Targeting Qubit States using Open-Loop Control

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Outline of Presentation

- Quantum bang-bang control
- Tailored open-loop control of decoherence
- New Results: Targeting Qubit States
  - Spin-Boson Model
  - Control Strategy
  - Arbitrary Target State
- Comparison with quantum feedback scheme

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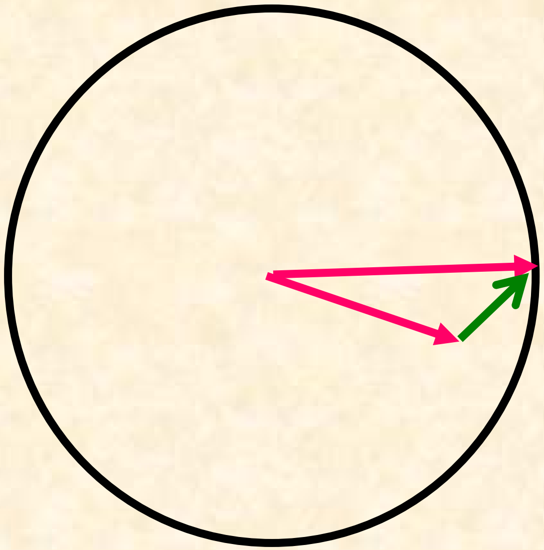
Motivation for Open-Loop Control

- **Quantum bang-bang control:**

Viola L. and Lloyd S., Phys. Rev. A **58**(4) 2733 (1998)

AIM: Maintain the state of a two-level system, using a **rapid** sequence of identical control pulses to counteract the effect of environmental **decoherence**



Previous Research at CESAR

Tailored open-loop control of decoherence

V. Protopopescu, R. B. Perez, C. D'Helon, and J. Schmulen

Preprint: quant-ph/0202141

- Decoherence and control are taken to act **simultaneously**
- Deals with general decoherence processes
- The required control is **tailored** to the (known) decoherence effects
- The state to be maintained must be **known** *a priori*

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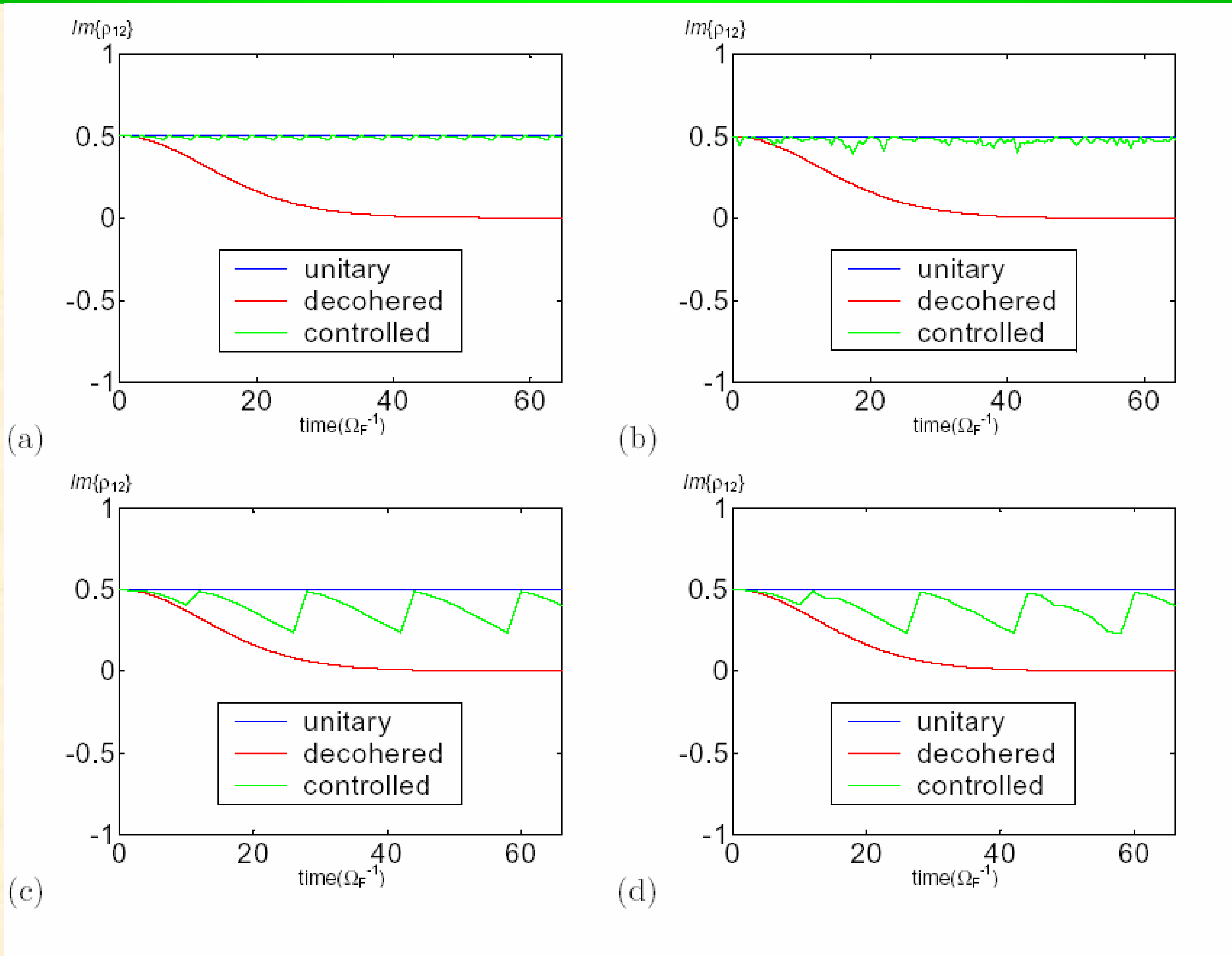
Control of Adiabatic Decoherence



