Adaptive Tracking and Regulation of a Wheeled Mobile Robot with Controller/Update Law Modularity*

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To appear in the *IEEE International Conference on Robotics and Automation*,
May 11-15, 2002, Washington, D.C.

Keywords: Mobile Robots, Adaptive Control, and Update Law Modularity

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* This research was supported in part by the Eugene P. Wigner Fellowship Program of the Oak Ridge National Laboratory (ORNL), managed by UT-Battelle, LLC, for the U.S. Department of Energy (DOE) under contract DE-AC05-00OR22725 and in part by the U.S. DOE Environmental Management Sciences Program (EMSP) project ID No. 82797 at ORNL, by ONR Project No. N00014-00-F-0485 at ORNL, and by U.S. NSF Grants DMI-9457967, DMI-9813213, EPS-9630167, ONR Grant N00014-99-1-0589, a DOC Grant, and an ARO Automotive Center Grant.

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Abstract: In this paper, we develop an adaptive torque control input for Wheeled Mobile Robots (WMRs) that can be utilized in a modular manner with parameter estimate update laws to enable unified tracking and regulation. That is, provided a prediction error-based update law ensures the parameter estimate vector is bounded, all of the signals are proven to be bounded. An additional stability analysis is provided to prove that if the adaptive update law is designed such that the prediction error is square-integrable and the estimated inertia matrix is positive-definite, the WMR tracking and regulation errors are globally asymptotically forced to a control term that can be made arbitrarily small.

I. Introduction

Over the last decade, the problem of regulating nonholonomic systems has been heavily targeted by control researchers due to the theoretically challenging nature of the problem. Specifically, due to the structure of the governing differential equations of the underactuated nonlinear system, the regulation problem cannot be solved via a smooth, time-invariant pure state feedback law due to the implications of Brockett's condition [2]. In addition to the regulation problem for the wheeled mobile robot (WMR), researchers have also targeted the more practical tracking control problem (which includes the path following problem as a subset). From a review of literature (see [3], [4], [8], [17], [19], [21], [22], [23] and the references therein), it can be observed that: (i) most of the tracking controllers do not solve the regulation problem due to restrictions on the reference model trajectory signals, (ii) most control designs rely heavily on the use of Barbalat's Lemma and its extensions during the kinematic stability analysis (i.e., the Lyapunov derivative is negative semi-definite in the system states as opposed to negative definite), (iii) some of the kinematic controllers are not differentiable (e.g., see the kinematic controller developed in [17]), and hence, the standard integrator backstepping procedure cannot be used to incorporate the mechanical dynamics (see the discussion in [17]), (iv) few results adaptively compensate for parametric uncertainty (e.g., payload mass, friction coefficients) in the dynamic model of the WMR, and (v) all of the adaptive control results rely on standard gradient adaptive update laws.

To address some of the above issues, Dixon et al. [9] developed a differentiable kinematic control law that utilizes a dynamic oscillator-like control term to obtain a global uniformly ultimately bounded solution for the unified WMR tracking and regulation problems. Since the proposed kinematic controller is differentiable, standard backstepping techniques were used to design a nonlinear robust controller that rejects uncertainty associated with the dynamic model. In [10], Dixon et al. redesigned the dynamic oscillator of [9] to achieve global adaptive tracking and regulation control. In [11], Dong et al. exploited the differentiable kinematic control structure proposed in [22] to construct a global adaptive asymptotic tracking control law for a class of nonholonomic systems; however, the Lyapunov derivative for the controllers in [10], [11] are negative semi-definite in the system states and gradient adaptive update laws were utilized.

In contrast to the adaptive controllers for WMR, several adaptive control results have been formulated for robot manipulators that explore new methods of parameter estimation. Most of this research has exploited Lyapunov-based techniques (i.e., the controller and the adaptive update law are designed in conjunction via a single Lyapunov function); however, the Lyapunov-based approach tends to restrict the design of the adaptive update law. For example, many of the previous adaptive controllers are restricted to utilizing position/velocity tracking error based gradient update laws. However, motivated by the fact that gradient update laws often exhibit slow parameter convergence (and hence, may retard the transient performance of the system) several researchers have explored control designs that incorporate other forms of update laws. Specifically, Slotine et al. [24] constructed a prediction error term as the difference between an estimated, filtered version of the robot dynamics and a filtered version of the input torque, and then developed a composite adaptive control law as the composite sum of a least-squares update law driven by the prediction error and a modified gradient update law driven by the link position/velocity tracking error. Although composite adaptive controllers have been experimentally proven (e.g., see [27]) to yield faster parameter convergence and improved transient response, the structure of the adaptive update law is still rather inflexible. In contrast to the Lyapunov-based approach given in [24], Leal et al. [14] utilized the flexibility[1] provided by previous passivity-based adaptive control designs to construct a modified least-squares update law with the link position/velocity tracking error as the input. A modified least-squares update law based on the link position/velocity tracking error was also proposed by Sadegh et al. [20] in the design of an exponentially stable desired compensation adaptation law (DCAL) based controller, provided the desired regression matrix satisfies a semi-persistency of excitation condition. In [25], Tang et al. developed an adaptive controller which included the standard gradient update law, the composite adaptation update law, and an averaging gradient update law as special cases.

