On Nearest Neighbor Implementation of Projective Fusers

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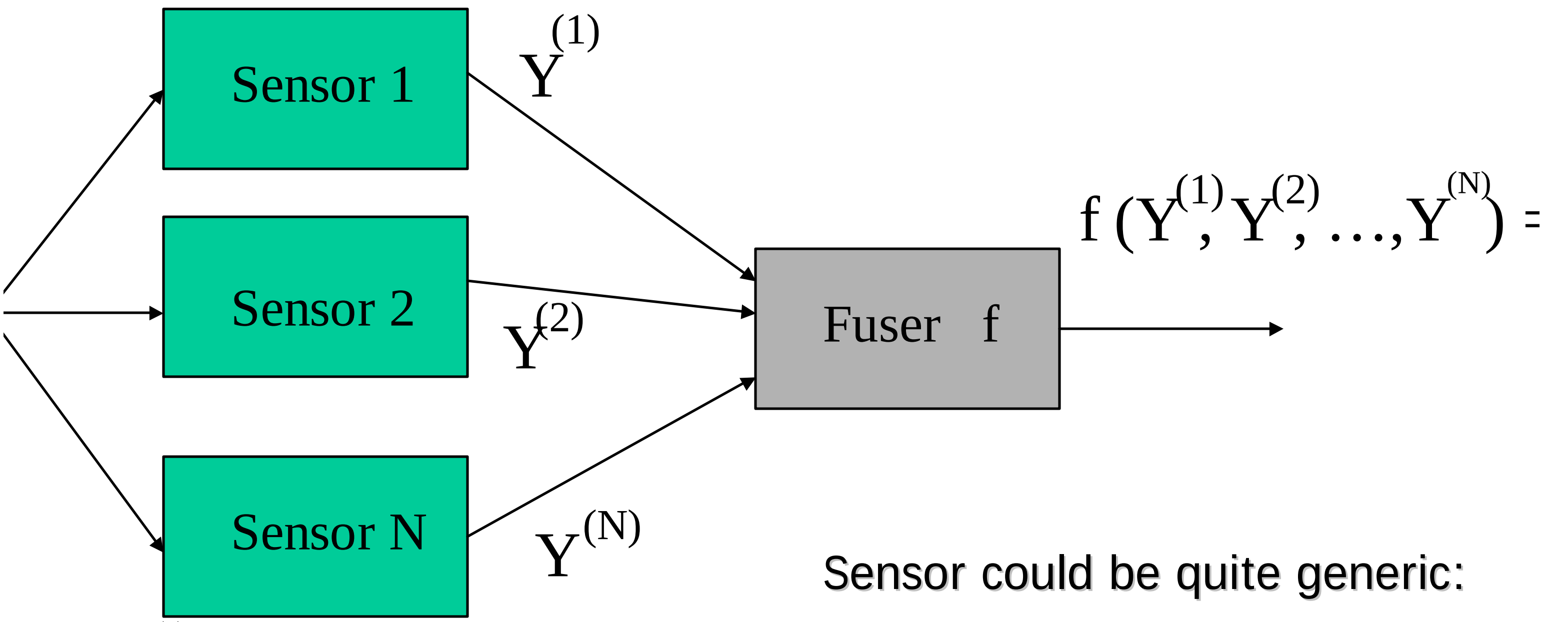
Outline of Presentation

- Overview of Generic Sensor Fusion Problem
  - Specific methods
  - Isolation fusers
- Projective Fusers
  - Class of projective fusers
- Nearest Neighbor Projective Fuser
  - Finite-sample performance bound
  - Application to sigmoid neural networks, methane hydrates exploration

Generic Sensor Fusion Problem



‹ and $Y^{(i)}$ are related by an
ınknown distribution $P_{X,Y^{(i)}}$

ıesign a fuser with
ıerformance guarantees
ıased on measurements

. **Rao, Journal of Franklin Institute, 1994,1999.**

Sensor could be quite generic:

- –Hardware device
  - •Ultrasonic and infrared sensors
- –Software module
  - •Function estimators
- –Database
  - •Archived measurements
- –Combination of hardware and s‹
  - •Camera and detection software

Finite Sample Guarantees

Known sensor distributions

- If all joint sensor distributions are known, optimal fuser can be obtained in principle
  - Not sufficient to know individual sensor distributions
  - Not guaranteed to be easily computable

Only measurements are given

- Performance guarantee limits:

  Cannot guarantee "close-to-optimal results" with probability one
- Best possible result:
  - With a high probability the measurement-based fuser is at least as good as the best

$$P[I(f) ? I(f^{*}) ? ?] ? ?$$

computed fuser

optimal fuser

  - Roughly speaking, fusion problem is estimation of regression (infinite-dimensional quantity) using only finite set of measurements

verview of Solutions: Finite Sample Guarantees

General Solution

- Showed that the problem is solvable in principle
- Under finiteness of scale-sensitive dimension of fuser class finite sample guarantees can be provided

Specific Fuser Methods

- Vector space methods
  - Linear fusers
  - Kurkova's neural networks
- Sigmoid neural networks
- Non-linear statistical estimators
  - Nadaraya-Watson estimator
  - regressograms

We developed finite sample guarantees for the fuser

Example: Fuser class forms a vector space of dimension d

- Sample size estimate

$$\frac{512}{?^{2}} ? d \ln ? \frac{64e}{?} ? \ln \frac{64e}{?} ? ? \ln 8/? ?$$

to ensure $P[I(f) ? I(f^{*}) ? ?] ? ?$

irrespective of sensor distributions

Isolation Fusers

Method:

1. choose fuser class with isolation property – contains an identi
   function for every input variable
2. compute empirically best fuser based on measurements

– Examples:
  - Linear and piecewise-linear fusers
  - Projective fusers

Performance

– Given enough sample size, fuser is at least as good as bes
  sensor in a probabilistic sense

N.S.V. Rao, Information Fusion,

IEEE Trans PAMI,

Function Estimation

roblem: To estimate a function $f:[0,1]^d \ ? \ [0,1]$

Based on sample $(X_1, f(X_1)),(X_2, f(X_2)),? \ ,(X_l, f(X_l))$

or estimate $\hat{f}$

Expected error is defined by

$$I(\hat{f}) \ ? \ ?(f(X) \ ? \ \hat{f}(X))^2 dP_X$$

user Design: Given N function estimators $\hat{f}_1, \hat{f}_2, ? \ , \hat{f}_N$

ompute fuser $f_F : [0,1]^N \ ? \ [0,1]$

uch that $f_F(X, \hat{f}_1, \hat{f}_2, ? \ , \hat{f}_N)$ approximates $f(X)$

Projective Fusers - Definition

<u>Method:</u> Fuser f_P using estimators $\hat{f}_1, \hat{f}_2, ? \;, \hat{f}_N$

Partition the input space in to blocks

$$P ? \{?_1, ?_2 ? \;, ?_k\}, ?_1 ? [0,1]^d, \mathop{?}_{i?1}^{k} ?_i ? [0,1]^d$$

Assign to each block $?_i$ an estimator $\hat{f}_j$ such that

$$f_P(X, \hat{f}_1, ? \;, \hat{f}_N) ? \hat{f}_j(X)$$

for all $X ? ?_i \hat{f}_i$

Informally:

– Divide the domain into blocks

– Use one of the estimators in each each block

Illustration

Optimal Fuser

- Compute error regressions of information sources: project one with lowest local error
- Fuser is better than best sub-combination