Graph 1. Control results for the initial state: i/√2|1> + 1/√2|2>

New Results:
Targeting Qubit States

- Drive the state of a two-level system to an arbitrary **pure target state**, and then maintain its coherence

- Comparison of performance vs. **quantum feedback scheme** for the spontaneous emission of a two-level atom

  Wang, Wiseman and Milburn, Phys. Rev. A 64 # 063810 (2001)

- Extending the applicability of the quantum feedback scheme to **arbitrary** pure target states

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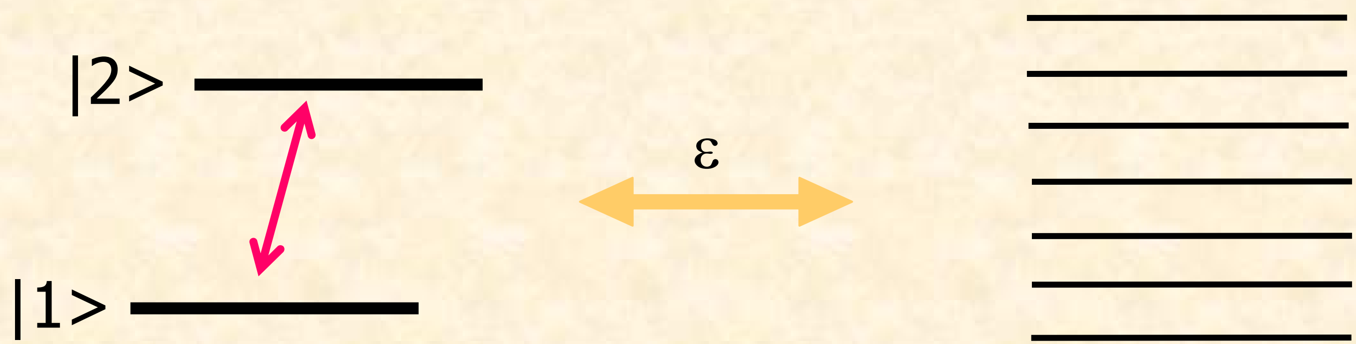
Qubit + Environment

- Hamiltonian: $$H = H_s + H_e + H_{se} + H_c,$$
- The whole system evolves unitarily:

$$\frac{\partial \rho}{\partial \tau} = -\frac{i}{\hbar}[H_I, \rho]$$



Two-level system Interaction Environment

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Spin-Boson Model

$$H_s = \hbar\omega_0\sigma_z$$

$$H_e = \hbar\sum_{q=1}^{\infty}\omega_{0q}a_q^{\dagger}a_q,$$

Thermal decoherence
(slower)

Adiabatic decoherence
(faster)

$$H_i = -\hbar\epsilon(\alpha_x\sigma_x + \beta_z\sigma_z)\sum_{q=1}^{\infty}(\Omega_q^{\star}a_q + \Omega_q a_q^{\dagger})$$

$$H_c = -\hbar\Omega_F V(t)[c_x\sigma_x + c_y\sigma_y],$$

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Evolution of the Qubit

- Transform to interaction-picture (rotating wave approx. and zero detuning)

- The evolution operator has a formal solution:

$$U_I(s,e,\tau) = \mathcal{T}[\exp\{-\frac{i}{\hbar}\int_0^\tau d\tau' H_I(\tau')\}],$$

- Use general Baker-Hausdorff theorem to expand the evolution operator into an infinite product of exponentials:

$$e^{-\frac{i}{\hbar}\int_0^\tau dt H_1(t)} \times e^{-\frac{i}{\hbar}\int_0^\tau dt H_2(t)} \times e^{-(\frac{i}{\hbar})^2\int_0^\tau dt \int_0^t dt' [H_1(t),H_2(t')]} \times \ldots$$

- **H_I=H_1+H_2** ;non-commuting operators

Evolution of the Qubit

- The evolution operator is approximated to first order in the magnitude of the control pulses, **V(t)**, and the coupling strength parameter, ε, of the system-environment interaction
- The environment is **traced out** to obtain explicit expressions for the elements of the reduced density matrix of the qubit

long timescale

Thermal decoherence
population change between levels

short timescale

Adiabatic decoherence
phase decay

Density Matrix Elements
Adiabatic Case

$$H_{Ic} = -\frac{\hbar}{2} V(\tau)\sigma_x,$$

$$\begin{aligned}
\rho_{11} &= \rho_{11}(0)\cos^2 I + \rho_{22}(0)\sin^2 I - i[\rho_{12}(0) - \rho_{21}(0)]e^{-g_{ad}}\cos I \sin I, \\
\rho_{22} &= \rho_{22}(0)\cos^2 I + \rho_{11}(0)\sin^2 I + i[\rho_{12}(0) - \rho_{21}(0)]e^{-g_{ad}}\cos I \sin I, \\
\rho_{12} &= \rho_{12}(0)e^{-g_{ad}}\cos^2 I + \rho_{21}(0)e^{-g_{ad}}\sin^2 I + i(\rho_{22}(0) - \rho_{11}(0))\cos I \sin I, \\
\rho_{21} &= \rho_{21}(0)e^{-g_{ad}}\cos^2 I + \rho_{12}(0)e^{-g_{ad}}\sin^2 I - i(\rho_{22}(0) - \rho_{11}(0))\cos I \sin I,
\end{aligned}$$

where g_{ad} is the decoherence function:

$$g_{ad}(\tau) = \gamma \int_0^\infty d\omega G(\omega)(1 - \cos\omega\tau)\coth\frac{\beta_0\omega}{2}$$

and I is the time integral of the control pulses:

$$I(\tau) = \frac{1}{2}\int_0^\tau d\tau' V(\tau')$$

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Density Matrix Elements Thermal Case

$$\begin{aligned}
\rho_{11} &= \frac{1}{2} + \frac{1}{2}(\rho_{11}(0) - \rho_{22}(0))e^{-2g_{th}} \cos(2C_x I) \cos(2C_y I) \\
&\quad - Re\{\rho_{12}(0)\}e^{-2g_{th}} \cos(2C_x I) \sin(2C_y I) - iIm\{\rho_{12}(0)\}e^{-g_{th}} \sin(2C_x I) \\
\rho_{12} &= [Re\{\rho_{12}(0)\} \cos(2C_y I) + Im\{\rho_{12}(0)\} \cos(2C_x I)]e^{-g_{th}} \\
&\quad + iRe\{\rho_{12}(0)\} \sin(2C_x I) \sin(2C_y I)e^{-2g_{th}} \\
&\quad + \frac{1}{2}(\rho_{11}(0) - \rho_{22}(0))[e^{-g_{th}} \sin(2C_y I) - ie^{-2g_{th}} \sin(2C_x I) \cos(2C_y I)] \\
\rho_{21} &= [Re\{\rho_{21}(0)\} \cos(2C_y I) + Im\{\rho_{21}(0)\} \cos(2C_x I)]e^{-g_{th}} \\
&\quad - iRe\{\rho_{21}(0)\} \sin(2C_x I) \sin(2C_y I)e^{-2g_{th}} \\
&\quad + \frac{1}{2}(\rho_{11}(0) - \rho_{22}(0))[e^{-g_{th}} \sin(2C_y I) + ie^{-2g_{th}} \sin(2C_x I) \cos(2C_y I)] \\
\rho_{22} &= \frac{1}{2} - \frac{1}{2}(\rho_{11}(0) - \rho_{22}(0))e^{-2g_{th}} \cos(2C_x I) \cos(2C_y I) \\
&\quad + Re\{\rho_{12}(0)\}e^{-2g_{th}} \cos(2C_x I) \sin(2C_y I) + iIm\{\rho_{12}(0)\}e^{-g_{th}} \sin(2C_x I)
\end{aligned}$$

where g_{th} is the decoherence function:

$$g_{th}(\tau) = \gamma \int_0^\infty d\omega \frac{1 - cos[(\omega_{12} - \omega)\tau]}{(\omega_{12} - \omega)^2} \omega^3 \coth(\beta_0 \omega/2) \exp(-\omega/\omega_c).$$

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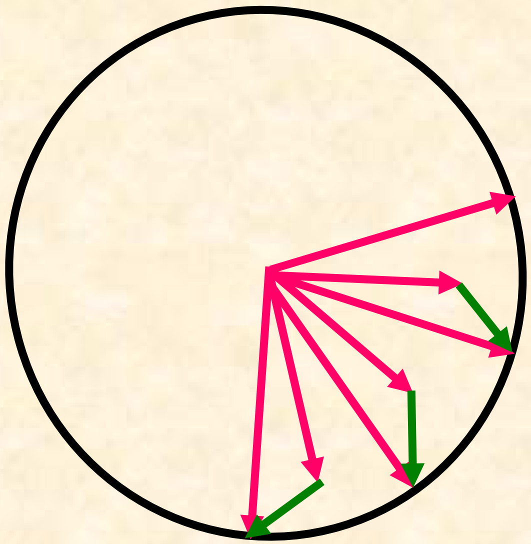
Open-Loop Control Strategy

- Assuming the decoherence function is known, we can explicitly calculate the external control that **drives** the initial value of a density matrix element, to its **target** value:

$$\rho_{ij}(g(t),I(t)) \rightarrow \rho_{ij}^{Target}$$

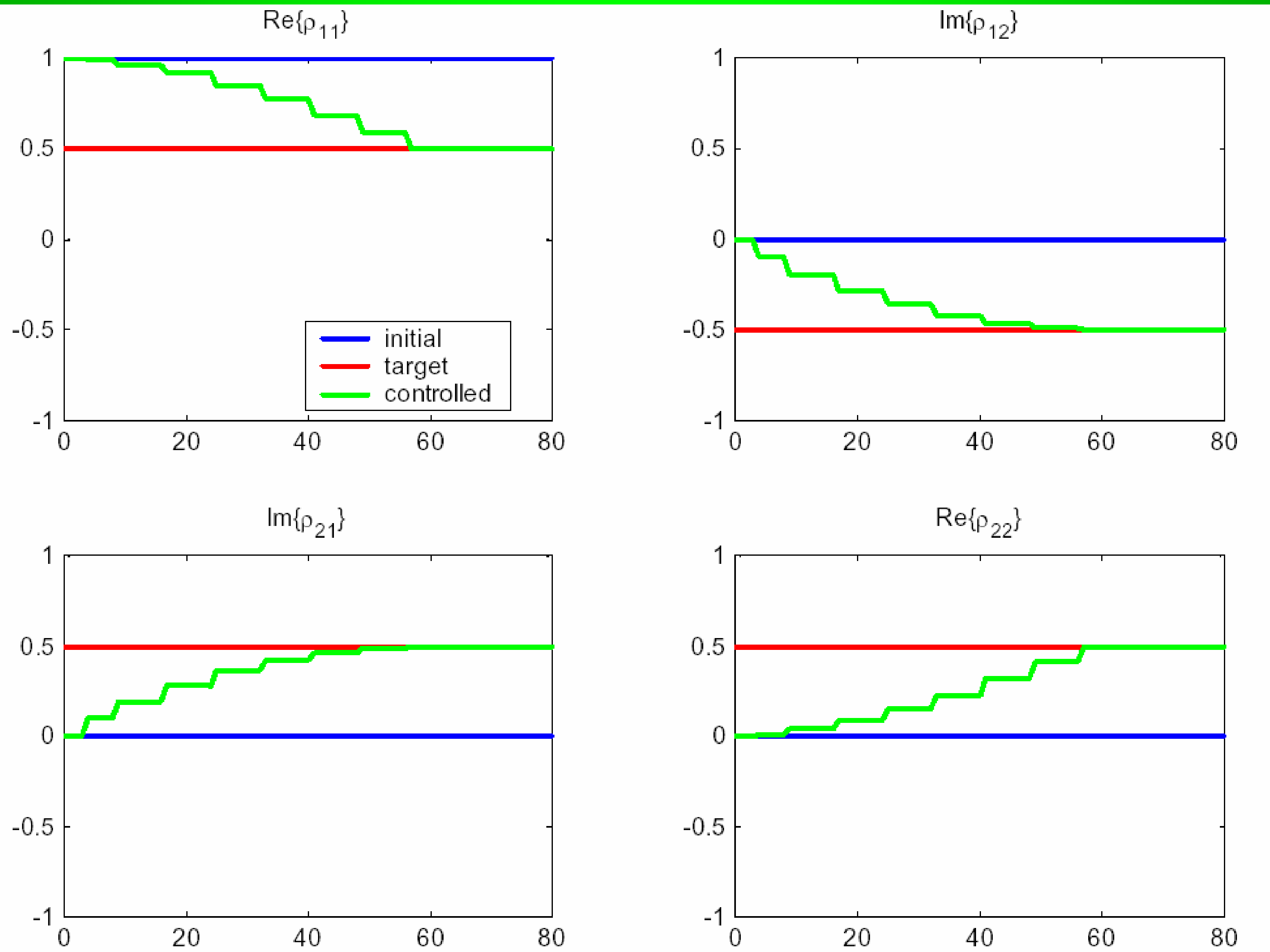
- This control is implemented as **a pulse acting on the two-level system**, and affects every density matrix element



Open-Loop Control Strategy

- The qubit is driven from the initial to the target states via a number of intermediate states
- Between 2 intermediate states, the equations for the density matrix elements determine the **control value, I(t),** required: $\rho_{ij}(g(t),I(t)) \rightarrow \rho_{ij}^{Intermediate}$
- Each density matrix element (real and imaginary parts) is solved for in turn
- A cycle of 8 steps will drive each of the density matrix elements **towards** **the next intermediate state**
- The target state is reached quickly for fast control pulse rates

Control of Adiabatic Decoherence

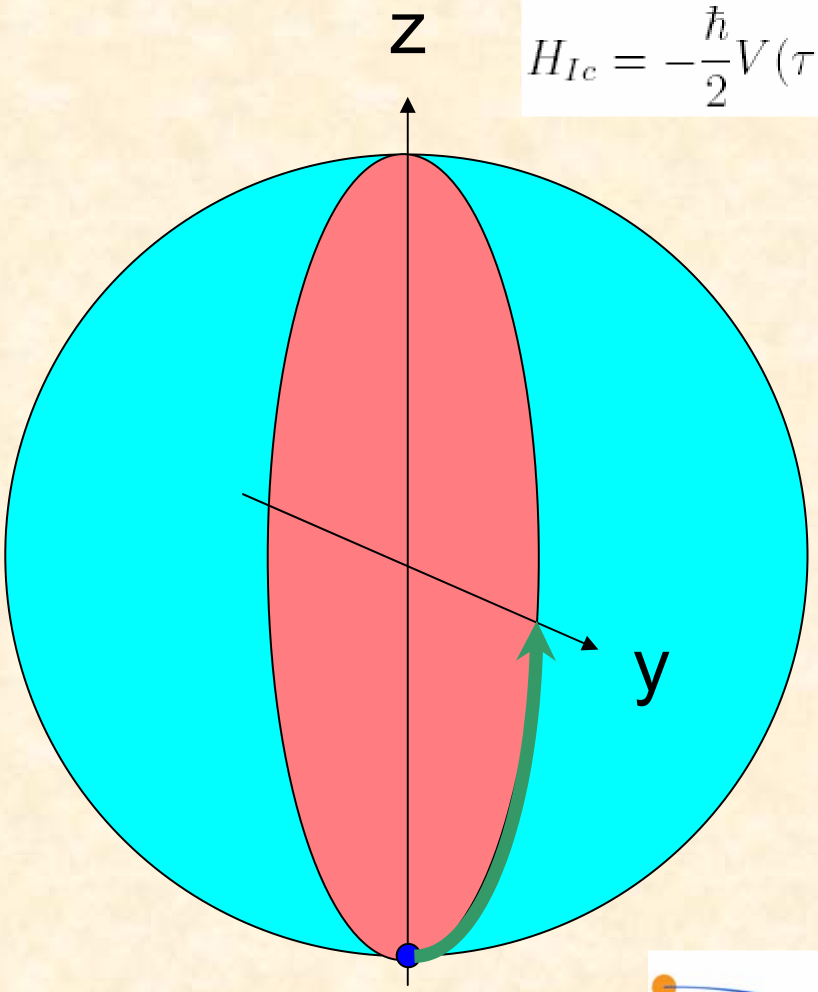


Graph 2. Control results for: |0> → 1/√2|1> + i/√2|2>

Control of Adiabatic Decoherence

- The qubit can be driven to any pure state in the y-z plane of the Bloch sphere
- Small number of intermediate states
- Low control pulse strength I(t)<0.1
- High fidelity >0.99 for final state
- Min. transition time ~ 100 control steps for diametrically-opposite states

