, important parameters for successful implementation of fractal encoding are discussed. Included in the discussion is the impact of image characteristics on various parameters or algorithm implementation choices in the context of two applications that have been successfully implemented.

**Keywords:** Fractal encoding, segmentation, defect detection, data reduction

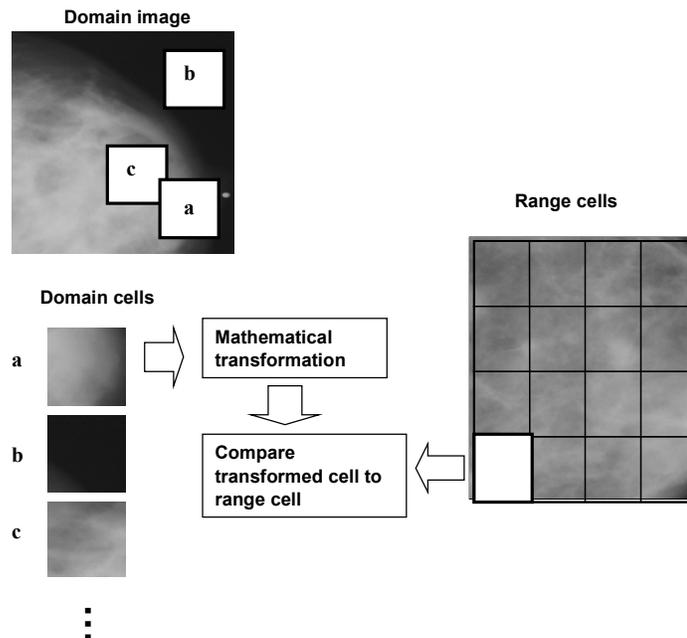
## 1. INTRODUCTION

The study of fractal encoding for image compression has been a fast developing research area since the early 1990s. The use of fractals for compression is based on the idea that individual regions of an image may exhibit similarity to each other at different scales. Though the primary focus of fractal encoding research has been on applications and methods for image compression, fractal techniques have been employed as an important step in a number of image processing, interpretation, or classification applications.

In this paper, two applications are described where fractal encoding was employed as a means of identifying anomalous regions in an image. Fractal encoding was used to eliminate normal tissue regions in a mammogram and to allow more rigorous examination of anomalous regions to locate microcalcifications that would be indicative of cancerous tissue. The second application used fractal encoding to re-detect and re-delineate the region of a semiconductor image that contained a defect. This data was used to extract feature measurements used in a content-based image retrieval system for semiconductor defects. In developing these two applications, some observations were made on the selection of some critical parameters and implementation options for the fractal encoding process that could impact the success of the technique as it is used for other image analysis applications.

## 2. FRACTAL ENCODING APPROACH

Fractal encoding techniques rely on the concept that parts of an image are very similar to other parts of the same image. More explicitly, the assumption is that one part of an image can be closely described by a scaled down copy of some other part of the image that has been translated and/or rotated according to a particular transformation. In preparation for encoding, an image is broken up into a set of non-overlapping "range" cells (i.e. image subregions). The concept behind fractal encoding is that a set of "domain" cells can be mathematically transformed in some way to closely resemble every range cell of the image being compressed. This is pictorially illustrated in Figure 1. The image to be encoded is broken into a number of subimages or range cells that cover an image completely in a non-overlapping manner. The domain cells may arise from a similarly broken down image, or may consist of a number of unrelated subimages, or may be created by using overlapping regions of an image. Each of these domain cells will undergo a number of transformations such as rotations, shrinkage, or intensity modifications to more closely match a given range image. A predefined set of transformations is applied to the domain cells to find the best map, in terms of minimizing an error



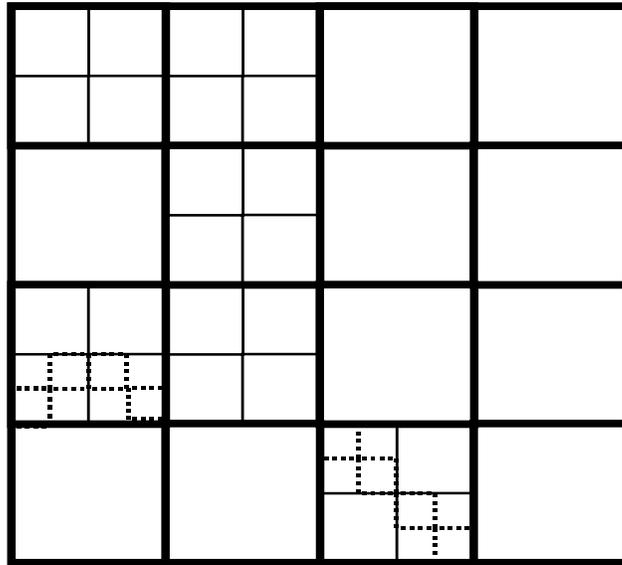
**Figure 1** The basis of fractal encoding is the comparison of a set of domain cells transformed to match the range cell.