In addition to the Lyapunov and passivity-based approaches given above, some research has exploited estimation-based approaches. Although these efforts have mainly targeted linear systems, estimation-based approaches allow for further flexibility in the construction of parameter update laws (e.g., prediction error-based gradient or normalized/unnormalized least squares update laws can be designed) due to the modular design of the controller and the update law. For example, Middleton et al. [18] utilized a modular estimation-based approach to augment the adaptive computed torque controller of [5] with additional terms which allowed the closed-loop error system to be written as a stable, strictly-proper, transfer function with the link position tracking error as the output and a prediction error related term as the input. The controller given in [18] enabled link position tracking and controller/update law modularity in the sense that any parameter update law could be used as long as its design ensured that: (i) the parameter estimates remain bounded, (ii) the prediction error is square integrable, and (iii) the estimated inertia matrix is positive-definite (i.e., a projection-type algorithm is required in the parameter update law). In [13], Krstic et al. utilized nonlinear damping [12] to extend previous linear estimation-based techniques to a class of parametric-strict-feedback nonlinear systems; however, this class of systems, does not encompass the robot dynamics due to coupling terms in the inertia matrix. However, motivated by the development given in [13], de Queiroz et al. [6] developed an adap-

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[1] The passivity-based adaptive controllers provide for some flexibility in the design of update law; however, the update law must be designed to satisfy a passive mapping condition.

tive link position tracking controller for robot manipulators which achieves controller/update law modularity. Although the result was similar to the result given in [18], the controller developed in [6] does not require the estimated inertia matrix to be positive-definite and does not require the online calculation of the inverse of the estimated inertia matrix.

Inspired by the result given in [6], in this paper we develop an adaptive torque control input for WMRs that can be utilized in a modular manner with parameter estimate update laws to enable unified tracking and regulation. That is, provided a prediction error-based update law ensures the parameter estimate vector is bounded, all of the signals are proven to be bounded. An additional stability analysis is then provided to prove that if the adaptive update law is designed such that the prediction error is square-integrable and the estimated inertia matrix is positive-definite, the WMR tracking and regulation errors are globally asymptotically forced to a control term that can be made arbitrarily small. To facilitate this result, we exploit the WMR kinematic control structure of [9]. The structure of the kinematic controller is crucial to the development of the modular adaptive controller because it has characteristics such as: (i) it is differentiable (enabling integrator backstepping to incorporate the dynamic model), (ii) it ensures the transformed states of the system are negative-definite in the time derivative of a radially unbounded nonnegative function (this is a key advantage over many of the current WMR designs that are negative semi-definite in the system states and require tools such as extended Barbalat's Lemma to prove stability), and (iii) it solves the unified regulation and tracking control results.

II. Kinematic Model

The kinematic model for the so-called kinematic wheel under the nonholonomic constraint of pure rolling and non-slipping is given as follows

$$\dot{q} = S(q)v \tag{1}$$

where $q(t)$, $\dot{q}(t) \in R^3$ are defined as

$$q = [x_c \quad y_c \quad \theta]^T \qquad \dot{q} = \begin{bmatrix} \dot{x}_c & \dot{y}_c & \dot{\theta} \end{bmatrix}^T \tag{2}$$

$x_c(t)$, $y_c(t)$, and $\theta(t) \in R$ denote the linear position and orientation, respectively, of the center of mass (COM) of the WMR, $\dot{x}_c(t)$, $\dot{y}_c(t)$ denote the Cartesian components of the linear velocity of the COM, $\dot{\theta}(t) \in R$ denotes the angular velocity of the COM, the matrix $S(q) \in R^{3\times 2}$ is defined as follows

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

and the velocity vector $v(t) \in R^2$ is defined as

$$v = [v_1 \quad v_2]^T = \begin{bmatrix} v_l & \dot{\theta} \end{bmatrix}^T \tag{4}$$

with $v_l(t) \in R$ denoting the linear velocity of the COM of the WMR.

III. Open-Loop Error System

To formulate the tracking control problem, we define the following time-varying reference model

$$\dot{q}_r = S(q_r)v_r \tag{5}$$

where $S(\cdot)$ was defined in (3), $q_r(t) = \begin{bmatrix} x_{rc}(t) & y_{rc}(t) & \theta_r(t) \end{bmatrix}^T \in R^3$ denotes the desired time-varying position and orientation trajectory, and $v_r(t) = \begin{bmatrix} v_{r1}(t) & v_{r2}(t) \end{bmatrix}^T \in R^2$ denotes the reference time-varying linear and angular velocity. With regard to (5), it is assumed that the signal $v_r(t)$ is constructed to produce the desired motion and that $v_r(t)$, $\dot{v}_r(t)$, $q_r(t)$, and $\dot{q}_r(t)$ are bounded for all time.

