Range Identification for Perspective Vision Systems*

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Abstract

In this paper, a new continuous observer is developed to determine range information (and hence the 3-dimensional (3D) coordinates) of an object feature moving with affine motion dynamics (or the more general Ricatti motion dynamics) with known motion parameters. The unmeasurable range information is determined from a single camera provided an observability condition is satisfied that has physical significance. To develop the observer, the perspective system is expressed in terms of the nonlinear feature dynamics. The structure of the proposed observer is inspired by recent disturbance observer results. The proposed technique facilitates a Lyapunov-based analysis that is less complex than the sliding-mode based analysis derived for recent discontinuous observer designs. The analysis demonstrates that the 3D task-space coordinates of the feature point can be asymptotically identified.

1 Introduction

The objective of most vision problems involves interpreting the motion of features of a 3-dimensional (3D) object through 2D images that are projected perspectively[1] from the 3D feature; hence, as stated in [5] vision systems are inherently perspective. Most research related to perspective systems have targeted the identification of the motion parameters (e.g., feature velocities) by using measureable state information. For example, for the following second order system [5]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{1}$$

the typical problem is to utilize the measureable states $x_1(t)$ and $x_2(t)$ to determine the unmeasurable parameters $a_i(t)$ $\forall i = 1, 2, 3, 4$ and possibly the unknown initial conditions $x_1(t_0)$ and $x_2(t_0)$. An excellent overview of research that has targeted this and similar problems (typically using an extended Kalman filter) is provided in [4, 5, 7].

In contrast to the class of perspective problems associated with using the measureable states to determine the parameters, several researchers have recently investigated the problem when the motion parameters are known along with the image-space feature coordinates, and the goal is to determine the unknown states (i.e., the actual 3D position of the feature). For example, a discontinuous recursive identifier based observer was developed in [7] to exponentially identify range information of features (i.e., points, lines, and planar curves) on an affine plane from successive images of a camera that is moving in a known manner (i.e., with known motion parameters). In [1], Chen and Kano develop a new discontinuous observer for a more general perspective system that exponentially forces the observation error to an arbitrarily small neighborhood (i.e., uniformly ultimately bounded (UUB)).

In this paper, we develop a continuous observer to determine range information (and hence, the 3D task-space coordinates) for an object feature moving with general affine motion dynamics with known motion parameters. As in [1, 10], the perspective system examined in this paper is more general than the skew-symmetric system examined in [7]. The unmeasurable range information is determined from a single camera provided an observability condition similar to [1, 7] is satisfied. As stated in [5], many geometric structures of a perspective system are lost if they are studied via linearization; hence, to develop the observer, the perspective system is transformed into a nonlinear dynamic system (i.e., the image-space feature dynamics). Based on the nonlinear dynamics of the image-space signals a continuous observer is designed that is inspired by the recent disturbance observer results in [3, 8]. The structure of the proposed observer facilitates a Lyapunov-based analysis that is less complex than the sliding-mode based analysis derived for the discontinuous observer design of [1] (and the unknown states can be exactly determined rather than "almost" determined

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[1] Other projective models (e.g., Orthographic Projection) have also been used in literature for vision research; however, the most commonly accepted model is the perspective projection.

as in the UUB result in [1]). The analysis demonstrates that the 3D task-space coordinates of the feature point can be asymptotically identified. The proposed observer can also be applied to object motion described by Ricatti dynamics and can be extended to n-dimensional perspective systems.

2 Perspective System

Consider an object feature undergoing an affine motion as follows [1, 10]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (2)$$

where $x_1(t),\ x_2(t),\ x_3(t) \in \mathbb{R}$ denote the unmeasurable task-space coordinates of an object feature along the X, Y, and Z axes of an inertial reference frame, respectively, with the Z axis being perpendicular with an image plane formed by a camera (i.e., the coordinate $x_3(t)$ denotes the depth from the image plane to the task-space object feature along the optical axis Z). In (2), the parameters $a_{i,j}(t) \in \mathbb{R}$ and $b_i(t)$ $\forall i, j = 1, 2, 3$ denote the known motion parameters [1, 10]. The affine motion dynamics introduced in (2) are expressed in a general form that describes an object motion that undergoes a rotation, translation, and linear deformation [10]. The measurable image-space coordinate of a feature, denoted by $y(t) \in \mathbb{R}^2$, is given as follows

$$y \triangleq \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} \frac{x_1}{x_3} & \frac{x_2}{x_3} \end{bmatrix}^T. \quad (3)$$