metric, onto the range cell. If the minimum error between the range cell and the closest domain cell is below the designated similarity threshold, the region is considered mapped and the location of the range cell, the index of the domain that maps to it, and the transformation parameters are recorded. If the error associated with the best match is greater than the similarity threshold, this range cell could be marked as not being matched or representing an anomalous region.

The first step in fractal encoding partitions the prospective image into a set of non-overlapping regions referred to as range cells. These cells can be of varying shape, or in any desirable arrangement, though rectangular grids, triangles, and quadtree partitioning schemes are found to be the most practical. The most commonly employed scheme, quadtree partitioning, is based on breaking an image subregion into 4 quadrants. Each quadrant may then be evaluated as a whole range cell or broken into 4 more sub-quadrants depending on the similarity threshold and two other parameters. These parameters are the minimum number of partitions, which would determine the largest image region considered, and the maximum number of partitions, which would determine the smallest image region considered. If the best match between the largest range cells and the transformed domain cells still has an error measurement that is greater than the similarity threshold, the range cell is further partitioned into four more range cells and re-evaluation continues similarly with each of these cells. Range cells that cannot be matched within the similarity threshold continue to be partitioned into 4 smaller range cells until the maximum number of partitions is reached. If this limit is reached and the closest domain cell does not match the range cell within the similarity threshold, this area would be marked as an anomalous region. This approach allows highly textured regions to be broken into smaller regions during the domain-to-range mapping. More uniform areas are still mapped to larger regions. In this manner, the part of the image that has the most information or detail is more accurately described. The minimum and maximum quadtree partition parameters determine the level of quadtree partitioning. These parameters are typically user selected based on the application of interest. An example pattern of image partitioning is shown in Figure 2.

A set of domain cells is generated for comparison to each of the range cells. The domain set can be a highly specified set of subimages or may be generated by partitioning an image in a manner similar to generating range cells. For these applications, the domain cells were generated using the original image or a slightly modified version, but their

composition can vary according to the application. For some applications, if a range image is known to have a number of anomalous regions, it may be advantageous to perform the encoding using a similar domain image known to contain no anomalies if such an image is available.



**Figure 2 Example of image with a minimum of 2 quadtree partitions and a maximum of 4 quadtree partitions.**

To select the most similar domain cell for each range cell, a set of affine transformations is applied to the domain cell and the resulting data is directly compared to the range cell. The transformations applied for mapping domain cells on to range cells were based on rotational and translational positioning, contractive scaling, and intensity scaling (depending on the application). With the quadtree partitioning scheme, if the measured error between any of the transformed regions and the range cell is less than a designated threshold, the range cell is considered mapped and further partitioning of the cell is not performed. If the range cell is not mapped after taking into consideration all the eligible domain cells, further partitioning of the cell continues until the pre-designated partition limit is reached. This approach differs slightly from that used in some compression algorithms where the best fit is found, not just the first domain that maps within tolerance to the range. More detailed explanations of fractal encoding can be found in other references.<sup>1,2</sup>

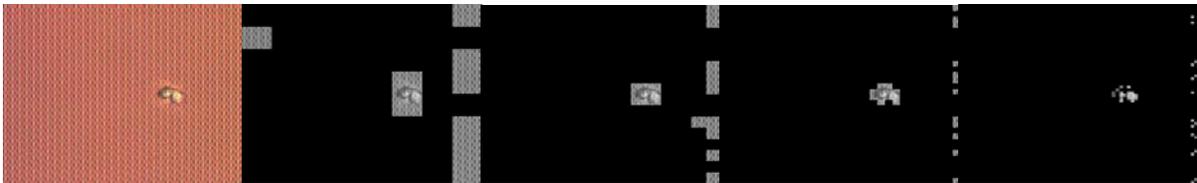
The next section discusses important implementation options of a fractal encoding scheme for segmentation purposes. It will also cover key parameters involved in the segmentation applications discussed in subsequent sections and the impact of these parameters on the fractal encoding process. Several examples illustrate the impact of adjusting these parameters and therefore provide guidance in their initial selection.