To facilitate the subsequent control synthesis and the corresponding stability proof, we define the following global invertible transformation [8]

$$\begin{bmatrix} w \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -\tilde{\theta}\cos\theta + 2\sin\theta & -\tilde{\theta}\sin\theta - 2\cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} \tag{6}$$

where $w(t) \in R$ and $z(t) = \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T \in R^2$ are auxiliary tracking error variables, and $\tilde{x}(t)$, $\tilde{y}(t)$, $\tilde{\theta}(t) \in R$ denote the difference between the actual Cartesian position and orientation of the COM and the reference position and orientation of the COM as follows

$$\tilde{x} = x_c - x_{rc} \qquad \tilde{y} = y_c - y_{rc} \qquad \tilde{\theta} = \theta - \theta_r. \tag{7}$$

After taking the time derivative of (6) and using (1-7), we can rewrite the tracking error dynamics in terms of the auxiliary variables defined in (6) as follows [8]

$$\begin{aligned} \dot{w} &= u^T J^T z + f \\ \dot{z} &= u \end{aligned} \tag{8}$$

where $J \in R^{2\times 2}$ is a skew-symmetric matrix defined as

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{9}$$

$f(z, v_r) \in R$ is defined as

$$f = 2\left(v_{r2}z_2 - v_{r1}\sin z_1\right), \tag{10}$$

and the auxiliary variable $u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \in R^2$ is defined in terms of the WMR position and orientation, linear and angular velocities, and the reference trajectory as follows

$$u = T^{-1}v - \begin{bmatrix} v_{r2} \\ v_{r1}\cos\tilde{\theta} \end{bmatrix} \qquad v = Tu + \Pi \tag{11}$$

where the global invertible matrix $T(q) \in R^{2\times 2}$ is defined as follows

$$T = \begin{bmatrix} (\tilde{x}\sin\theta - \tilde{y}\cos\theta) & 1 \\ 1 & 0 \end{bmatrix} \tag{12}$$

and $\Pi(q, q_r, v_r) \in R^2$ is defined as

$$\Pi = \begin{bmatrix} v_{r1}\cos\tilde{\theta} + v_{r2}(\tilde{x}\sin\theta - \tilde{y}\cos\theta) \\ v_{r2} \end{bmatrix}. \tag{13}$$

IV. Dynamic Model

The WMR dynamic model can be expressed in the following form [8]

$$M^*\dot{u} + V_m^* u + N^* = B^*\tau \tag{14}$$

where

$$\begin{aligned} M^* &= T^T M T, \qquad B^* = T^T B, \\ V_m^* &= T^T M \dot{T}, \qquad N^* = T^T\left(F + M\dot{\Pi}\right) \end{aligned} \tag{15}$$

$M \in R^{2\times 2}$ represents the constant, positive-definite inertia matrix, $F(u, q, q_r, v_r) \in R^2$ represents the friction effects, $\tau(t) \in R^2$ represents the torque input vector, $B \in R^{2\times 2}$ represents a known, constant global invertible input matrix that governs torque transmission to the wheels (see [8] for explicit examples of B) and $T(q)$ and $\Pi(q, q_r, v_r)$ are defined in (12) and (13), respectively. The dynamic equation of (14) exhibits the following properties which will be employed during the subsequent control development and stability analysis.

Property 1: The transformed inertia matrix $M^*(q)$ is symmetric, positive-definite, and satisfies the following inequalities [8]

$$m_1\|\xi\|^2 \le \xi^T M^*\xi \le m_2(z, w)\|\xi\|^2 \quad \forall \xi \in R^2 \tag{16}$$

where $m_1 \in R$ is a known positive constant, $m_2(z, w) \in R$ is a known, positive bounding function which is assumed to be bounded provided $z(t)$ and $w(t)$ are bounded ($\dot{m}_2(z, w, \dot{z}, \dot{w})$ is also assumed to be bounded provided $z(t), \dot{z}(t), w(t)$, and $\dot{w}(t)$ are bounded), and $\|\cdot\|$ is the standard Euclidean norm.

Property 2: A skew-symmetric relationship exists between the transformed inertia matrix and $V_m^*(q, \dot{q})$ as follows

$$\xi^T\left(\frac{1}{2}\dot{M}^* - V_m^*\right)\xi = 0 \qquad \forall \xi \in R^2 \tag{17}$$

where $\dot{M}^*(q)$ represents the time derivative of the transformed inertia matrix.

Property 3: The robot dynamics given in (14) can be linearly parameterized as follows

$$Y\vartheta = M^*\dot{u} + V_m^* u + N^* \tag{18}$$

where $\vartheta \in R^p$ contains the unknown constant mechanical parameters (i.e., inertia, mass, and friction effects) and $Y(u, \dot{u}) \in R^{2\times p}$ denotes the known regression matrix. We also note that the following linear parameterization can be formulated

$$Y_s \vartheta = M^* \dot{u}_d + V_m^* u_d + N^* \tag{19}$$

where $Y_s(u, u_d, \dot{u}_d) \in R^{2\times p}$ denotes a known regression matrix, ϑ is the same unknown constant parameter vector given in (18), and $\dot{u}_d(t) \in R^2$ represents the time derivative of the subsequently designed input $u_d(t) \in R^2$.

