

# Control of Friction at the Nanoscale

Y. Braiman, J. Barhen, and V. Protopopescu  
Center for Engineering Science Advanced Research  
Computing and Computational Sciences Directorate  
Oak Ridge National Laboratory, Oak Ridge, TN 37831

## Abstract

We propose a terminal attractor algorithm to control frictional dynamics towards a desired value of the sliding velocity. This control algorithm shows very robust control ability and also significantly reduces the transient time to reach the terminal behavior. Moreover, only knowledge of the sliding velocity is required to apply the proposed control.

Ability to control and manipulate friction during sliding is of significant importance for a large variety of technological applications. The outstanding challenge deals with complex dynamical systems with many degrees of freedom, under strict size confinement, and having only very limited control access. Despite great progress made through the past half century, many basic issues in fundamental tribology such as origin of friction and failure of lubrication have remained unsolved. Moreover, the current reliable knowledge related to friction and lubrication is mainly applicable to the macroscopic systems and machinery and, most likely, will be of only of limited use (if any at all) for micro- and nano-systems. Indeed, when the thickness of the lubrication film is of the same order as the molecular or atomic size, the behavior of the lubricant becomes significantly different from the behavior of macroscopic (bulk) lubricant [1].

Better understanding of the intimate mechanisms of friction, lubrication, and other interfacial phenomena at the atomic and molecular scales is expected to provide designers and engineers the required tools and capabilities to control and monitor friction, reduce unnecessary wear, and predict mechanical faults and failure of lubrication in MEMS and nano-devices [2].

In addition to conventional dissipation mechanisms (e.g., photonic [3, 4] and electronic [3, 5]), friction of the nonlinear system can be significantly affected by the dynamical properties of the sliding system such as, for example, the fluctuations of each individual element from the center of mass motion. A nonlinear system driven far from equilibrium can exhibit a variety of complex spatial and temporal behaviors, each resulting in different patterns of motion and corresponding to different friction coefficients [6].

Friction can be manipulated by applying small adjustments (perturbations) to accessible elements and parameters of the sliding system. This operation requires a-priori knowledge of the strength and timing of the perturbations. Recently the groups of J. Israelachvili [7] (experimental) and U. Landman [8] (full-scale molecular dynamics computer simulation) showed that friction in thin-film boundary lubricated junctions can be reduced by coupling of small amplitude (of order of 1Å) directional mechanical oscillations of the confining boundaries to the molecular degree of freedom of the sheared interfacial lubricating fluid. Using a surface force apparatus, modified for measuring friction forces while simultaneously inducing normal (out-of-plane) vibrations between two boundary-lubricated sliding surfaces, load- and frequency-dependent transitions between a number of "dynamical friction" states have been observed [7]. In particular, regimes of vanishingly small friction at interfacial oscillations were found. Extensive grand-canonical molecular dynamics simulations [8] revealed the nature of the dynamical states of confined sheared molecular films, their structural mechanisms, and the molecular scale mechanisms underlying transitions between them. Significant changes in frictional responses were observed in the two-plate model [9] by modulating the normal response to lateral motion [10]. In addition, the surface roughness and the thermal noise are significant factors in making decisions about control on the micro and

the nano-scale [11, 12]. These results point to a completely new direction for realizing ultra-low friction in mechanical devices.

In this paper, we address some fundamental issues related to targeting and control of friction in small driven nonlinear particle arrays. We propose a feedback control scheme, based on the properties of terminal attractors [13, 14]. This type of control has been successfully implemented in modifying the dynamical behavior of artificial neural networks [13, 14]. The main advantage of terminal attractor algorithms consists in their robustness and ability to significantly reduce the transient times (we will later on discuss the properties of terminal attractors).