### **3. FRACTAL ENCODING OPTIONS AND PARAMETERS**

In fractal encoding there are a number of important decisions and options to be selected regarding range and domain generation and parameters used in the encoding process. The most important of these, (1) the minimum and maximum levels of partitioning, (2) the value of the similarity threshold, (3) the choice of contractive vs. non-contractive mapping, and (4) the choice of overlapping vs. non-overlapping domain cells are discussed in this section.

### 3.1. Minimum/maximum Partitioning Level

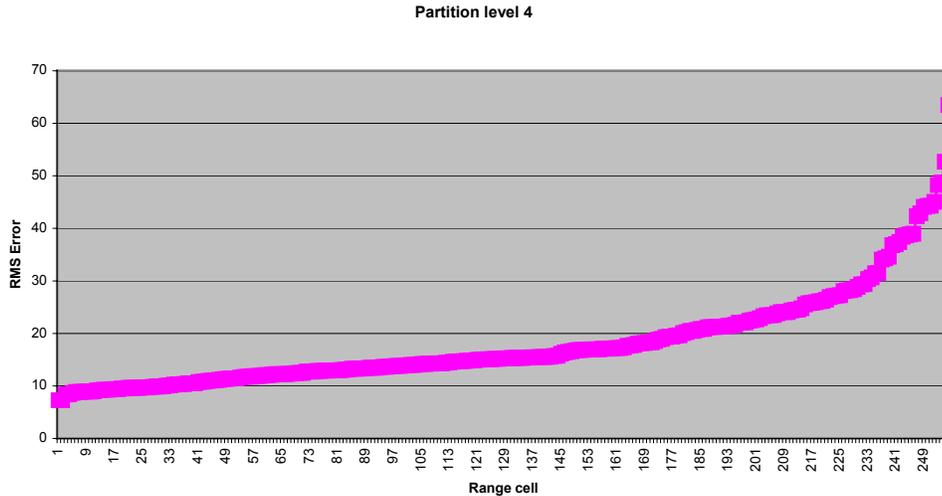
The quadtree partitioning scheme was chosen because it allows image regions with many details to be mapped more precisely using smaller cells and large smooth regions can be mapped more efficiently to large cells. The choice of a value for minimum and maximum level of quadtree partitioning is governed by (1) the size of typical structures found in the image and (2) the expected size of the anomaly or the object to be segmented. The finer and more uniform the background structure, the less sensitive the algorithm performance is to these parameters. This is shown in Figure 3 where a defect on a finely textured background could be accurately isolated over a number of different partition sizes. With all other fractal segmentation parameters held constant, the effect of increasing the partition level depth is observed. Because of the simplicity and relatively small size of the patterned background, this anomaly can be segmented out over a range of maximum partition sizes. Complex patterned images with large structures may be more sensitive to the initial choice of these parameters, and depending on the expected size of the anomaly to be segmented, may begin to be mapped into the regular background if partition sizes become too small.



**Figure 3 Segmentation of anomaly on a semiconductor image at a maximum partition level of 3, 4, 5, and 6 with all other parameters unchanged.**

### 3.2. Similarity Threshold

In the segmentation scheme for these two applications, a region is labeled as being representative of an anomaly if the transformed domain with the lowest root mean square (rms) error value for that range is above the designated similarity threshold; cells with an rms error below the similarity threshold are labeled as being normal or expected. The values of the rms error for the best match of each range cell of an image at a single partition level (all range cells are the same size) were ordered and plotted as shown in the graph in Figure 4. The y-axis represents the best match rms error-value for a range cell. The x-axis is representative of individual range cell indices that have been ordered in terms of their rms error. The optimal similarity threshold value depends on the range of rms error values over which anomalies tend to occur and fluctuates based on the partition size selected. If the similarity threshold is too high, everything in the image is mapped and no segmentation of anomalous regions occurs; similarly, if the value is too low, nothing in the image is mapped and the entire image is seen as being representative of the anomaly. The curve in figure 4 is fairly typical for semiconductor applications and a similarity threshold is selected near the point at which partitions display a dramatic change in rms error level. For the image analyzed in Figure 4, the defect was best isolated using a threshold value of approximately 45 at that partition level. At values of 50 and higher the defect began to be broken up and thresholds above 70 would map all the partition values into a single entity. A similarity threshold below 8 would not map any of the partition values. For defect or anomaly detection, fractal encoding will be most effective if the range partitions in which the defect fall have a high rms error when mapped relative to most of the other range partitions in the image. Note that the plot in Figure 4 could also be used to automatically select the value of this similarity threshold by choosing a value where the rms error from one partition to the next makes a relatively large change.