V. Control Development

A. Control Objective

The objective in this paper is to design an adaptive controller that solves the unified tracking and regulation problems for a WMR under the additional constraint of parametric uncertainty for the robot dynamics of (14) such that a modularity is achieved in the design of the controller and the parameter update law. To quantify the error between the parameter estimate generated by the modular adaptive update law and the actual parameters of the WMR dynamic model, we define a parameter estimation error vector as follows

$$\tilde{\vartheta} = \vartheta - \hat{\vartheta} \tag{20}$$

where $\hat{\vartheta}(t) \in R^p$ is a dynamic estimate of ϑ, defined in (18). Motivated by the desire to facilitate a modular adaptive control scheme that is independent of acceleration measurements, we also define a measurable prediction error $\varepsilon(t) \in R^2$ as follows

$$\varepsilon = Y_f \tilde{\vartheta} = Y_f \vartheta - Y_f \hat{\vartheta} = \tau_f - Y_f \hat{\vartheta} \tag{21}$$

where the filtered regression matrix [18] $Y_f(u) \in R^{2\times p}$ is defined by the following differential equation and initial condition

$$\dot{Y}_f + \beta Y_f = \beta Y, \qquad Y_f\left(u(0)\right) = 0 \tag{22}$$

where $Y(u, \dot{u})$ was defined in (18) (see [16] for details regarding an acceleration independent formulation of $Y_f(u)$), $\beta \in R$ is a positive control term, and the filtered torque $\tau_f(t) \in R^2$ is generated by the following differential equation and initial condition

$$\dot{\tau}_f + \beta \tau_f = \beta B^* \tau, \qquad \tau_f(0) = 0 \tag{23}$$

where $\tau(t)$ and $B^*(q)$ are defined in (14) and (15), respectively.

To achieve the objective of simultaneously solving the tracking and regulation problems, we will employ a control strategy that exploits a dynamic oscillator-like structure. To facilitate this control design, we define an auxiliary error signal $\tilde{z}(t) \in R^2$ as the difference between the subsequently designed dynamic oscillator-like signal $z_d(t) \in R^2$ and the transformed variable $z(t)$, defined in (6), as follows

$$\tilde{z} = z_d - z. \tag{24}$$

Moreover, since we will also employ the integrator backstepping technique [1] to incorporate the dynamic model, we define a backstepping error signal $\eta(t) \in R^2$ to quantify the mismatch between the kinematic velocity signal $u(t)$ and the subsequently designed desired kinematic velocity input, denoted by $u_d(t)$, as follows

$$\eta = u_d - u. \tag{25}$$

B. Control Design

Based on the open-loop kinematic system given in (8) and the subsequent stability analysis, we design $u_d(t)$ as follows [9]

$$u_d = u_a - k_2 m_2 z \tag{26}$$

where the auxiliary control term $u_a(t) \in R^2$ is defined as

$$u_a = \left(\frac{k_1 m_2 w + f}{\delta_d^2} \right) J z_d + \Omega_1 z_d, \tag{27}$$

the auxiliary signal $z_d(t)$ is defined by the following dynamic oscillator-like relationship

$$\begin{aligned} \dot{z}_d &= \frac{\dot{\delta}_d}{\delta_d} z_d + \left(\frac{k_1 m_2 w + f}{\delta_d^2} + w \Omega_1 \right) J z_d \\ z_d^T(0) z_d(0) &= \delta_d^2(0), \end{aligned} \tag{28}$$

the auxiliary terms $\Omega_1(w, z, v_r) \in R$ and $\delta_d(t) \in R$ are defined as

$$\Omega_1 = k_2 m_2 + \frac{\dot{\delta}_d}{\delta_d} + w \left(\frac{k_1 m_2 w + f}{\delta_d^2} \right) \tag{29}$$

and

$$\delta_d = \alpha_0 \exp(-\alpha_1 t) + \varepsilon_1 \tag{30}$$

respectively, k_1, k_2, α_0, α_1, $\varepsilon_1 \in R$ are positive, constant control gains, $f(z, v_r)$ was defined in (10), and $m_2(z, w)$ was given in (16). Furthermore, based on the transformed dynamic model given by (14) and the subsequent stability analysis, we design the control torque input $\tau(t)$ as follows

$$\tau = (B^*)^{-1} \left(\tau^* + Y_s \hat{\vartheta} + k_a m_2 \eta \right) \tag{31}$$