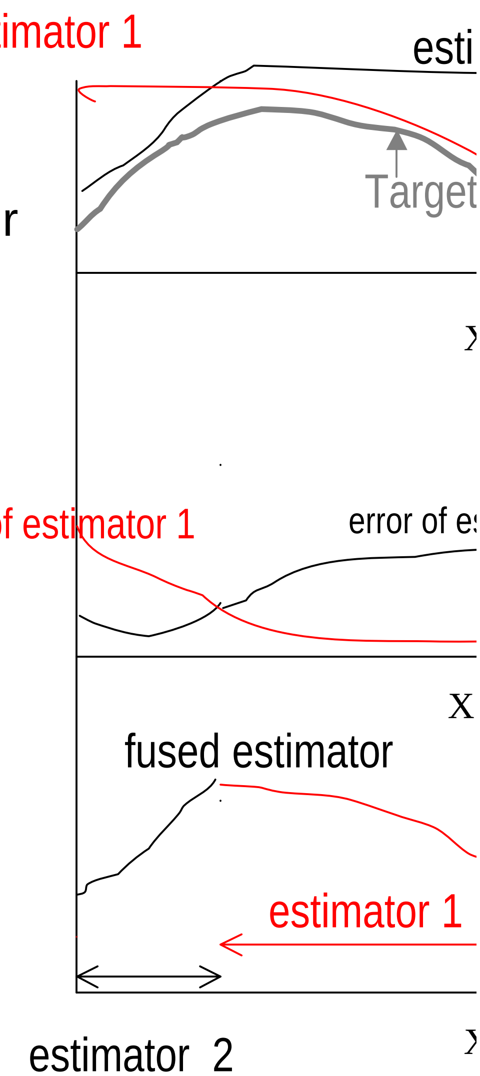
Challenges:

- In practice only finite measurements are given: error regressions cannot be exactly known

Our results: **measurement-based approximation**

– Cellular decomposition method

– Nearest neighbor projective fuser



Optimal Projective Fuser f_{LE}

ethod:

Compute error regressions of sensors

Project the sensor corresponding to lower envelope of regressions

Blocks are decided by the lower envelope

operties:

– At least as good as when applied on a subset of sensors

– Possess isolation property – hence the performance guarante

– Complementary performance compared to linear fusers

Performance Results

– Need to know the error distributions to exactly compute it

– Based on iid sample only an approximation can be computed

- Asymptotic result: As simple size goes to infinity, fuser is at least as good as best subset of sensors [Fusion 1999, MFI 99]
- Finite sample result: Given sufficient sample size, fuser is close to optimal with a probability [Fusion 2002]

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Projective vs. Linear Fusers

Complementary Performance:

- Projective fuser is better if sensors are locally efficient
- Linear fuser is better if errors are "symmetric"

ample-Based Projective Fuser

- ethod:
  - stimate the errors using adaptive cells method
  - oject estimators according to lower envelope of estimated regressions
- symptotic Result $I(\hat{f}_{LE}) ?\ I(f_{LE})$ as $l ?\ ?$
  – As simple size goes to infinity, fuser is at least as good as subset of sensors [Fusion 1999, MFI 99]
- roperties
  - Cells sizes must be carefully chosen
  - Does not provide any guarantee for finite sample

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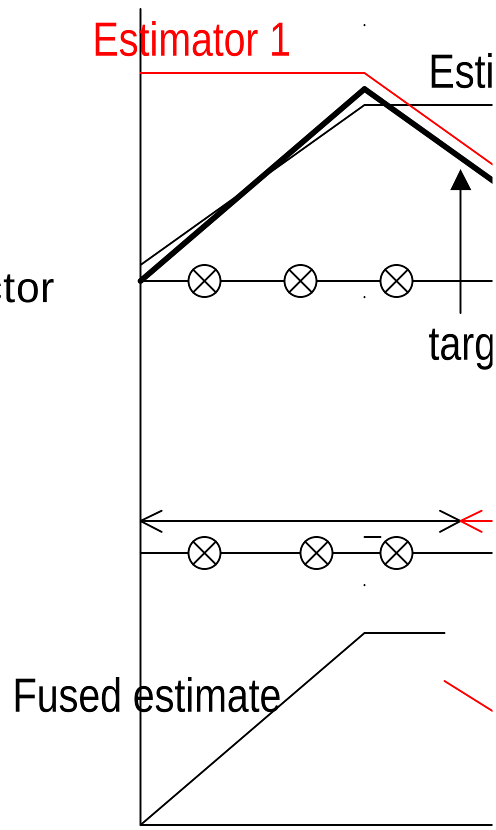
Nearest Neighbor Projective Fuser