We will demonstrate friction control on a phenomenological model of friction [9, 15, 16, 17, 18]. Despite their relative simplicity, phenomenological models [9, 15, 16, 17, 18] show a fair agreement with some of the experimental results using the friction force apparatus [19, 20] and quartz microbalance experiments [11, 21]. The basic equations for the driven dynamics of a one dimensional particle array of  $N$  identical particles moving on a surface are given by a set of coupled nonlinear equations of the form [22]:

$$m\ddot{x}_n + \gamma\dot{x}_n = -\partial U / \partial x_n - \partial V / \partial x_n + f_n + \eta(t), \quad n=1, \dots, N \quad (1)$$

where  $x_n$  is the coordinate of the  $j$ th particle,  $m$  is its mass,  $\gamma$  is the linear friction coefficient representing the single particle energy exchange with the substrate,  $f_n$  is the applied external force, and  $\eta(t)$  is Gaussian noise. The particles are subjected to a periodic potential  $U(x_n + a) = U(x_n)$  and interact with each other via a pair-wise potential  $V(x_n - x_j)$ ,  $n, j = 1, 2, \dots, N$ .

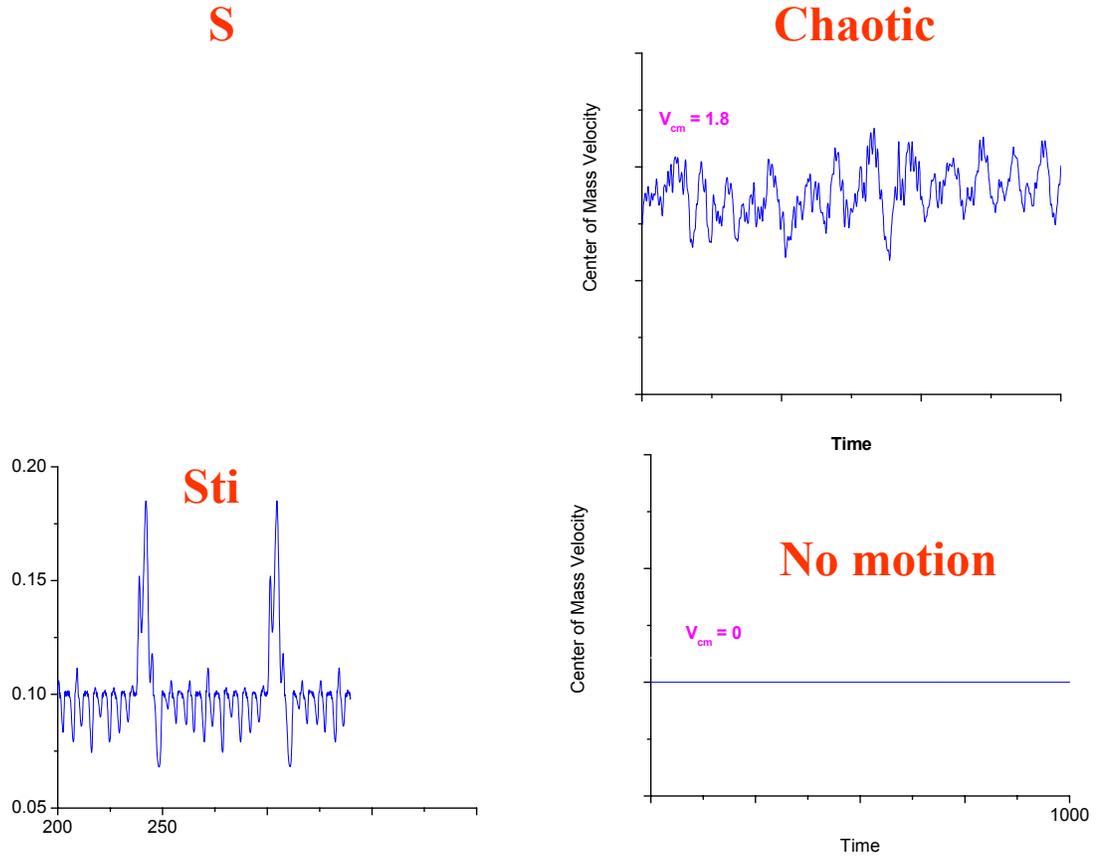
System (1) provides a general framework of modeling friction although the amount of details and complexity varies in different studies from simplified 1D models [17, 23] through 2D and 3D models [18, 24, 26,] to a full set of molecular dynamics simulations [27].