where $\tau^*(t) \in R^2$ is defined as follows

$$\begin{aligned} \tau^* \;\; = \;\; & \left[\frac{1}{\beta} Y_f \dot{\hat{\vartheta}} + \left(\dot{\hat{M}^*} - \hat{V}_m^* \right) \eta \right] + k_n \left\| Y_s \right\|_{i\infty}^2 \eta \\ & + k_n \left\| \frac{1}{\beta} Y_f \dot{\hat{\vartheta}} \right\|^2 \eta + k_n \left\| \left(\dot{\hat{M}^*} - \hat{V}_m^* \right) \eta \right\|^2 \eta \\ & + k_n \left\| J z w \right\|^2 \eta + k_n \left\| \tilde{z} \right\|^2 \eta \end{aligned} \tag{32}$$

where $m_2(z, w)$, $Y_s(u, u_d, \dot{u}_d)$, $\hat{\vartheta}(t)$, and $Y_f(u)$ are defined in (16), (19), (20), and (22), respectively, $\dot{\hat{\vartheta}}(t) \in R^p$ (which includes $\dot{\hat{M}^*}(\cdot)$ and $\hat{V}_m^*(\cdot)$) denotes the time derivative of the dynamic estimate $\hat{\vartheta}(t)$, and $k_a, k_n \in R$ are positive constant control gains.

Remark 1: Motivation for the structure of (28) is obtained by taking the time derivative of $z_d^T(t) z_d(t)$ as follows

$$\frac{d}{dt}\left(z_d^T z_d \right) = 2 z_d^T \left(\frac{\dot{\delta}_d}{\delta_d} z_d + \left(\frac{k_1 m_2 w + f}{\delta_d^2} + w \Omega_1 \right) J z_d \right) \tag{33}$$

where (28) has been utilized. After noting that the matrix J of (9) is skew symmetric, we can rewrite (33) as follows

$$\frac{d}{dt}\left(z_d^T z_d \right) = 2 \frac{\dot{\delta}_d}{\delta_d} z_d^T z_d. \tag{34}$$

As result of the selection of the initial conditions given in (28), it is easy to verify that

$$z_d^T z_d = \left\| z_d \right\|^2 = \delta_d^2 \tag{35}$$

is a unique solution to the differential equation given in (34) (see [9] for motivation regarding the design of $\delta_d(t)$). The bracketed control terms given in (32) are incorporated in the control design to cancel similar terms in the subsequent stability analysis. The remaining terms of (32) are incorporated to facilitate the input-to-state stability property of the closed-loop system with respect to $\tilde{\vartheta}(t)$.

C. Closed-Loop Error Systems

To facilitate the closed-loop error system development, we inject the auxiliary control input $u_d(t)$ into the open-loop dynamics of $w(t)$ given by (8) by adding and subtracting the term $u_d^T(t) J z(t)$ to the right-side of (8) and utilizing (25) to obtain the following expression

$$\dot{w} = -u_d^T J z + \eta^T J z + f. \tag{36}$$

After substituting (26) for $u_d(t)$, adding and subtracting $u_a^T(t) J z_d(t)$ to the resulting expression, utilizing (24), and exploiting the skew symmetry of J defined in (9), we can rewrite the dynamics for $w(t)$ as follows

$$\dot{w} = -u_a^T J z_d + u_a^T J \tilde{z} + \eta^T J z + f. \tag{37}$$

Finally, by substituting (27) for only the first occurrence of $u_a(t)$ in (37) and then utilizing the equality given by (35), the skew symmetry of J defined in (9), and the fact that $J^T J = I_2$ (Note that I_2 denotes the standard 2×2 identity matrix), we can obtain the final expression for the closed-loop error system for $w(t)$ as follows

$$\dot{w} = -k_1 m_2 w + u_a^T J \tilde{z} + \eta^T J z. \tag{38}$$

To determine the closed-loop error system for $\tilde{z}(t)$, we take the time derivative of (24), substitute (28) for $\dot{z}_d(t)$, and then substitute (8) for $\dot{z}(t)$ to obtain

$$\dot{\tilde{z}} = \frac{\dot{\delta}_d}{\delta_d} z_d + \left(\frac{k_1 m_2 w + f}{\delta_d^2} + w\Omega_1 \right) J z_d + \eta - u_d \quad (39)$$

where the auxiliary control input $u_d(t)$ was injected by adding and subtracting $u_d(t)$ to the right-side of (39), and (25) was utilized. After substituting (26) for $u_d(t)$ and then substituting (27) for $u_a(t)$ in the resulting expression, we can rewrite (39) as follows

$$\dot{\tilde{z}} = \frac{\dot{\delta}_d}{\delta_d} z_d + w\Omega_1 J z_d - \Omega_1 z_d + k_2 m_2 z + \eta. \quad (40)$$