The affine dynamics introduced in (2) and the image-space signal introduced in (3) define the perspective system [1]. After taking the time derivative of (3) and utilizing (2), the image-space trajectory of the object feature can be obtained as follows

$$\dot{y}_1 = \frac{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1}{x_3} - \frac{x_1\left(a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_3\right)}{x_3^2} \quad (4)$$

$$\dot{y}_2 = \frac{a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2}{x_3} - \frac{x_2\left(a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_3\right)}{x_3^2}. \quad (5)$$

To facilitate subsequent analysis, the time derivative of the inverse of $x_3(t)$ is determined as follows

$$\frac{d}{dt}\left(\frac{1}{x_3}\right) = \frac{-a_{31}x_1 - a_{32}x_2 - a_{33}x_3 - b_3}{x_3^2}. \quad (6)$$

By utilizing (3), the expressions given in (4-6) can be rewritten as follows

$$\dot{y}_1 = a_{13} + \left(a_{11} - a_{33}\right)y_1 + a_{12}y_2 - a_{31}y_1^2 - a_{32}y_1y_2 + f_1 \quad (7)$$

$$\dot{y}_2 = a_{23} + a_{21}y_1 + \left(a_{22} - a_{33}\right)y_2 - a_{32}y_2^2 - a_{31}y_1y_2 + f_2 \quad (8)$$

$$\frac{d}{dt}\left(\frac{1}{x_3}\right) = -\frac{1}{x_3}\left(a_{31}y_1 + a_{32}y_2 + a_{33}\right) - \frac{b_3}{x_3^2} \quad (9)$$

where $f_1(x_3,\ y_1),\ f_2(x_3,\ y_2) \in \mathbb{R}$ are unmeasurable signals[2] defined as follows

$$f_1 \triangleq \frac{1}{x_3}\left(b_1 - b_3y_1\right) \quad (10)$$

$$f_2 \triangleq \frac{1}{x_3}\left(b_2 - b_3y_2\right). \quad (11)$$

For the perspective system given in (2) and (3), the following assumptions are made [1].

Assumption 1: The known motion parameters $a_{i,j}(t)$ and $b_i(t)$ $\forall i, j = 1, 2, 3$ introduced in (2) are bounded functions of time, the parameters $a_{i,j}(t)$ are first order differentiable, and the parameters $b_i(t)$ are second order differentiable.

Assumption 2: The image-space feature coordinates $y_1(t)$ and $y_2(t)$ are bounded functions of time (i.e., $y_1(t),\ y_2(t) \in \mathcal{L}_\infty$).

Assumption 3: The object feature motion avoids the degenerate case where the feature intersects the image plane. That is, $x_3(t) > \varepsilon_0$ where $\varepsilon_0 \in \mathbb{R}$ is an arbitrarily small positive constant, and hence, $\frac{1}{x_3(t)} \in \mathcal{L}_\infty$. Moreover, we also assume that $x_3(t) \in \mathcal{L}_\infty$.

Remark 1 *Assumptions 2 and 3 are standard assumptions (see also [1, 7]) that are practically properties of the physical system rather than assumptions.*

Remark 2 *Based on Assumptions 1-3, the expressions given in (2) and (7-11) can be used to determine that* $\dot{x}_3(t),\ \dot{y}(t),\ \frac{d}{dt}\left(\frac{1}{x_3(t)}\right),\ f_1(x_3,\ y_1),\ f_2(x_3, y_2) \in \mathcal{L}_\infty$. *Given that these signals are bounded, the development provided in the Appendix can be used along with Assumptions 1-3 to also determine that* $\dot{f}_1(\cdot),\ \dot{f}_2(\cdot),\ \ddot{f}_1(\cdot)$, *and* $\ddot{f}_2(\cdot) \in \mathcal{L}_\infty$.