Basic Idea

- Decompose into Voronoi regions of measurements
- Given a test point
  - Identify Voronoi region that contains it
  - Use the estimator with least error as a predictor



Performance

<u>Computational:</u> polynomial-time computable

<u>Finite-sample result</u>: given finite sample, fuser performs almost as good as optimal with a high probability

- first finite sample result for projective fusers

Definition: Nearest Neighbor Projective Fusers:

<u>oronoi Decomposition:</u>

Decompose domain into blocks $V(X_2),? \ \ ,V(X_l)$

$V(X_j)$ consists of all points closer to X_j than any other X_i

iven X:

$NN(X)$: index to $V(X_k)$ such that $X \ ? \ V(X_k)$

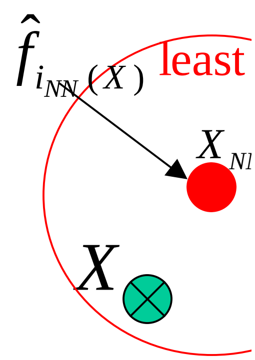
- simply the Voronoi cell that contains X

$i_{NN}(X)$:estimator with lowest error at nearest sample point $X_{NN(}$

$$i_{NN}(X) \ ? \ \underset{i?1,2,? \ \ ,N}{\operatorname{argmin}}[f(X_{NN(X)}) \ ? \ \hat{f}_i(X_{NN(X)})]^2$$

<u>earest Neighbor Projective Fuser</u> $\hat{f}_{NN}$

$$\hat{f}_{NN}(X, \hat{f}_1,? \ \ , \hat{f}_N) \ ? \ \hat{f}_{i_{NN}(X)}(X)$$



Finite-Sample Results: Projective Fusers

General Condition: Each estimator $\hat{f}_i$ has finite total variation

This results includes wide class of estimators:

- Smooth estimators:
  - sigmoid neural networks with bounded weights
  - potential functions
  - radial basis functions
  - smooth kernel estimators
- Non-smooth estimators
  - K-nearest neighbors
  - Regression trees
  - Nadaraya-Watson estimator
  - Regressograms

Performance Guarantee

$$P ? I(\hat{f}_{NN}) ? I(f_{LE}) ? ? ? ? ?$$

Functions of Bounded Variation

Informally:

Distance covered by walking along the function is finite

- Examples:
  - functions on compact domain with finite number of jumps
  - functions on compact domain with bounded derivative
  - Lipschitz functions on compact domains

One dimensional: $h:[-A,A]\to\mathbb{R}$

Partition $-A = y_0 < y_1 < \dots < y_n = A$

Set of all partitions: $\Pi[-A,A]$

For all partitions we have bounded $\sum_{k=1}^{n} |h(y_k) - h(y_{k-1})| \le M$

For multi-dimensional, bounded in each dimension

Sample-Size Estimate: Projective Fusers

et V_i be the total variation of $\hat{f}_i$ and $V ? \overset{N}{\underset{i?1}{?}} V_i$

Given sample of size

$$\frac{256}{?^2} ? 18 ? 1 ? \frac{128V}{?} ? \ln^2(128/?) ? \ln(16/?) ?$$

We guarantee

$$P ? I(\hat{f}_{NN}) ? I(f_{LE}) ? ? ? ? ?$$

irrespective of the underlying distributions, I.e. estimators c be arbitrarily correlated.