**Figure 4 Plot of the rms values for the partitions of an image at a partition level of 4.**

### 3.3. Contractive vs. Non-contractive Mapping

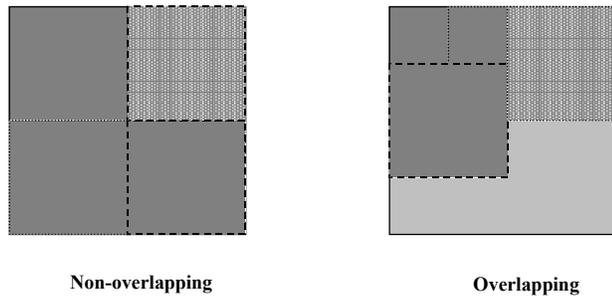
Fractal based compression techniques use contractive mappings, which shrink the dimensions of the domain cell for comparison with range cells. For these applications the domain was created using regions from the original image. Because a contractive mapping was used, domain partitioning always began and terminated at one level higher than range partitioning to ensure that there were always domain cells that could be contractively mapped to range cells. Since the ultimate goal of the fractal encoding in these applications was segmentation, not compression, a non-contractive mapping was a reasonable approach for the domain transformation. In cases where it was believed that a non-contractive mapping would provide better results, a non-contractive mapping was emulated by (1) creating the domain using an expanded interpolated image of double the width and height and (2) then using the pre-described contractive mappings. While a contractive domain mapping worked well for the amorphous like features of mammograms, a non-contractive mapping was found to be advantageous for images with larger structures like semiconductor images.

Intensity normalization is often a part of the mapping transformation. This normalization increases the similarity of two fairly uniform areas of differing intensity. From an encoding perspective this will improve the compression performance, but may have a cost in the decoded image quality. For the purposes of a segmentation algorithm, intensity normalization may be needed if the image is part of a natural scene or object, but not desirable if the image is some type of man-made pattern such as a semi-conductor layer. Another consideration for selecting image normalization is whether the domain and range cells come from a common source image. If their source is common, it may not be necessary or desirable to perform this operation. If the domain and range cells come from different sources, this normalization will probably be required.

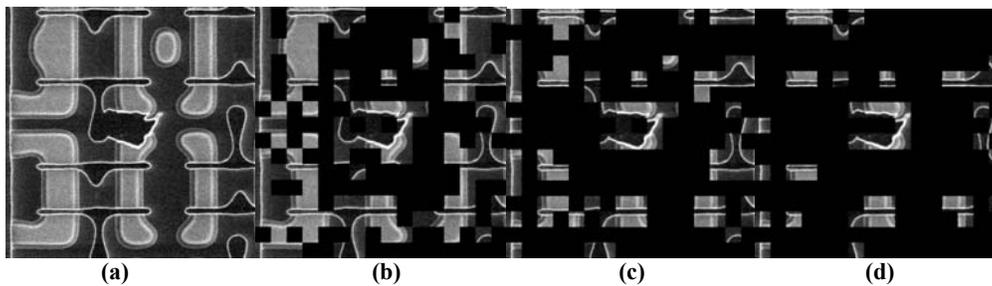
### 3.4. Overlapping vs. Non-overlapping Domain Cells

Domains can also be created using overlapping or non-overlapping cells as shown in Figure 5. Overlapping cells will result in much larger domain sets than using non-overlapping cells. For example, a domain consisting of the four quadrants of an image would expand to 9 domain cells when a simple 50% overlapping domain is used or by 25 domain cells when using domains that overlap by 75%. When there are relatively large structures in an image, an overlapping domain may more readily identify similar regions lying in different orientations or locations within a partition. This can be illustrated in the semiconductor application shown in Figure 6 where the use of overlapping partitions reduced the

irrelevant clutter, but allowed the detection of the defect. With the use of overlapping domains comes an increase in the number of comparisons to make and consequently, a longer computation time.



**Figure 5** Image broken down into 4 non-overlapping domains and the first 4 domain cells for the same image with 50% overlap of domain cells.