To better present our ideas, we simplify this model to the case where the substrate potential has a simple periodic form, there is a zero misfit length between the array and the substrate, the same force  $f$  is applied to each particle, and the interparticle coupling is linear. The coupling with the substrate is, however, strongly nonlinear. For this case, using the dimensionless phase variable  $\phi = 2\pi x / a$ , the equation of motion reduces to the dynamic Frenkel-Kontorova model

$$\ddot{\phi}_n + \gamma\dot{\phi}_n + \sin(\phi_n) = f + k(\phi_{n+1} - 2\phi_n + \phi_{n-1}). \quad (2)$$

A quantity that will play an important role in the control algorithm is the average(center of mass) velocity, defined as:  $v_{cm} := \sum_{n=1}^N \dot{\phi}_n$ . In frictional dynamics, both stick-slip and sliding motions can be achieved without changing the values of the system

parameters. This feature allows us to efficiently control frictional dynamics. In Figure 1 we demonstrate the time series of the center of mass velocity for a chain consisting of  $N = 15$  particles. We observed four types of motion: periodic sliding, chaotic and periodic stick-slip, and static motion. All motion types are obtained by only changing the initial conditions of the particle's positions and velocities. However, the values of the average velocity of the center of mass in the "natural" uncontrolled motion (to the friction coefficient of the array,  $\eta = (f / v_{cm} - \gamma) / \gamma$ ), may have only a limited range of values. These values can be estimated as: (i)  $v = f / \gamma$  for sliding motion, (ii)  $v = 0$  (non-sliding), and (iii)  $v = nv_0$ , where  $n$  is an integer, and  $v_0 = \frac{2\pi}{nN\gamma} \sqrt{\frac{\pi - \cos^{-1} f}{\pi} (\kappa - \kappa_c)^{1/2}}$ , for periodic stick-slip motion [17]. In the range of parameters of our consideration, we observed only one single value of the average velocity of the center of mass for chaotic stick-slip.



**Figure 1.** “Natural motion” of the uncontrolled array of  $N=15$  particles. We observe smooth sliding motion (top left), chaotic (top right) and periodic (bottom right) stick slip and static (bottom left). The parameters are:  $f = 0.3$ ,  $\gamma = 0.1$ , and  $\kappa = 0.26$ . All four types of motion are obtained by changing the initial positions and velocities in the array.

Our goal is: (i) to achieve any targeted value of the average sliding velocity using only (relatively) small values of the control and (ii) to significantly reduce the transient time to reach the desired behavior. To that effect, we propose the following control algorithm:

$$\ddot{\phi}_n + \gamma \dot{\phi}_n + \sin(\phi_n) = f + \kappa(\phi_{n+1} - 2\phi_n + \phi_{n-1}) + C(t) \quad (3)$$

where the control  $C(t)$  is given by:

$$C(t) = \alpha(v_{target} - v_{cm})^\beta \quad (4)$$

where  $v_{target}$  is the targeted velocity for the center of mass,  $\beta = 1/(2n+1)$ , and  $n = 1, 2, 3, \dots$ . In the following, we shall use  $\beta = 1/3$ . The above expression for the control utilizes the concept of "terminal attractors" [13, 14]. System (3) can be written as a  $2N$ -dimensional first order system:

$$\dot{\phi} - F_n(\phi_1, \phi_2, \dots, \phi_{2N}) = 0, \quad n = 1, 2, 3, \dots, 2N \quad (5)$$

where, for simplicity, we maintained the same notation for the (now different) unknown functions. At equilibrium, the fixed points of this  $2N$ -dimensional, dissipative dynamical system are defined as its constant solutions,  $\phi_n(\infty)$ . If the real part of the eigenvalues  $\mu_e$  of the Jacobian matrix,  $M_{nm} = \partial F_n / \partial \phi_m$ , at a fixed points are all negative (that is  $Re \mu_e < 0$ ) then these points are locally asymptotically stable. Such points are conventional static attractors: each motion along the phase curve that gets close enough to  $\phi(\infty)$  (i.e., enters a basin of attractor), approaches the corresponding constant value as  $t$  tends to infinity.