After substituting (29) for only the second occurrence of $\Omega_1(t)$ in (40) and using the fact that $JJ = -I_2$, we can cancel common terms and rearrange the resulting expression to obtain

$$\dot{\tilde{z}} = -k_2 m_2 \tilde{z} + wJ \left[\left(\frac{k_1 m_2 w + f}{\delta_d^2} \right) J z_d + \Omega_1 z_d \right] + \eta \quad (41)$$

where (24) has been utilized. Since the bracketed term in (41) is equal to $u_a(t)$ defined in (27), we can obtain the final expression for the closed-loop error system for $\tilde{z}(t)$ as follows

$$\dot{\tilde{z}} = -k_2 m_2 \tilde{z} + wJu_a + \eta. \quad (42)$$

To develop the closed-loop error system for $\eta(t)$, we take the time derivative of (25), substitute (14) for $\dot{u}(t)$, add and subtract $V_m^* (q, \dot{q}) u_d(t)$, and then rearrange the resulting expression as follows

$$M^* \dot{\eta} = Y_s \vartheta - V_m^* \eta - B^* \tau \quad (43)$$

where (19) and (25) were utilized. After substituting (31) and (32) into (43), the following expression is obtained

$$\begin{aligned} M^* \dot{\eta} &= -V_m^* \eta + Y_s \tilde{\vartheta} - k_a m_2 \eta - \frac{1}{\beta} Y_f \dot{\hat{\vartheta}} - \left(\dot{\hat{M}^*} - \hat{V}_m^* \right) \eta \quad (44) \\ &\quad -k_n \left(\left\| \frac{1}{\beta} Y_f \dot{\hat{\vartheta}} \right\|^2 + \left\| \left(\dot{\hat{M}^*} - \hat{V}_m^* \right) \eta \right\|^2 \right. \\ &\quad \left. + \|Y_s\|_{i\infty}^2 + \|Jzw\|^2 + \|\tilde{z}\|^2 \right) \eta. \end{aligned}$$

where (20) was utilized.

VI. Input-to-State Stability Result

Theorem 2: Given the closed-loop error system in (38), (42), and (44), if $\tilde{\vartheta}(t) \in L_\infty [0, t_f)$ then all signals are bounded under closed-loop operation for $[0, t_f)$ where t_f denotes the final time.

Proof: To prove Theorem 2, we define a non-negative function $V_1(t) \in R$ as follows

$$V_1 = \frac{1}{2} \eta^T M^* \eta + \frac{1}{2} w^2 + \frac{1}{2} \tilde{z}^T \tilde{z} \quad (45)$$

where (45) is upper and lower bounded as follows

$$\lambda_1 \|\xi\|^2 \leq V_1 \leq \lambda_2 m_2 \|\xi\|^2 \quad (46)$$

where $\lambda_1, \lambda_2 \in R^1$ are positive bounding constants, $m_2(z, w)$ was given in (16), and $\xi(t) \in R^5$ is defined as follows

$$\xi = \begin{bmatrix} w, & \tilde{z}^T, & \eta^T \end{bmatrix}^T. \quad (47)$$

After taking the time derivative of (45), substituting for the closed-loop error systems given in (38), (42), and (44), utilizing (17), and then cancelling common terms, the following expression is obtained

$$\begin{aligned} \dot{V}_1 &= -k_a m_2 \eta^T \eta - k_1 m_2 w^2 - k_2 m_2 \tilde{z}^T \tilde{z} \quad (48) \\ &\quad + \left[\|Y_s\|_{i\infty} \|\eta\| \left\| \tilde{\vartheta} \right\| - k_n \|Y_s\|_{i\infty}^2 \|\eta\|^2 \right] \\ &\quad + \left[\left\| (\dot{\hat{M}^*} - \hat{V}_m^*) \eta \right\| \|\eta\| - k_n \left\| (\dot{\hat{M}^*} - \hat{V}_m^*) \eta \right\|^2 \|\eta\|^2 \right] \\ &\quad + \left[\left\| \frac{1}{\beta} Y_f \dot{\hat{\vartheta}} \right\| \|\eta\| - k_n \left\| \frac{1}{\beta} Y_f \dot{\hat{\vartheta}} \right\|^2 \|\eta\|^2 \right] \\ &\quad + \left[\|Jzw\| \|\eta\| - k_n \|Jzw\|^2 \|\eta\|^2 \right] \\ &\quad + \left[\|\tilde{z}\| \|\eta\| - k_n \|\tilde{z}\|^2 \|\eta\|^2 \right] \end{aligned}$$

where the notation $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a signal. After completing the squares for the bracketed terms of (48), the following upper bound can be formulated

$$\dot{V}_1 \leq -k_a m_2 \|\eta\|^2 - k_1 m_2 w^2 - k_2 m_2 \|\tilde{z}\|^2 + \frac{1}{k_n} \left\| \tilde{\vartheta} \right\|^2 + \frac{1}{k_n} \quad (49)$$

which can be further upper bounded by the use of (46) as follows

$$\dot{V}_1 \leq -\gamma_1 V_1 + \gamma_2 \quad (50)$$

where $\gamma_1, \gamma_2 \in R$ are positive constants defined as follows

$$\gamma_1 = \frac{\min(k_a, k_1, k_2)}{\lambda_2} \quad (51)$$

$$\gamma_2 = \frac{1}{k_n} \sup_t \left\| \tilde{\vartheta}(t) \right\|^2 + \frac{1}{k_n} \quad (52)$$

where the assumption that $\tilde{\vartheta}(t) \in L_\infty [0, t_f)$ (i.e., a constant bounded value for $\sup_t \left\| \tilde{\vartheta}(t) \right\|^2$ exists) was utilized. After solving the differential inequality given in (50), the following upper bound can be formulated

$$\begin{aligned} V_1(t) &\leq V_1(0) e^{-\gamma_1 t} + e^{-\gamma_1 t} \int_0^t e^{\gamma_1 \xi} \gamma_2 d\xi \quad (53) \\ &\leq V_1(0) e^{-\gamma_1 t} + \frac{\gamma_2}{\gamma_1}. \end{aligned}$$