$$H_{Ic} = -\frac{\hbar}{2} V(\tau)\sigma_x,$$



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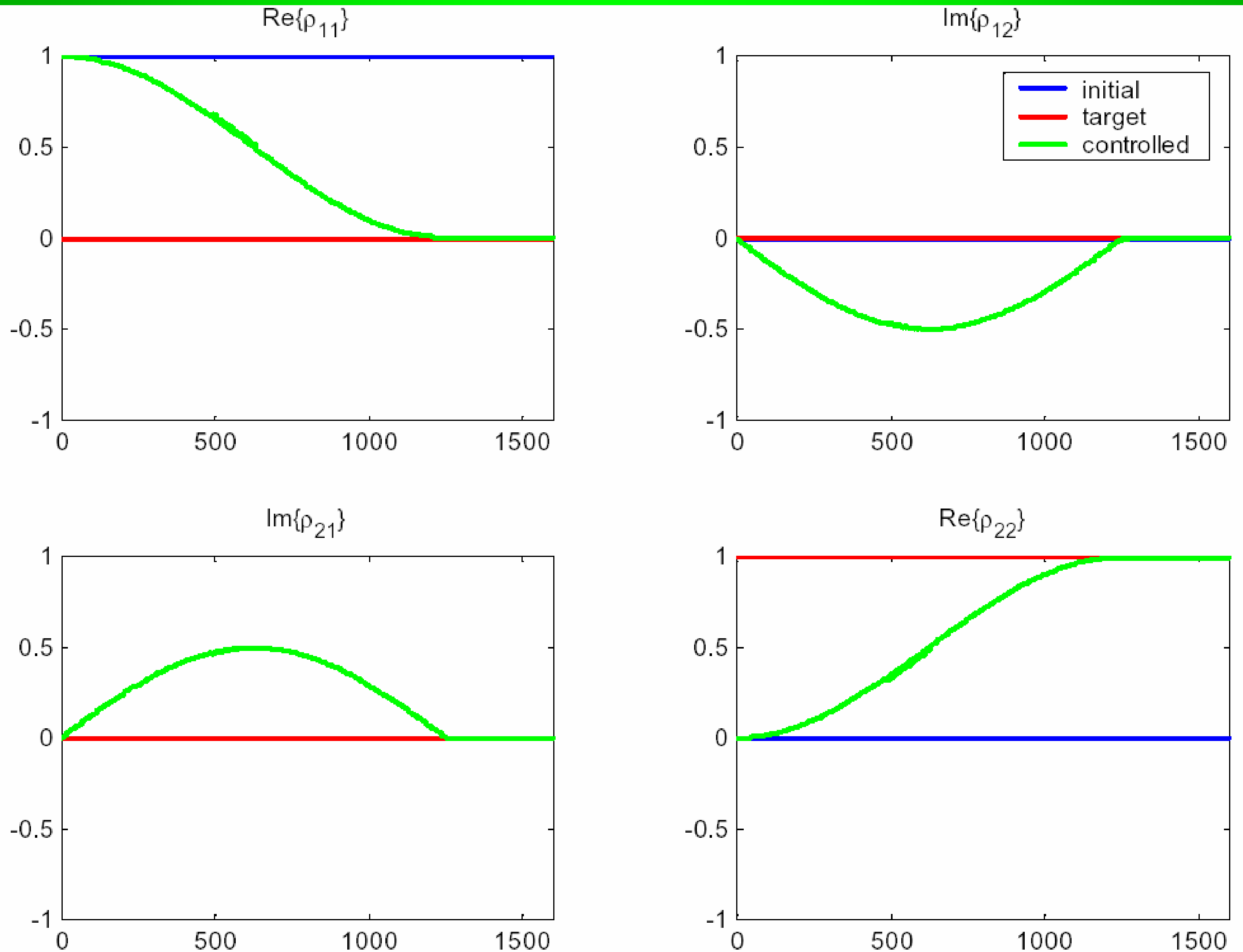
Control of Thermal Decoherence