3 Observation Problem

3.1 Objective

The objective in this paper is to determine the unmeasurable state $x_3(t)$ of the perspective vision system described by (2) and (3). From (3) and the fact that $y_1(t)$ and $y_2(t)$ are measurable, it is clear that if $x_3(t)$ is identified then the complete 3D task-space coordinate of the feature can be determined. To achieve this objective, an observer is constructed based on the unmeasurable image-space dynamics for $y(t)$. To quantify the performance of the

[2] The signals $f_1(x_3,\ y_1)$, and $f_2(x_3,\ y_2)$ are unmeasurable due to a dependence on the unmeasurable state $x_3(t)$.

observer, a measurable observer estimation error signal, denoted by $e(t) \in \mathbb{R}^2$, is defined as follows

$$e \triangleq \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T = \begin{bmatrix} y_1 - \hat{y}_1 & y_2 - \hat{y}_2 \end{bmatrix}^T \tag{12}$$

where $\hat{y}(t) \triangleq [\hat{y}_1(t), \hat{y}_2(t)]^T \in \mathbb{R}^2$ denotes a subsequently designed observer signal. To facilitate the subsequent development, a filtered observation error signal, denoted by $r(t) \in \mathbb{R}^2$, is designed as follows

$$r \triangleq \begin{bmatrix} r_1 & r_2 \end{bmatrix}^T = \begin{bmatrix} \dot{e}_1 + \alpha_1 e_1 & \dot{e}_2 + \alpha_2 e_2 \end{bmatrix}^T \tag{13}$$

where $\alpha_1,\ \alpha_2 \in \mathbb{R}$ denote positive constant gains. Based on the dynamics in (7) and (8) and the definitions introduced in (12) and (13), it is clear that $r(t)$ is unmeasurable due to the fact that $\dot{y}(t)$ is a function of the unmeasurable disturbance terms $f_1(x_3,\ y_1)$ and $f_2(x_3,\ y_2)$. The subsequent development will target the design of estimates for $f_1(x_3,\ y_1)$ and $f_2(x_3,\ y_2)$ based on the strategy that if the mismatch between the estimates and the disturbance terms $f_1(x_3,\ y_1)$ and $f_2(x_3,\ y_2)$ can be driven to zero, then $x_3(t)$ can be identified by exploiting the fact that $b_i(t)\ \forall i = 1, 2, 3$ and the states $y_1(t)$ and $y_2(t)$ are measurable. Specifically, from (10) and (11), the inverse of the square of $x_3(t)$ can be determined as follows

$$\left(\frac{1}{x_3}\right)^2 = \frac{f_1^2 + f_2^2}{(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2}\ . \tag{14}$$

Based on the structure of (14), it is clear that the following observability condition must be satisfied

$$(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2 > 0\ . \tag{15}$$

That is, $x_3(t)$ can be identified once the mismatch between the disturbance terms $f_1(x_3,\ y_1)$ and $f_2(x_3,\ y_2)$ and the respective estimates are driven to zero.

Remark 3 *The observability condition introduced in (15) is not required by the subsequent analysis to prove that the observer design remains bounded. That is, the subsequent analysis can be used to prove that $f_1(x_3,\ y_1)$ and $f_2(x_3, y_2)$ can be identified independently of (15); however, (15) is required to prove that $x_3(t)$ can be identified. In [7], a discussion is provided regarding the physical justification of (15) with regard to the focus of expansion.*

