Robust control of decoherence in realistic quantum gates

V. Protopopescu, R. B. Perez, C. D’Helon, and J. Schmulen

Center for Engineering Science Advanced Research, Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6355 USA
Phone:(865) 574-4722 Fax:(865) 241-0381 Email:protopopesva@ornl.gov

We present an open-loop (bang-bang) control scheme, to eliminate the effects of decoherence in a two-level quantum system (NOT gate). The controls are determined exactly from the condition to eliminate decoherence. Numerical simulations show excellent performance and robustness of the proposed control scheme.

The realization of any quantum computer relies on maintaining *quantum coherence* in the system, for a period of time spanning at least the duration of the desired computation. Unfortunately, due to coupling to the environment, the evolution within the quantum computer loses its unitary character; in other words, the system decoheres [1-6]. Several approaches have been proposed to eliminate or mitigate the undesirable effects of decoherence in open quantum systems, including open loop (quantum bang-bang) control [7-9], decoherence free subspaces (DFS) [10], and quantum feedback [11].

We focus on open-loop control for a two-level system. Our implementation follows in the steps of Lloyd and Viola's results [7-9], but has several new aspects: (i) decoherence and control are consistently taken into account from the beginning within a realistic model; (ii) the required control is directly related to and calculated from the decoherence effects, which presents the practical advantage of maintaining the frequency and amplitude of the required controls at manageable levels; (iii) the effect of imperfect controls is assessed.

The two-level system is described within the spin-boson model [12-15], with the external control included from the beginning in the Hamiltonian as an independent interaction term. Adiabatic and thermal decoherence arise from the coupling of the system to the outside world through the matrices σ_z and σ_x, respectively. After tracing out the environment modes, reduced equations are obtained for the two-level system in which the effects of both decoherence and external control appear explicitly. By equating the elements of the reduced density matrix in the presence of decoherence and (unknown) control with the elements of the density matrix undergoing a unitary evolution, we determine the control that cancels out the effect of the decoherence and momentarily restores unitarity. The control scheme advances through a cycle of eight time steps, to handle in turn the real and imaginary parts of the matrix elements of the two-level system. In general, resetting any one of the matrix elements to its corresponding unitary value implies that the cumulative effect of the control pulses applied since the last correction cancels out the effect of decoherence. We note two important things for the applicability of the scheme. First, the knowledge of the decoherence function is needed only for a finite period of time. Second, after initial transients, controls will stabilize and the whole cycle will repeat. This behavior has indeed been observed: the values of the control pulses were stabilized rapidly after the first cycle of eight time steps, as shown in Figure 1, in the case of adiabatic decoherence.

The relative size of the time step between control pulses determines the deviation of the matrix elements from their unitary values. Also, the size and frequency of the control needed to restore the ideal situation depends on the strength of the decoherence. For γ=1 (extremely strong decoherence), the frequency of the control can be decreased only to ~2 times the Rabi frequency. If we keep decreasing the frequency, we cannot restore exact unitarity, at least not by using this scheme. For γ=0.1 (strong to medium decoherence), we can decrease the frequency to ~0.2 of the Rabi frequency. Of course, higher frequency results in better restorations of unitarity.

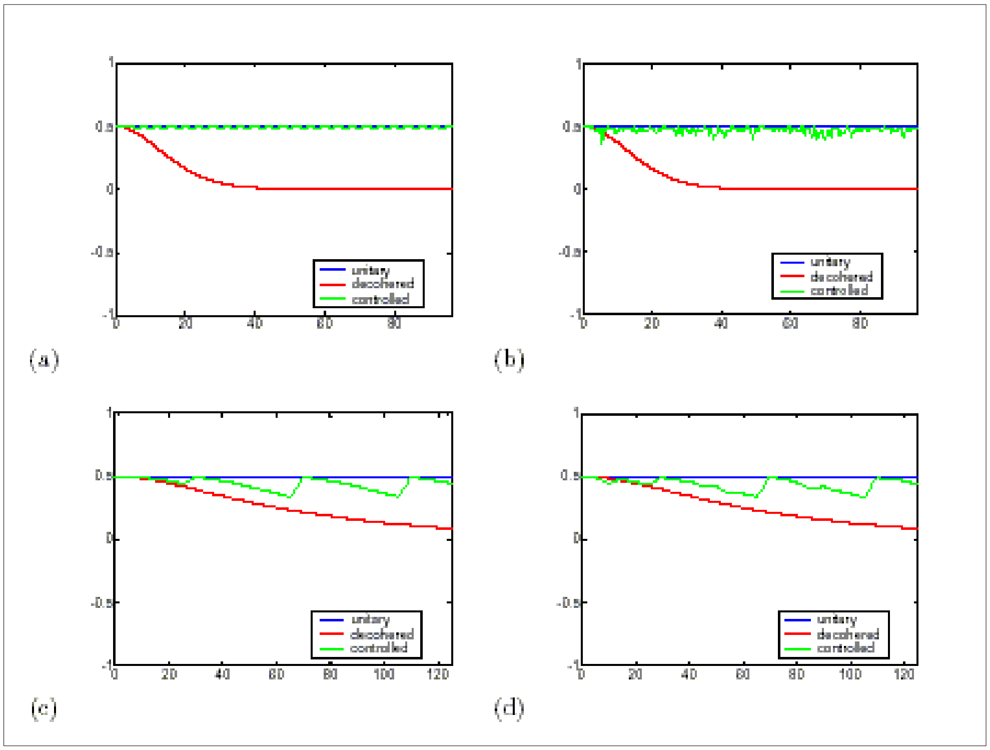

Figure 1. The unitary, adiabatically decohered, and controlled evolution of the Im{ρ_{12}} element of the density matrix for the initial state i/√2|0> + 1/√2|1>. There are three parameters characterizing these plots: the time between control pulses, T (scaled in terms of the Rabi frequency), the decoherence rate, γ (dimensionless), and the standard deviation, ΔI, of the control pulses after adding normally-distributed noise. (a) T=0.5, γ=1, Δ I=0; (b) T=0.5, γ=1, ΔI=0.1; (c) T=5, γ=0.1, ΔI=0; (d) T=5, γ=0.1, ΔI=0.1.

For thermal decoherence, a similar result is obtained. Since decoherence affects all elements of the density matrix, the amount of control needed to restore unitarity is slightly higher than in the adiabatic case. If the control pulsess are perturbed by noise, the control is still very efective.

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