Nonlinear dynamical system such as Eq. (5) satisfy the Lipschitz condition, that is  $|\partial F_n / \partial \phi_m| \leq K < \infty$ . This condition guarantees the existence of a unique solution for each initial configuration. As a result, a transient solution cannot intersect the asymptotic solution to which it tends. Therefore, the time spent to reach conventional attractors and, in our case, achieving precise control, is unacceptably large.

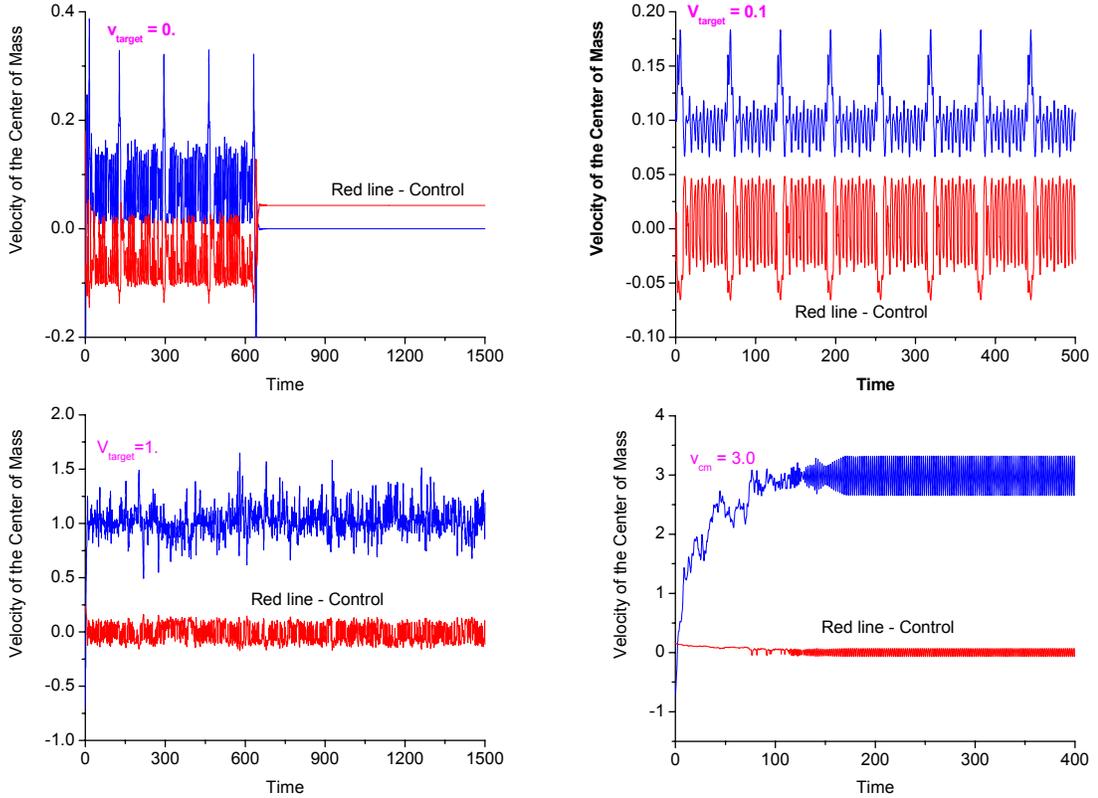
In contrast, the terminal attractor dynamics that we are utilizing violates the Lipschitz condition. As a result, trajectories reach the terminal attractor in finite time. To illustrate the simplest example of a terminal attractor, consider the equation  $\dot{\phi} = -\phi^{1/3}$ . At the equilibrium point  $\phi = 0$  the Lipschitz condition is violated, since  $\partial \dot{\phi} / \partial \phi = -(1/3)\phi^{-2/3}$  tends to minus infinity as  $\phi$  tends to zero. Since here,  $Re(\mu) \rightarrow -\infty$ , the equilibrium point  $\phi = 0$  is an attractor with "infinite" local stability.