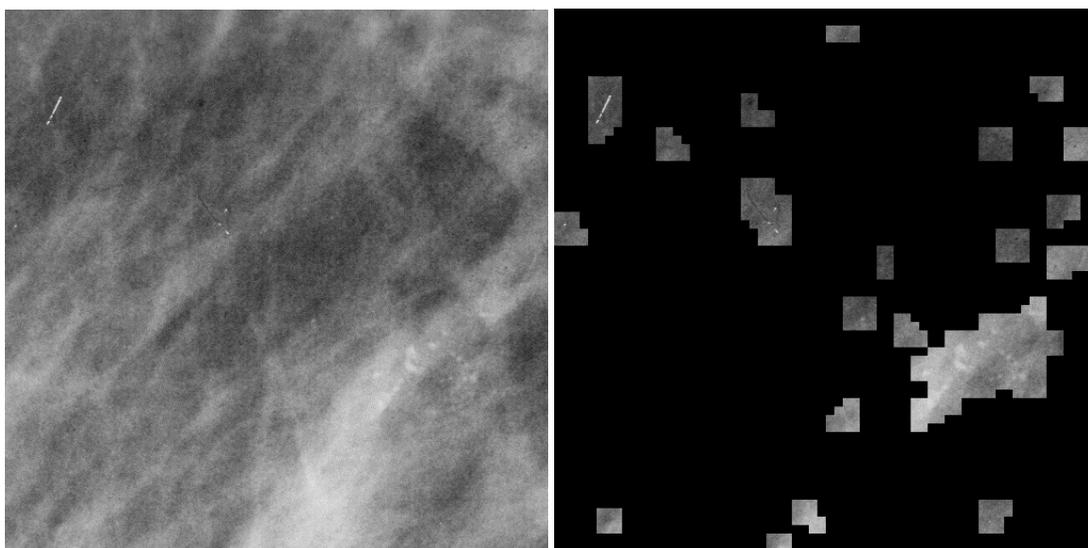
**Figure 6** Detection of a defect on a high-resolution image of a (a) semiconductor wafer using (b) non-overlapping domains, (c) domains overlapping by 50%, and (d) domains overlapping by 75%.

## 4. EXAMPLE APPLICATIONS

### 4.1. Computer Aided Diagnosis for Digital Mammography

Use of computer-aided diagnosis of mammograms is growing as an important screening aid for radiologists. One of the issues with these systems is throughput. A single mammogram image is on the order of 15-30 MB, and for a normal screening at least 4 mammogram images are generated. Both the handling of such large images and the processing applied to them can be quite computationally intensive. In this application, fractal encoding techniques are used to create “focus of attention” regions in which more complex image manipulations can be applied. There has also been research into using lossless compression techniques in these “focus of attention” areas and lossy techniques in other regions to minimize data loss while optimizing compression rates.

The concept of applying fractal encoding techniques to mammograms arose due to their visual similarity to clouds, which had been successfully modeled by fractals. As a segmentation tool, the fractal encoding finds a “match” to normal breast tissue and tissue that has a differing structure cannot be matched within tolerance to a domain region. Important abnormalities such as microcalcifications will appear different from the normal breast tissue. The application of this technique does not pinpoint the location of microcalcifications or other anomalies; it merely defines regions of unusual appearance. It is effective however in reducing the number of regions in which a more complex algorithm is applied.<sup>3</sup> An example of this reduction effect is shown in Figure 7.

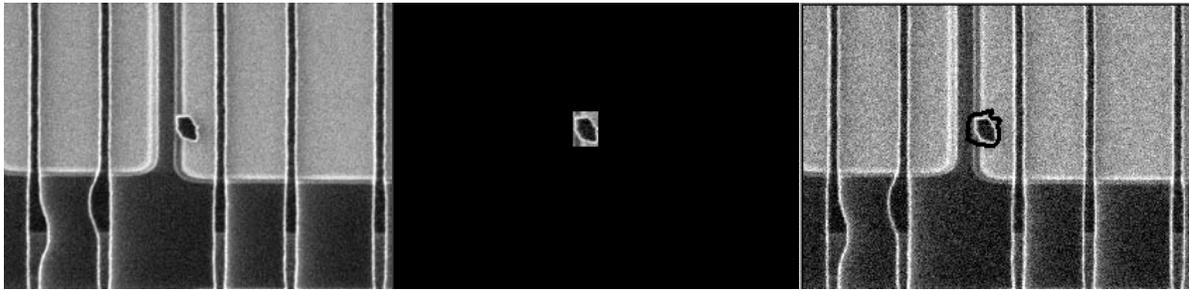


**Figure 7 Original mammogram and mammogram displaying only focus of attention regions. Note that the “anomalies” within the image, the microcalcifications (white speckles), have been highlighted in the focus of attention regions.**