3.2 Observer Design and Error System

By taking the time-derivative of (12) the following error dynamics can be obtained for $e(t)$

$$\dot{e} = \dot{y} - \dot{\hat{y}}\ . \tag{16}$$

Based on the structure of (7), (8), and (16), the elements of the observer signal $\hat{y}(t)$ are designed as follows

$$\begin{aligned} \dot{\hat{y}}_1 \ &= \ a_{13} + (a_{11} - a_{33})\, y_1 \\ &\quad + a_{12} y_2 - a_{31} y_1^2 - a_{32} y_1 y_2 + \hat{f}_1 \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{\hat{y}}_2 \ &= \ a_{23} + a_{21} y_1 + (a_{22} - a_{33})\, y_2 \\ &\quad - a_{32} y_2^2 - a_{31} y_1 y_2 + \hat{f}_2 \end{aligned} \tag{18}$$

where $\hat{f}_1(t),\ \hat{f}_2(t) \in \mathbb{R}$ denote subsequently designed estimates for the unmeasurable signals $f_1(t)$ and $f_2(t)$ introduced in (7) and (8). After substituting (7), (8), (17), and (18) into (16), the following error dynamics are obtained

$$\dot{e} = \begin{bmatrix} f_1 - \hat{f}_1 & f_2 - \hat{f}_2 \end{bmatrix}^T. \tag{19}$$

By taking the time-derivative of (13) the following error dynamics can be obtained for $r(t)$

$$\dot{r} = \begin{bmatrix} \dot{f}_1 - \dot{\hat{f}}_1 + \alpha_1 \left(f_1 - \hat{f}_1\right) \\ \dot{f}_2 - \dot{\hat{f}}_2 + \alpha_2 \left(f_2 - \hat{f}_2\right) \end{bmatrix} \tag{20}$$

where (19) and the time derivative of (19) have been utilized. Based on the structure of (20) and the subsequent analysis, the estimates $\hat{f}_1(t)$ and $\hat{f}_2(t)$ are designed as follows[3]

$$\dot{\hat{f}}_1 = -(k_{s1} + \alpha_1)\hat{f}_1 + \gamma_1 sgn(e_1) + \alpha_1 k_{s1} e_1 \tag{21}$$

$$\dot{\hat{f}}_2 = -(k_{s2} + \alpha_2)\hat{f}_2 + \gamma_2 sgn(e_2) + \alpha_2 k_{s2} e_2 \tag{22}$$

where $k_{s1},\ k_{s2},\ \gamma_1,\ \gamma_2 \in \mathbb{R}$ denote constant observer gains and the notation $sgn(\cdot)$ is used to indicate the standard, signum function. After substituting (21) and (22) into (20) and then adding and subtracting the terms $k_{s1} f_1(x_3,\ y_1)$ and $k_{s2} f_2(x_3,\ y_2)$, the following expression can be obtained

$$\dot{r} = \eta - \begin{bmatrix} k_{s1} r_1 + \gamma_1 sgn(e_1) \\ k_{s2} r_2 + \gamma_2 sgn(e_2) \end{bmatrix} \tag{23}$$

where $\eta(t) \triangleq \begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix}^T \in \mathbb{R}^2$ is defined as follows

$$\eta \triangleq \begin{bmatrix} \dot{f}_1 + (k_{s1} + \alpha_1)\, f_1 \\ \dot{f}_2 + (k_{s2} + \alpha_2)\, f_2 \end{bmatrix}. \tag{24}$$

Remark 4 *The time derivative of (24) can be determined as follows*

$$\dot{\eta} = \begin{bmatrix} \ddot{f}_1 + (k_{s1} + \alpha_1)\, \dot{f}_1 \\ \ddot{f}_2 + (k_{s2} + \alpha_2)\, \dot{f}_2 \end{bmatrix}. \tag{25}$$

From (24) and (25), the statements in Remark 2 can be used to conclude that $\eta(t),\ \dot{\eta}(t) \in \mathcal{L}_\infty$.

Remark 5 *The structure of the disturbance observer given by (21) and (22) contains discontinuous terms; however, it is interesting to note that the overall structure of the observer is not discontinuous. That is, after a close examination of (17) and (18), it is clear that $\dot{\hat{y}}_1(t)$ and*

[3] The design of the estimates $\hat{f}_1(t)$ and $\hat{f}_2(t)$ is inspired by the development given in [3, 8].