This is precisely the effect we seek to achieve with the control term  $C(t)$ . For  $\beta = 1/3$ , we have:

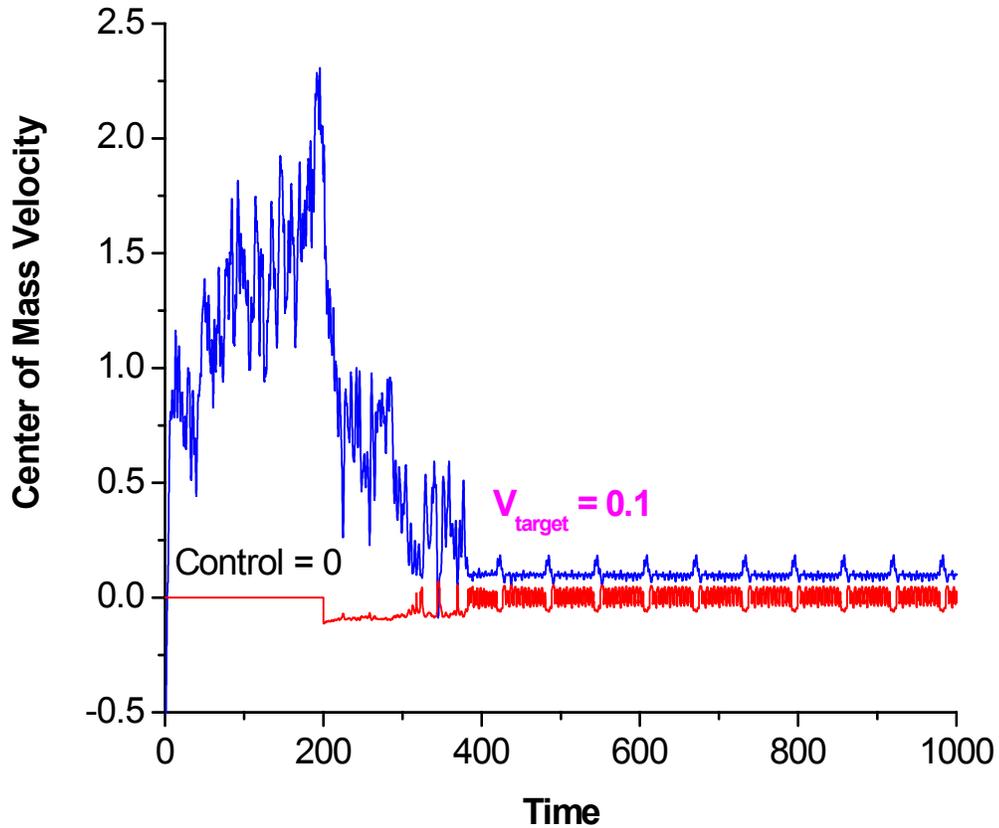
$$\frac{dC}{dv_{cm}} = -(1/3)\alpha(v_{target} - v_{cm})^{-2/3}. \quad (6)$$

Thus  $dC / dv_{cm} \rightarrow -\infty$  as  $v_{cm} \rightarrow v_{target}$ .

Note that this control strategy only requires the knowledge of the average velocity of the chain, and the control is the same for all particles in array. In Figure 2, we illustrate the performance of the control algorithm. We have chosen four target velocities, namely:  $v = 0, 0.1, 1.$ , and  $3$ . Red color lines demonstrate the time series of the control, while the blue lines show the time series of the velocity of the center of mass. In all cases, we obtained the desired values of the average velocity for rather small values of the control. In Figure 3, we show the convergence time needed to reach the desired value of the average velocity: convergence is very fast and, again, the strength of the control is small.



**Figure 2.** Performance of the control algorithm. We picked four values of the average velocities:  $v = 0, 0.1, 1.0,$  and  $3.0$  for  $N=15$  particle array. Blue lines show time series of the center of mass velocities while red lines show the control. In all cases, the desired behavior was achieved. The parameters are the same as in Figure 1.



**Figure 3.** Transient time to reach the desired asymptotic behavior. The blue line shows the time series of the center of mass velocity while the red line shows the control. We observe very short transient time. All the parameters are the same as in Figure 1.

In summary, we have proposed a new type of algorithm to control friction of the sliding nano-objects. This algorithm is based on the concept of "terminal attractor" and requires only the knowledge of the instant average velocity of the center of mass. We have demonstrated the ability to control the chain towards the desired sliding velocity and this control was achieved in a short transient time.

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