Informally, error of $\hat{f}_{NN}$ is within optimal, namely $I(f_{LE})$, with probability $1 ? ?$

Implies asymptotic result

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Outline of Finite-Sample Bound

Embed fusers, $\hat{f}_{NN}$ and f_{LE}, in function class of bounded variation F_V

Use Vapnik's argument to show

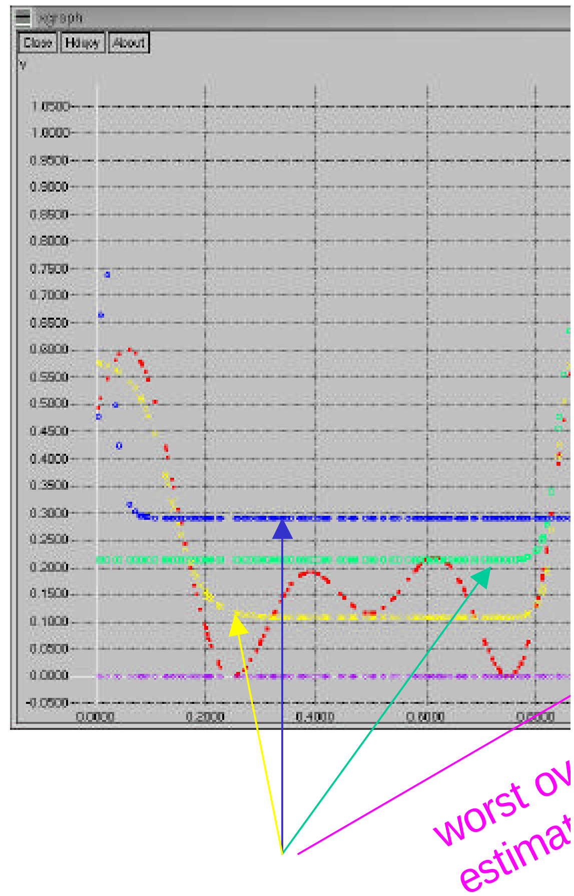
$$P\{ I(\hat{f}_{NN}) ? I(f_{LE}) ? ? \} ? P\left\{ \sup_{g ? F_V} | \hat{I}(g) ? I(g) | ? ? / 2 \right\}$$

Use fat-shattering index of F_V to obtain the sample size

$$\frac{256}{?^2}\left[18\, fat_{F_V}(? / 256) \ln^2(128/?^2) ? \ln(16/?) \right]$$

$$fat_{F_V}(?) ? 1 ? \frac{V}{2?}$$

pplication: Sigmoid Neural Network Estimators



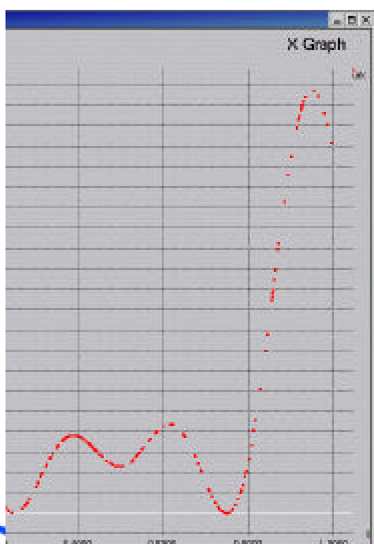
ng neural networks: for function timation

raining problem is NP-hard

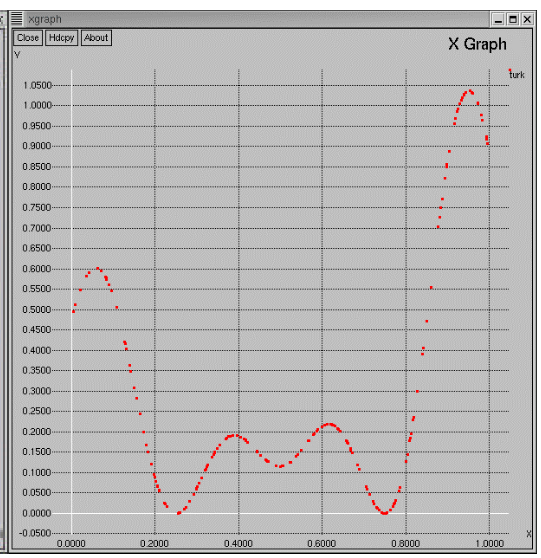
lost training algorithms yield sub optimal results