$\dot{\hat{y}}_2(t)$ *only contain the low pass filtered outputs* $\hat{f}_1(t)$ *and* $\hat{f}_2(t)$ *of the discontinuous terms in (21) and (22). Therefore, since the observer strategy only utilizes expressions that are functions of* $\hat{f}_1(t)$ *and* $\hat{f}_2(t)$*, the applied observer signals are not discontinuous.*

4 Analysis

The following theorem and associated proof can be used to conclude that the observer design of (17), (18), (21), and (22) can be used to identify the unmeasurable state $x_3(t)$.

Theorem 1 *Given the perspective system in (2) and (3), the unmeasurable state* $x_3(t)$ *(and hence, the 3D task-space coordinates of the object feature) can be asymptotically determined using the observer design given in (17), (18), (21), and (22) provided the constants* γ_1 *and* γ_2 *introduced in (21) and (22) are selected according to the following sufficient conditions*

$$\gamma_1 \geq |\eta_1| + |\dot{\eta}_1| \qquad \gamma_2 \geq |\eta_2| + |\dot{\eta}_2| \tag{26}$$

where $\eta(t)$ *is defined in (24), and the observability condition introduced in (15) is satisfied.*

Proof: To prove Theorem 1, we first define a non-negative function $V(t)$ as follows

$$V \triangleq \frac{1}{2} r^T r \ . \tag{27}$$

After taking the time derivative of (27) and substituting for the error system dynamics given in (23), the following expression can be obtained

$$\begin{aligned} \dot{V} = & -k_{s1} r_1^2 - k_{s2} r_2^2 \\ & + (\dot{e}_1 + \alpha_1 e_1)(\eta_1 - \gamma_1 sgn(e_1)) \\ & + (\dot{e}_2 + \alpha_2 e_2)(\eta_2 - \gamma_2 sgn(e_2)) \,. \end{aligned} \tag{28}$$

After integrating (28) and exploiting the fact that

$$\xi \cdot sgn(\xi) = |\xi| \ ,$$

the following inequality can be obtained

$$\begin{aligned} V(t) \leq & \ V(t_0) - \int_{t_0}^{t} \left(k_{s1} r_1^2(\sigma) + k_{s2} r_2^2(\sigma) \right) d\sigma \\ & + \alpha_1 \int_{t_0}^{t} |e_1(\sigma)| \left(|\eta_1(\sigma)| - \gamma_1 \right) d\sigma + \Omega_1 \\ & + \alpha_2 \int_{t_0}^{t} |e_2(\sigma)| \left(|\eta_2(\sigma)| - \gamma_2 \right) d\sigma + \Omega_2 \end{aligned} \tag{29}$$

where the auxiliary terms $\Omega_1(t), \Omega_2(t) \in \mathbb{R}$ are defined as follows

$$\begin{aligned} \Omega_1 \triangleq & \int_{t_0}^{t} \dot{e}_1(\sigma)\, \eta_1(\sigma)\, d\sigma \\ & - \gamma_1 \int_{t_0}^{t} \dot{e}_1(\sigma)\, sgn(e_1(\sigma)) d\sigma \end{aligned} \tag{30}$$

$$\begin{aligned} \Omega_2 \triangleq & \int_{t_0}^{t} \dot{e}_2(\sigma)\, \eta_2(\sigma)\, d\sigma \\ & - \gamma_2 \int_{t_0}^{t} \dot{e}_2(\sigma)\, sgn(e_2(\sigma)) d\sigma . \end{aligned} \tag{31}$$

After evaluating the integral expressions in (30), the following expressions can be obtained

$$\begin{aligned} \Omega_1 = & \ e_1(\sigma)\, \eta_1(\sigma)|_{t_0}^{t} d\sigma - \int_{t_0}^{t} e_1(\sigma)\, \dot{\eta}_1(\sigma)\, d\sigma \\ & - \gamma_1 |e_1(\sigma)|\, |_{to}^{t} \\ = & \ e_1(t)\, \eta_1(t) - \int_{t_0}^{t} e_1(\sigma)\, \dot{\eta}_1(\sigma)\, d\sigma - \gamma_1 |e_1(t)| \\ & - e_1(t_0)\, \eta_1(t_0) + \gamma_1 |e_1(t_0)| \,. \end{aligned} \tag{32}$$