Hence, from (45) and (53), the following inequality can be obtained

$$\|\xi(t)\| \leq \sqrt{\frac{\lambda_2 m_2(z(0), w(0))}{\lambda_1} \|\xi(0)\|^2 e^{-\gamma_1 t} + \frac{\gamma_2}{\gamma_1 \lambda_1}} < \infty. \quad (54)$$

Based on (54) and (47), it is straightforward to see that $w(t)$, $\tilde{z}(t)$, $\eta(t) \in L_\infty [0, t_f)$. After utilizing (24), (35), and the fact that $\tilde{z}(t)$, $\delta_d(t) \in L_\infty [0, t_f)$, we can also conclude that $z(t)$, $z_d(t) \in L_\infty [0, t_f)$. From (8), (10), (25-29), (38), and (42), we can prove that $f(t)$, $u_d(t)$, $u_a(t)$, $\dot{z}_d(t)$, $\dot{\tilde{z}}(t)$, $\dot{z}(t)$, $\dot{w}(t)$, $\dot{\eta}(t)$, $\Omega_1(t)$, $u(t) \in L_\infty [0, t_f)$. Given that $w(t)$, $z(t)$, $u(t) \in L_\infty [0, t_f)$, we can utilize (1-6), (12), and (13) to prove that $q(t)$, $\dot{q}(t)$, $v(t)$, $\tilde{x}(t)$, $\tilde{y}(t)$, $\tilde{\theta}(t)$, $T(q)$, $\Pi(q, q_r, v_r) \in L_\infty [0, t_f)$. Based on the previous bounding arguments, we can prove that $\dot{u}_d(t) \in L_\infty$, and hence, $Y_s(u, u_d, \dot{u}_d)$, $Y_f(u) \in L_\infty [0, t_f)$. Since $\dot{\eta}(t)$, $\dot{u}_d(t) \in L_\infty [0, t_f)$, we can utilize the time derivative of (25) to prove that $\dot{u}(t) \in L_\infty [0, t_f)$; hence, $Y(u, \dot{u}) \in L_\infty [0, t_f)$. Based on the assumption that $\tilde{\vartheta}(t) \in L_\infty [0, t_f)$, we can now utilize (20) and (21) to prove that $\hat{\vartheta}(t)$, $\varepsilon(t)$, $\tau_f(t) \in L_\infty [0, t_f)$. As described in [7], given that $u(t)$, $\varepsilon(t) \in L_\infty [0, t_f)$, we can now prove that $\dot{\hat{\vartheta}}(u, \varepsilon) \in L_\infty [0, t_f)$; hence, we can now utilize (31) and (32) to prove that $\tau(t)$, $\tau^*(t) \in L_\infty [0, t_f)$. ■

VII. Tracking and Regulation Result

Theorem 3: Given the control law (31), (32), and any update law, denoted by $\dot{\hat{\vartheta}}(u, \varepsilon)$, that ensures $\hat{\vartheta}(t) \in L_\infty [0, t_f)$, $\hat{M}^*(q)$ is positive-definite, and that $\varepsilon(t) \in L_2 [0, t_f)$, all signals are bounded during closed-loop operation for $t \in [0, \infty)$ and the position and orientation tracking errors $|\tilde{x}(t)|, |\tilde{y}(t)|, \left|\tilde{\theta}(t)\right|$ asymptotically approach a positive control term as follows

$$\lim_{t \to \infty} |\tilde{x}(t)|, |\tilde{y}(t)|, \left|\tilde{\theta}(t)\right| = \rho \varepsilon_1 \quad (55)$$

where $\rho \in R$ is a positive bounding constant and ε_1, given in (30), can be made arbitrarily small.