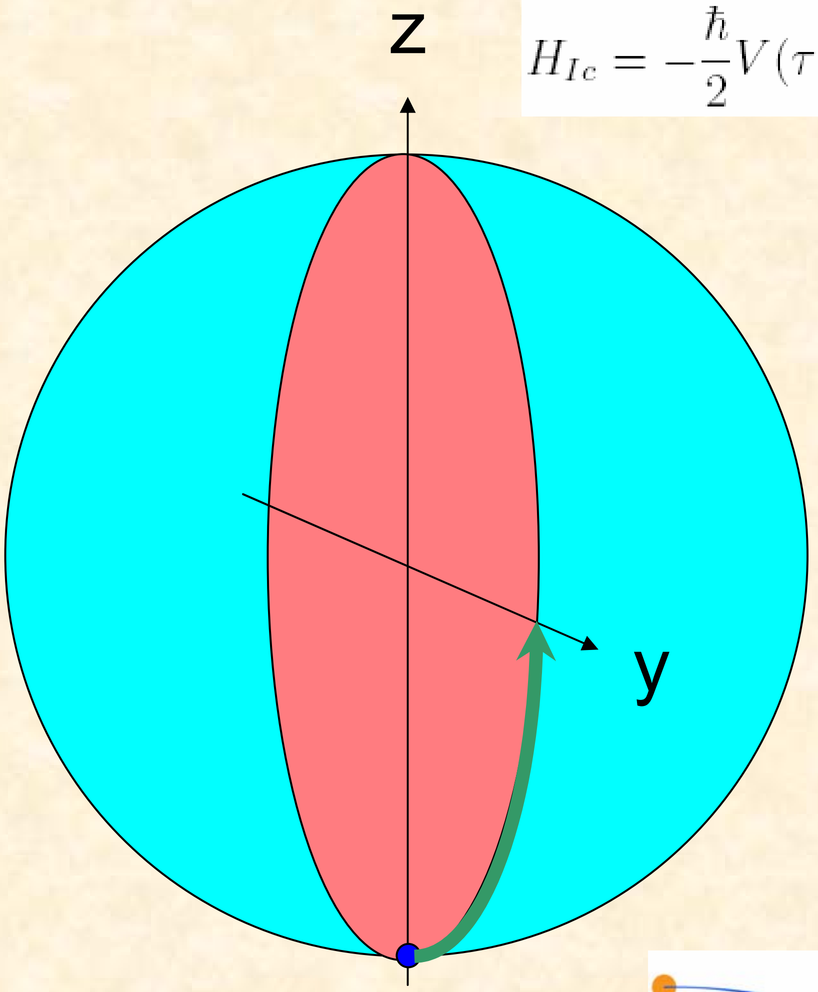


Graph 3. Control results for |0> → |1>

Control of Thermal Decoherence

- The qubit can be driven to any pure state in the y-z plane of the Bloch sphere
- 100 intermediate states for smooth transition
- Low control pulse strength I(t)<0.01
- High fidelity >0.99 for final state
- Transition time ~ 1000 control steps for diametrically-opposite states

$$H_{Ic} = -\frac{\hbar}{2} V(\tau)\sigma_x,$$



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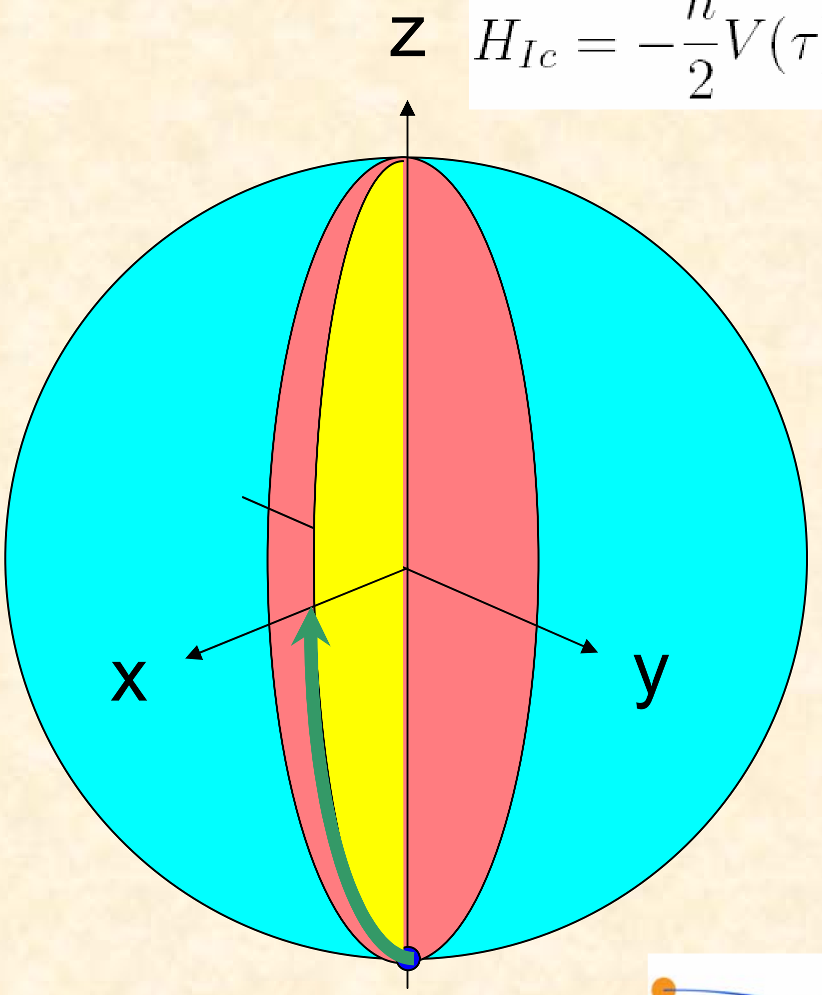
Control of Thermal Decoherence

- The qubit can be driven to any pure state in the x-z plane of the Bloch sphere
- 100 intermediate states for smooth transition
- Low control pulse strength I(t)<0.01
- High fidelity >0.99 for final state
- Transition time ~ 1000 control steps for diametrically-opposite states