By performing the same operations, $\Omega_2(t)$ can be evaluated as follows

$$\begin{aligned} \Omega_2 = & \ e_2(t)\, \eta_2(t) - \int_{t_0}^{t} e_2(\sigma)\, \dot{\eta}_2(\sigma)\, d\sigma \\ & - \gamma_2 |e_2(t)| - e_2(t_0)\, \eta_2(t_0) \\ & + \gamma_2 |e_2(t_0)| \,. \end{aligned} \tag{33}$$

After substituting (32) and (33) into (29) and performing some algebraic manipulation, the following inequality can be obtained

$$\begin{aligned} V(t) \leq & \ V(t_0) - \int_{t_0}^{t} \left(k_{s1} r_1^2(\sigma) + k_{s2} r_2^2(\sigma) \right) d\sigma \\ & + \Omega_3 + \zeta_0 \end{aligned} \tag{34}$$

where the auxiliary terms $\Omega_3(t), \zeta_0 \in \mathbb{R}$ are defined as follows

$$\begin{aligned} \Omega_3 \triangleq & \ \alpha_1 \int_{t_0}^{t} |e_1(\sigma)| \left(|\eta_1(\sigma)| + |\dot{\eta}_1(\sigma)| - \gamma_1 \right) d\sigma \\ & + \alpha_2 \int_{t_0}^{t} |e_2(\sigma)| \left(|\eta_2(\sigma)| + |\dot{\eta}_2(\sigma)| - \gamma_2 \right) d\sigma \\ & + |e_1(t)| \left(|\eta_1(t)| - \gamma_1 \right) + |e_2(t)| \left(|\eta_2(t)| - \gamma_2 \right) \end{aligned} \tag{35}$$

$$\begin{aligned} \zeta_0 \triangleq & - e_1(t_0)\, \eta_1(t_0) + \gamma_1 |e_1(t_0)| \\ & - e_2(t_0)\, \eta_2(t_0) + \gamma_2 |e_2(t_0)| \,. \end{aligned} \tag{36}$$

Provided the constants γ_1 and γ_2 are selected according to the inequalities introduced in (26), $\Omega_3(t)$ will always be negative or zero; hence, the following upper bound can be developed

$$V(t) \leq V(t_0) - \int_{t_0}^{t} \left(k_{s1} r_1^2(\sigma) + k_{s2} r_2^2(\sigma) \right) d\sigma + \zeta_0 \ . \tag{37}$$

From (27) and (37), the following inequalities can be determined

$$V(t_0) + \zeta_0 \geq V(t) \geq 0; \tag{38}$$

hence, $r(t) \in \mathcal{L}_\infty$. The expression in (37) can be used to determine that

$$\begin{aligned} \int_0^\infty \left(k_{s1} r_1^2 + k_{s2} r_2^2\right) d\sigma &\leq V(t_0) + \zeta_0 - V(\infty) \quad (39) \\ &\leq V(t_0) + \zeta_0 < \infty \ . \end{aligned}$$

By definition, (39) can now be used to prove that $r(t) \in \mathcal{L}_2$. From the fact that $r(t) \in \mathcal{L}_\infty$, (12) and (13) can be used to prove that $e(t)$, $\dot{e}(t)$, $\hat{y}(t)$, and $\dot{\hat{y}}(t) \in \mathcal{L}_\infty$. The expressions in (17), (18), (21), and (22) can be used to determine that $\hat{f}_1(t)$, $\hat{f}_2(t)$, $\dot{\hat{f}}_1(t)$, and $\dot{\hat{f}}_2(t) \in \mathcal{L}_\infty$. Based on the facts that $f_1(x_3, y_1)$, $f_2(x_3, y_2)$, $\dot{f}_1(\cdot)$, and $\dot{f}_2(\cdot) \in \mathcal{L}_\infty$, the expressions in (23) and (24) can be used to prove that $\eta(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$. Based on the fact that $r(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$ and that $r(t) \in \mathcal{L}_2$, Barbalat's Lemma [9] can be used to prove that

