NEW SUMMING ALGORITHM USING ENSEMBLE COMPUTING

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We propose an ensemble algorithm, which provides a new approach for summing up a set of function samples. The query complexity of the algorithm depends only on the scaling of the measurement sensitivity with the number of distinct spin sub-ensembles. From a practical point of view, the proposed algorithm may result in an exponential speedup, compared to known quantum and classical summing algorithms.

In this paper, we propose an ensemble algorithm for evaluating and summing an arbitrary function, as an alternative to the quantum algorithms that are currently believed to be the most efficient algorithms available, in terms of query complexity.

To date, a handful of quantum algorithms have been proposed to solve specific mathematical problems much faster than the best available classical algorithms. Among them, Shor's prime factoring algorithm [1] and Grover's unsorted search algorithm [2] are the best known, as they address problems of direct practical relevance.

The possibility of speeding up the evaluation and summing of a large number of function samples was first noted by Abrams and Williams [3], who suggested calculating numerical integrals and stochastic processes using quantum algorithms. Quantum algorithms exploit the inherent parallelism offered by entangled quantum states, to perform certain computational tasks much more efficiently than classical devices, using either *pure* or *pseudopure* quantum states [4,5] that are tensor products of multiple qubits. The numerical value of an integral is evaluated by employing either the mean estimation algorithm devised by Grover [6] to calculate the mean of a discrete set of numbers, or by using the quantum counting algorithm proposed by Brassard, Hoyer, and Tapp [7] to determine the number of elements that fulfill a specified condition. Both of these approaches rely on a generalization of Grover's search algorithm, resulting in a quadratic speedup in comparison with classical randomized (Monte Carlo) algorithms, and an exponential speedup in comparison with classical deterministic algorithms for a single processor.

An alternative paradigm for computing has been suggested by Bruschweiler *et al.* [8], which operates on ensembles i.e., mixed states of identical spin systems. This exploits the parallelism available by acting simultaneously on linear combinations of many different input states in an ensemble of spins.

This type of parallelism is classical in nature, though the evolution of the spin systems is governed by quantum dynamics. In contrast, quantum computing with pure states relies on the parallelism of entangled states, to perform operations in a Hilbert space that is the tensor product of multiple qubits.

While requiring an exponentially larger set of memory resources to encode the same number of distinct input states, ensemble computing can accommodate decoherence times that are exponentially shorter than that for an equivalent computation involving entangled states. Ensemble algorithms can also be exponentially faster, for adequate measurement sensitivities e.g., Bruschweiler [9] has proposed a new strategy for searching an unsorted database, using the ensemble computing paradigm in the context of NMR technology. The ensemble search algorithm employs binary partition of the N elements in the database, to find the desired element after $O(log(N))$ oracle queries.

We present a new approach to summing up function samples, using an ensemble algorithm, and discuss its query complexity. The algorithm has three main steps. The first step consists of preparing an ensemble mixture of N input states representing the sampling domain. In the second step a function f is applied to the input states, using a single transformation U_f to perform the function evaluation for every input state at once. This parallel application results in an ensemble mixture, which contains all of the function values in the output register. Finally, measurement of the output register automatically averages the contributions from the entire ensemble, giving a signal proportional to the sum of the function samples S_f.

Up to this point, the query complexity of the proposed algorithm is one i.e., only one function evaluation is required. If the measurement sensitivity of the experiment is adequate to distinguish between distinct normalized output signals with a precision equal to or better than $1/N$, then only one run of the algorithm is required to determine the sum S_f. In this case, the ensemble summing algorithm may give an exponential speedup over all known quantum and classical summing algorithms. For a signal-to-noise ratio $SNR \approx 10^4$ in an NMR implementation, the measurement sensitivity is adequate to allow a single application of the ensemble summing algorithm for up to ≈ 100 function samples.

However as the number of sample points N increases, the measurement sensitivity eventually becomes inadequate. Thus significant differences between normalized output signals, differences as small as $1/N$, will not be detectable in a single experimental trial. To enhance the sensitivity, the algorithm is repeated a number of times. For example, in an NMR implementation the proposed algorithm will have to be repeated N^2 times, taking into account the square-root scaling of the signal-to-noise ratio with the number of experimental trials.

The ensemble summing algorithm has an exponential advantage in terms of query complexity, relative to the implementation of the quantum summing algorithm using Grover's search algorithm with pseudopure states in

NMR, as the two algorithms have the same scaling for the measurement sensitivity. Similarly, the query complexity of the ensemble search algorithm is exponentially smaller than that required for Grover's search algorithm using pseudopure states in NMR.

However a comparison with the (theoretical) implementation of Grover's search algorithm using pure states shows that the ensemble summing algorithm will be more efficient only for a total number of samples below a threshold value determined by the measurement sensitivity. The threshold value for the number of function samples, N_t, is given by $N_t \approx (SNR)^{4/3}$.

Finally we note, that the ensemble summing algorithm presented here is applicable to estimating the mean of a continuous function, or the definite integral of a continuous multi-dimensional function.

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References

1. P. Shor, *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, ed. S. Goldwasser (IEEE Computer Society, Los Alamitos, CA, 1994).
2. L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
3. D. S. Abrams and C. P. Williams, quant-ph/9908083 (1999).
4. D. P. DiVincenzo, quant-ph/0002077 (2000).
5. L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood and I. L. Chuang, Nature **414**, 883 (2001).
6. L.K. Grover, quant-ph/9711043 (1997).
7. G. Brassard, P. Hoyer and A. Tapp, Lect. Notes in Comp. Science **1443**, 820 (1998).
8. Z. L. Madi, R. Bruschweiler and R. R. Ernst, J. Chem. Phys. **109**, 10603 (1998).
9. R. Bruschweiler, Phys. Rev. Lett. **85**, 4815 (2000).