$$H_{Ic} = -\frac{\hbar}{2} V(\tau)\sigma_y,$$



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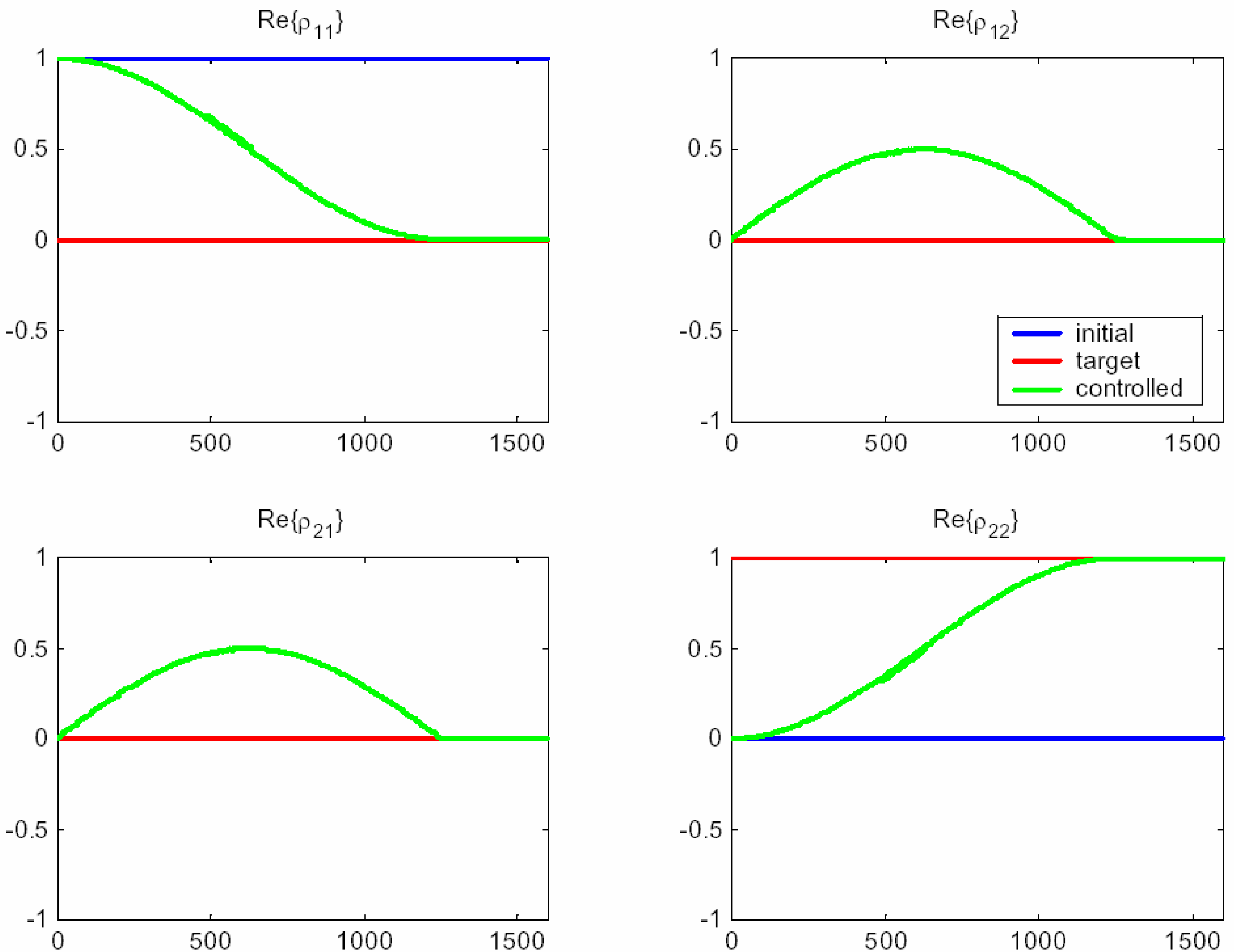
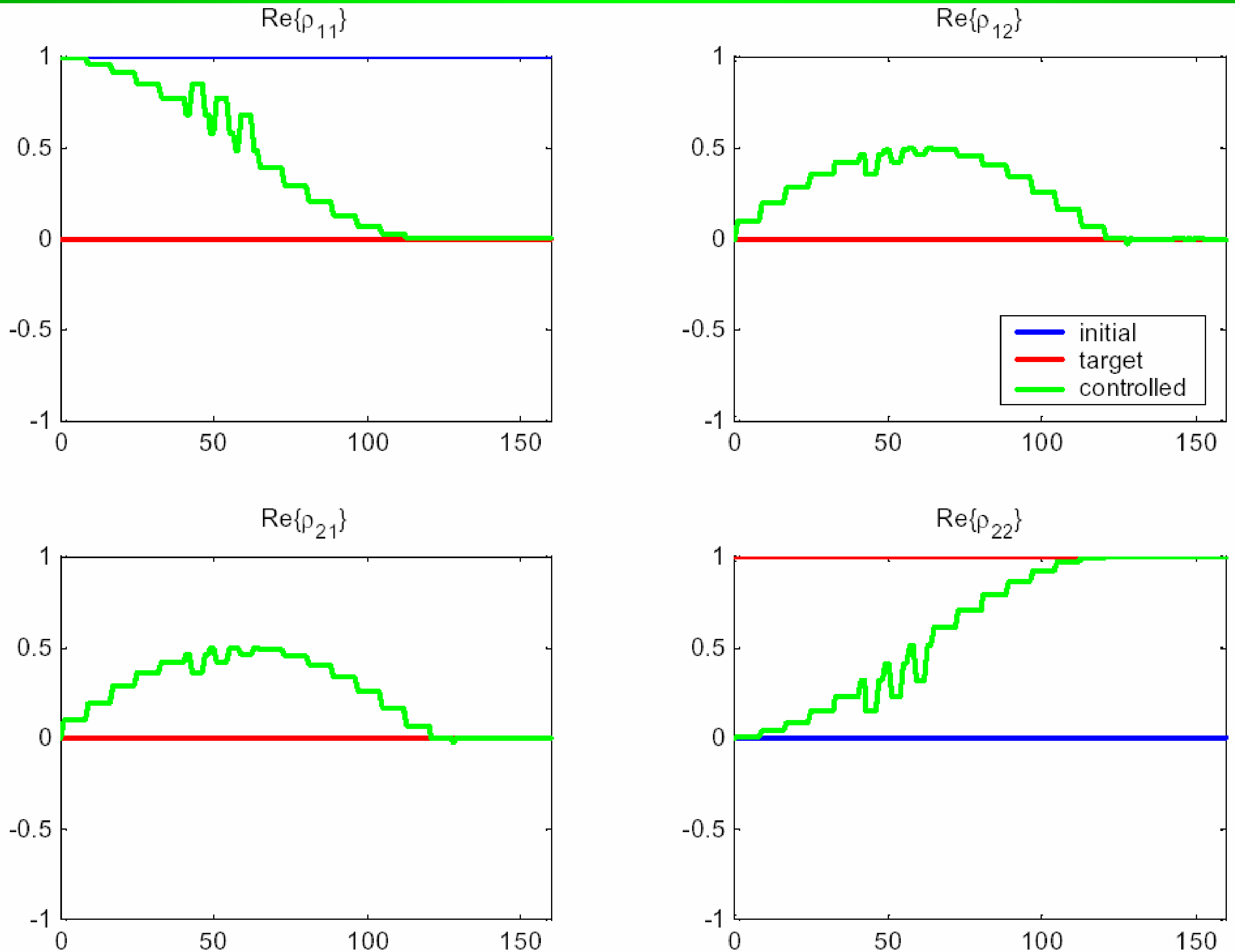
Control of Thermal Decoherence