ackpropagation algorithm is sensitive to starting weights and learning rate

ng data



test data



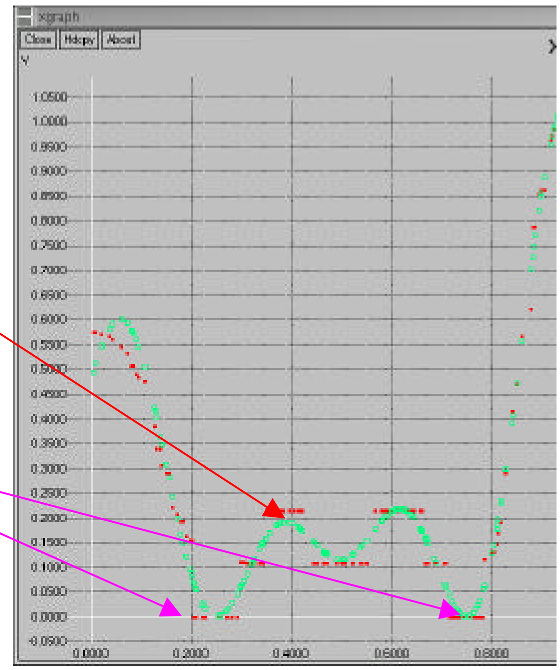
Sigmoid neural network different starting weigh and learning rates

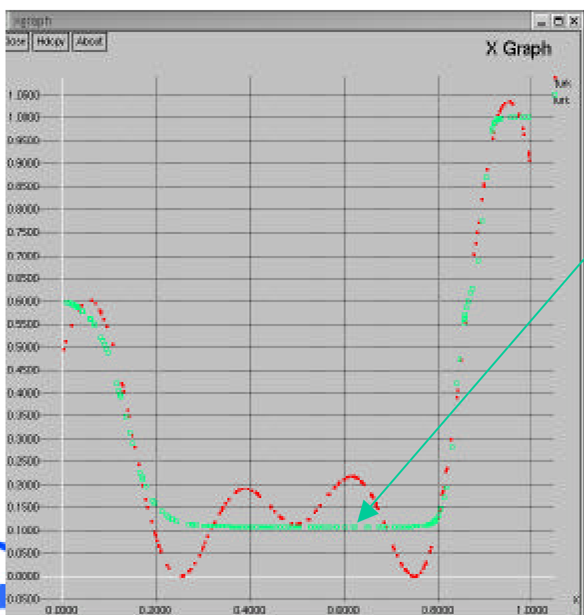
d Neural Network Estimators

est neighbor projective fuser

lses locally best estimators

lote the worst overall estimator is good at certain parts





Linear fuser

Picks a single weight for entire domain

Embrittlement Predictions

aborator : Jy-An Wang (Nuclear Sci. and Tech. Division, ORNL

<u>Overall Goal:</u> Predict residual defects in materials due to neutron-indu damage in light-water reactors

Transition temperature shift – vital indicator of embrittlement lev

- Several predictors available
  - Fluence-based models
  - Eason's models
  - Reg. Guide 1.99 model
  - Feedforward neural network models
  - Nearest-neighbor model

Fusion Approach: Combine all the predictors

- General Electric boiling water reactor data
  - **Isolation Fuser** (linear least squares)
    - 56.5% and 32.8% reduction in uncertainty plate and weld data, respectively, over model
  - **Nearest Neighbor Projective Fuser**
    - 67.3% and 52.4% reduction in uncertainty in plate and weld data, respectively, ov best model

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Conclusions

resented class of projective fusers:

- Optimal projective fuser corresponds to lower envelope of regressi
- Only an approximation can be computed based on sample

earest Neighbor Projective Fuser:

- sample based approximation to optimal projective fuser
- polynomial-time computable
- provides finite sample guarantee

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