$$\lim_{t\to\infty} r(t) = 0 \ . \tag{40}$$

From (40), Lemma 1.6 of [2] can be used to prove that

$$\lim_{t\to\infty} e(t), \dot{e}(t) = 0 \ . \tag{41}$$

Given the result in (41), the expression given in (12) can be used to determine that

$$\lim_{t\to\infty} \hat{y}_1(t) = y_1 \qquad \lim_{t\to\infty} \hat{y}_2(t) = y_2 \tag{42}$$

and (19) can be used to determine that

$$\lim_{t\to\infty} \hat{f}_1(t) = f_1 \qquad \lim_{t\to\infty} \hat{f}_2(t) = f_2 \ . \tag{43}$$

If the observability condition given in (15) is satisfied (i.e., if either $f_1(x_3, y_1)$ or $f_2(x_3, y_2)$ are nonzero), then the result in (43), the fact that the parameters $b_i(t)$ $\forall i = 1, 2, 3$ are assumed to be known, and the fact that the image-space signal $y(t)$ is measurable can be used to identify the unknown task-space parameter $x_3(t)$ from (14). Once $x_3(t)$ is identified, the complete 3D task-space coordinates of the object feature can be determined from (3). □

Remark 6 *In addition to the general affine motion model considered in (2), several results in literature have examined the following Riccati motion dynamics (e.g., [1, 4, 6])*

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (44) \\ &+ \begin{bmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & c_1 & 0 & c_2 & c_3 & 0 \\ 0 & 0 & c_1 & 0 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_1 x_3 \\ x_2^2 \\ x_2 x_3 \\ x_3^2 \end{bmatrix} . \end{aligned}$$

For these dynamics, the same expressions given in (7-11) can be obtained, and hence, the observer design in (17), (18), (21), and (22) still applies for this motion field. Note that the discontinuous observer developed in [1] for the affine motion dynamics in (2) can also be applied to the motion dynamics in (44). As in [1], the observer system design in this paper can also be extended to general n-dimensional perspective systems.

5 Conclusion

In this paper a new continuous observer inspired by the development in [8] was developed to identify an unmeasurable range signal (and hence the 3D task-space coordinates of an object feature) via a single camera given the motion parameters of a general affine system (or Ricatti system). To develop the observer the affine perspective system is transformed into the nonlinear feature dynamics. Through a Lyapunov-based analysis, the observer was proven to asymptotically regulate the observation errors. The impact of these results are that a continuous observer can be used to enable a monocular vision system to identify the range parameter (even in the presence of sensor noise) of an object moving (with known motion parameters) with an affine or Riccati motion dynamics.

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A Appendix

To prove that $\dot{f}_1(\cdot),\ \dot{f}_2(\cdot) \in \mathcal{L}_\infty$, the time derivative of (10) and (11) is determined as follows

$$\dot{f}_1 = \frac{1}{x_3}\left(\dot{b}_1 - \dot{b}_3 y_1 - b_3\dot{y}_1\right) + \frac{d}{dt}\left(\frac{1}{x_3}\right)(b_1 - b_3 y_1) \quad (45)$$

$$\dot{f}_2 = \frac{1}{x_3}\left(\dot{b}_2 - \dot{b}_3 y_2 - b_3\dot{y}_2\right) + \frac{d}{dt}\left(\frac{1}{x_3}\right)(b_2 - b_3 y_2)\ . \quad (46)$$