Graph 4. Control results for |0> → |1>

Control of Thermal Decoherence

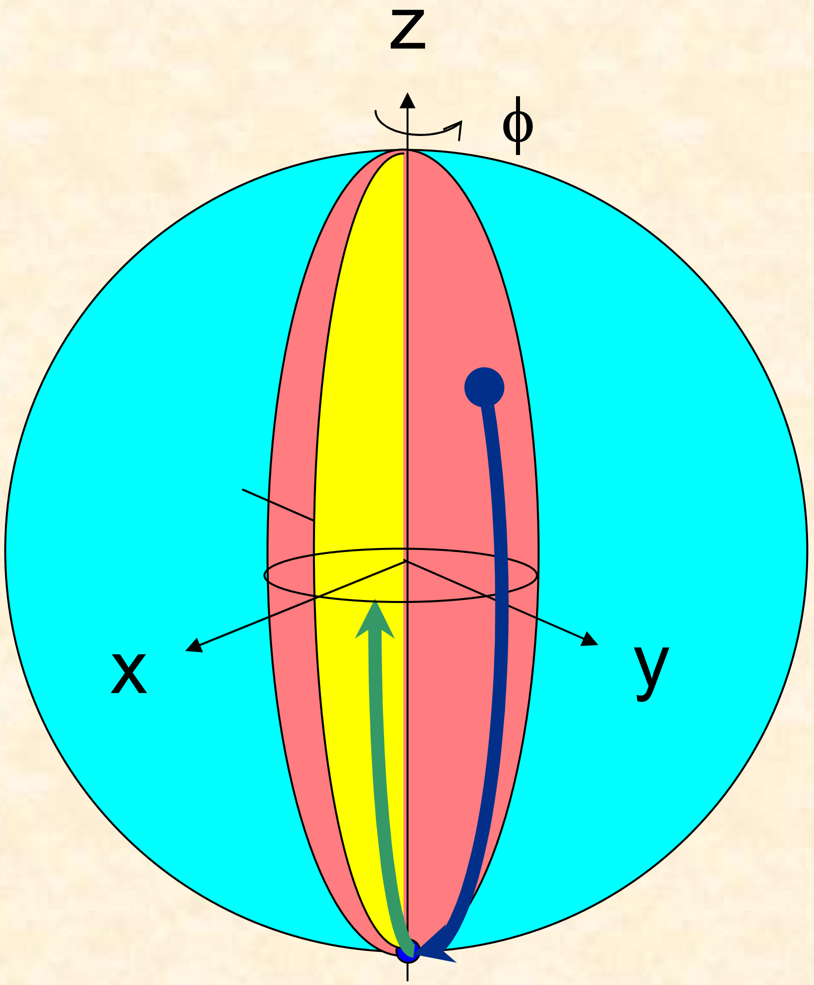


Graph 5. Ten intermediate states for |0> → |1>

Targeting an Arbitrary Pure State

- The state of a two-level system, can be driven reversibly from an arbitrary pure state to a pure target state, by using a sequence of **two** different control Hamiltonians,

$$\mathbf{H_c \propto \sigma_x \sin\phi + \sigma_y \cos\phi}$$



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Targeting an Arbitrary Pure State

- The control Hamiltonian $\mathbf{H_c}(\phi)$ drives the qubit along the edge of the **plane S_ϕ**:

$$x \sin\phi + y \cos\phi = 0,$$

which always contains the z axis of the Bloch sphere

- S_ϕ **rotated by an angle ϕ** around the z-axis with respect to the reference x-z plane
- Order and sign of the Hamiltonians is **reversed** to return to the initial state

Open-Loop Control vs. Quantum Feedback

- Target fidelity of the final state fluctuates, but very close to **unity**
- Smoothness and length of transition determined by the **number of intermediate states**, and the **rate of the control pulses**
- Open-loop control is qualitatively similar to the **Quantum Feedback Scheme** of Wang et al., at the **ensemble level** (in the thermal decoherence regime)
- Less stringent requirements:
  - **high** control pulse rate, **low** control pulse strength
  - *a priori* knowledge of **decoherence function**

Future Research

1. Driving between **mixed** initial and target states
2. Open-loop control to drive a **2-qubit system** to a maximally entangled state
3. Use **global control** for a system of N independent qubits

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