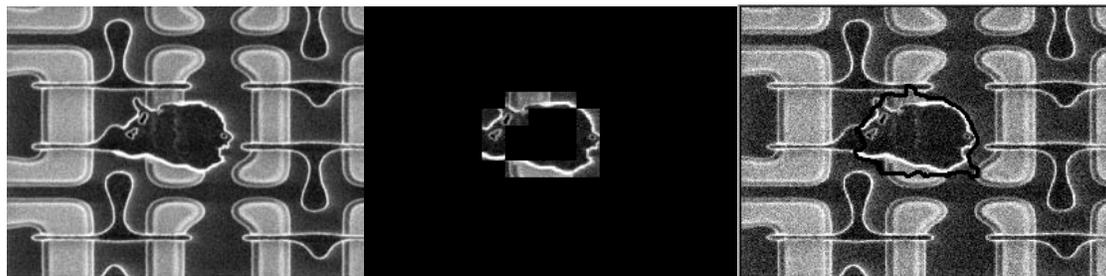
#### **4.2. Automated Semiconductor Defect Detection**

Semiconductor manufacturers maintain large databases of historical defect imagery that are used for diagnostic purposes. Because of storage requirements, reference images of non-defective imagery are rarely stored. Identifying these defects without the reference image is relatively easy to do by a human, but may require specific processing techniques and parameters highly tuned to the expected data for an automated vision system. Re-identifying and detecting such defects manually is cost prohibitive when a large number of images is involved. In order to automatically locate defects in multiple images from these databases, a technique was developed that used the defect image itself as a reference to segment out the approximate location of the defect. A secondary-processing step used deformable contours to more accurately pinpoint the boundaries of the defect. The concept behind the segmentation algorithm that makes fractal-encoding techniques useful is the expectation that the defect area will appear different from the normal image background structure.<sup>4</sup>

Typically, the images in these databases have been generated by an inspection tool that has located the defect and has centered the defect in its field of view. The fractal encoding technique used often isolates a number of non-matching regions, but these are evaluated or eliminated based on expectations of proximity to the center of the image and size. Examples of several Scanning Electron Microscope defect images and their corresponding fractal region segmented out as containing the defect are shown in Figures 8 and 9. The secondary step in this process uses deformable contours to better define the boundaries of the defect. Example results after this processing step are also shown in the previous figures. The borders were compared to manually delineated defect borders to evaluate the effectiveness of the algorithm.



**Figure 8 Original image (left), regions identified by fractal encoding (middle), image with defect contour outlined (right).**



**Figure 9 Original image (left), irregular borders detected with fractal encoding (middle), image with defect contour outlined in black (right).**

## 5. CONCLUSION

Use of fractal encoding as a segmentation tool can be useful in some specialized applications. For these applications proper selection of parameters is imperative for a successful implementation. Guidelines for the selection of the partitioning size based on the structure of the image were discussed. The selection of threshold criteria and various techniques to create the set of domain cells were also reviewed. The most critical parameters in determining algorithm performance, however, were the selection of the minimum and maximum partitioning levels and the value of the similarity threshold.

For the mammography application, encoding was not used in a strict sense for segmentation of anomalies, but for the elimination of normal data from consideration. For the semiconductor applications, fractal encoding was successful when it was able to map domain cells to range cells with irregular contours that often characterized defects. Defects that did not display irregular borders, an unusual orientation, or visible gray level variations were less likely to be properly detected. Expectations that the defect would fall near the center of the image were also useful in eliminating some areas that appeared anomalous. In general fractal encoding can be a useful image segmentation tool in cases where image characteristics tend to be somewhat predictable and repetitive and in which objects to be segmented tend to be irregular or different from the normal background structure.

There are a number of ways that fractal encoding can be implemented to perform segmentation. This paper has reviewed several options to examine in implementing this type of algorithm and has furnished some guidelines on parameter selection based on the image data used. This information is valuable in developing fractal encoding as a tool for other segmentation type applications.

## REFERENCES

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