The facts that $\dot{y}(t),\ \frac{d}{dt}\left(\frac{1}{x_3(t)}\right) \in \mathcal{L}_\infty$ can be used along with Assumptions 1-3 to conclude from (45) and (46) that $\dot{f}_1(\cdot)$, and $\dot{f}_2(\cdot) \in \mathcal{L}_\infty$. To prove that $\ddot{f}_1(\cdot),\ \ddot{f}_2(\cdot) \in \mathcal{L}_\infty$, the time derivative of (45) and (46) can be determined as follows

$$\begin{aligned}\ddot{f}_1 = {} & \frac{d}{dt}\left(\frac{1}{x_3}\right)\left(\dot{b}_1 - \dot{b}_3 y_1 - b_3\dot{y}_1\right) \\ & + \frac{1}{x_3}\left(\ddot{b}_1 - \ddot{b}_3 y_1 - 2\dot{b}_3\dot{y}_1 - b_3\ddot{y}_1\right) \\ & + \frac{d^2}{dt^2}\left(\frac{1}{x_3}\right)(b_1 - b_3 y_1) \\ & + \frac{d}{dt}\left(\frac{1}{x_3}\right)\left(\dot{b}_1 - \dot{b}_3 y_1 - b_3\dot{y}_1\right)\end{aligned} \quad (47)$$

$$\begin{aligned}\ddot{f}_2 = {} & \frac{d}{dt}\left(\frac{1}{x_3}\right)\left(\dot{b}_2 - \dot{b}_3 y_2 - b_3\dot{y}_2\right) \\ & + \frac{1}{x_3}\left(\ddot{b}_2 - \ddot{b}_3 y_2 - 2\dot{b}_3\dot{y}_2 - b_3\ddot{y}_2\right) \\ & + \frac{d^2}{dt^2}\left(\frac{1}{x_3}\right)(b_2 - b_3 y_2) \\ & + \frac{d}{dt}\left(\frac{1}{x_3}\right)\left(\dot{b}_2 - \dot{b}_3 y_2 - b_3\dot{y}_2\right)\end{aligned} \quad (48)$$

where

$$\begin{aligned}\ddot{y}_1 = {} & \dot{a}_{13} + (\dot{a}_{11} - \dot{a}_{33})\, y_1 + (a_{11} - a_{33})\, \dot{y}_1 \\ & + \dot{a}_{12} y_2 + a_{12}\dot{y}_2 - \dot{a}_{31} y_1^2 - 2a_{31} y_1 \dot{y}_1 \\ & - \dot{a}_{32} y_1 y_2 - a_{32}\dot{y}_1 y_2 - a_{32} y_1 \dot{y}_2 + \dot{f}_1\end{aligned} \quad (49)$$

$$\begin{aligned}\ddot{y}_2 = {} & \dot{a}_{23} + \dot{a}_{21} y_1 + a_{21}\dot{y}_1 + \\ & (\dot{a}_{22} - \dot{a}_{33})\, y_2 + (a_{22} - a_{33})\, \dot{y}_2 \\ & - \dot{a}_{32} y_2^2 - 2a_{32} y_2 \dot{y}_2 - \dot{a}_{31} y_1 y_2 \\ & - a_{31}\dot{y}_1 y_2 - a_{31} y_1 \dot{y}_2 + \dot{f}_2\end{aligned} \quad (50)$$

$$\begin{aligned}\frac{d^2}{dt^2}\left(\frac{1}{x_3}\right) = {} & -\frac{d}{dt}\left(\frac{1}{x_3}\right)(a_{31} y_1 + a_{32} y_2 + a_{33}) \\ & - \frac{\dot{b}_3}{x_3^2} + 2\frac{b_3 \dot{x}_3}{x_3^3} \\ & - \frac{1}{x_3}\left(\dot{a}_{31} y_1 + a_{31}\dot{y}_1 + \dot{a}_{32} y_2 + a_{32}\dot{y}_2 + \dot{a}_{33}\right)\end{aligned} \quad (51)$$

and the expressions given in (7-11) were utilized. The expressions given in (47-51), Assumptions 1-3, and the facts that $\dot{x}_3(t),\ \dot{y}(t),\ \frac{d}{dt}\left(\frac{1}{x_3(t)}\right),\ f_1(x_3,\ y_1),\ f_2(x_3, y_2) \in \mathcal{L}_\infty$ can now be used to prove that $\ddot{f}_1(\cdot),\ \ddot{f}_2(\cdot) \in \mathcal{L}_\infty$.