Proof: Theorem 2 can be directly applied to prove that all signals are bounded on $[0, t_f)$ during closed-loop operation. As in [13], the bounds are dependent only on the initial conditions, control gains, and the reference trajectory, (i.e., not dependent on t_f); hence, due to the independence of time, t_f can be expanded to ∞. To prove (55), we can utilize (18) and (19) to obtain the following expression

$$Y \tilde{\vartheta} = Y_s \tilde{\vartheta} - \tilde{M}^* \dot{\eta} - \tilde{V}_m^* \eta \quad (56)$$

where $\tilde{M}^*(q), \tilde{V}_m^*(q, \dot{q}) \in R^{2 \times 2}$ are defined as follows

$$\tilde{M}^* = M^* - \hat{M}^*, \qquad \tilde{V}_m^* = V_m^* - \hat{V}_m^* \quad (57)$$

and $Y_s(u, u_d, \dot{u}_d)\tilde{\vartheta}(t)$ is defined in (19). With the intent of writing the term $Y_s(u, u_d, \dot{u}_d)\tilde{\vartheta}(t)$ in terms of $\varepsilon(t)$, we utilize (20-22) to obtain the following expression

$$\begin{aligned}\frac{1}{\beta}\dot{\varepsilon}+\varepsilon &= \frac{1}{\beta}\dot{Y}_f\tilde{\vartheta}+Y_f\tilde{\vartheta}-\frac{1}{\beta}Y_f\dot{\hat{\vartheta}} \\ &= Y\tilde{\vartheta}-\frac{1}{\beta}Y_f\dot{\hat{\vartheta}}.\end{aligned} \tag{58}$$

After substituting (56) into (58) for $Y(u, \dot{u})\tilde{\vartheta}(t)$, the following expression can be obtained

$$Y_s\tilde{\vartheta} = \frac{1}{\beta}\dot{\varepsilon}+\varepsilon+\tilde{M}^*\dot{\eta}+\tilde{V}_m^*\eta+\frac{1}{\beta}Y_f\dot{\hat{\vartheta}}. \tag{59}$$

By substituting (59) into (44) and utilizing (57), we can rewrite the closed-loop system given in (44) as follows

$$\hat{M}^*\dot{\eta} = -A\eta+\frac{1}{\beta}\dot{\varepsilon}+\varepsilon-\dot{\hat{M}}^*\eta \tag{60}$$

where $A(t) \in R$ is a positive, time-varying signal defined as follows

$$\begin{aligned}A &= k_a m_2 + k_n\left(\|Y_s\|_{i\infty}^2+\left\|\frac{1}{\beta}Y_f\dot{\hat{\vartheta}}\right\|^2\right. \\ &\quad \left.+\left\|\left(\dot{\hat{M}}^*-\hat{V}_m^*\right)\eta\right\|^2+\|Jzw\|^2+\|\tilde{z}\|^2\right).\end{aligned} \tag{61}$$

To facilitate the subsequent analysis, we define a variable transformation $\chi(t) \in R^2$ as follows [6]

$$\chi = \hat{M}^*\eta - \frac{1}{\beta}\varepsilon. \tag{62}$$

After taking the time derivative of (62) and substituting (60) into the resulting expression, we obtain

$$\dot{\chi} = -A\eta+\varepsilon. \tag{63}$$

By utilizing (62), (63) can be rewritten as follows

$$\dot{\chi} = -A\left(\hat{M}^*\right)^{-1}\chi+\left(I_n-\frac{1}{\beta}A\left(\hat{M}^*\right)^{-1}\right)\varepsilon. \tag{64}$$

To examine the stability of $\chi(t)$, we define another transformation, denoted by $\psi(t) \in R$, as follows [6]

$$\psi = \sqrt{\frac{1}{2}\chi^T\chi}. \tag{65}$$

After squaring (65) and then taking the time derivative of the resulting expression, we obtain the following expression

$$\begin{aligned}\frac{d}{dt}\left(\psi^2\right) &= 2\psi\dot{\psi} \le -2A\lambda_{\min}\left\{(\hat{M}^*)^{-1}\right\}\psi^2 \\ &\quad +\sqrt{2}\left\|(I_n-\frac{1}{\beta}A(\hat{M}^*)^{-1})\right\|_{i\infty}\|\varepsilon\|\psi\end{aligned} \tag{66}$$

where (64) and (65) were utilized, and $\lambda_{\min}\{\cdot\}$ denotes the minimum eigenvalue of the argument. Based on the definition given in (65), we have that $\psi(t) \ge 0$; hence, we can use (66) to obtain the following inequality

$$\dot{\psi} \le -\zeta_1\psi+\zeta_2\|\varepsilon\| \tag{67}$$

where the positive constants ζ_1, $\zeta_2 \in R$ are defined as follows[2]

$$\begin{aligned}\zeta_1 &= \inf_t\{A\}\lambda_{\min}\left\{\left(\hat{M}^*\right)^{-1}\right\} \\ \zeta_2 &= \frac{\sqrt{2}}{2}\sup_t\left\|\left(I_n-\frac{1}{\beta}A\left(\hat{M}^*\right)^{-1}\right)\right\|_{i\infty}.\end{aligned} \tag{68}$$

[2]Provided $\hat{M}(q)$ is positive definite, $\hat{M}^{-1}(q)$ will also be positive definite, and hence, the minimum eigenvalue will be positive. Moreover, since all signals are bounded from the previous analysis, the time-varying signals $A(t)$ and $\hat{M}^{-1}(q(t))$ can be bounded by constants.

The solution of the differential inequality of (67) is given by

$$\psi(t) \le \psi(0)e^{-\zeta_1 t}+\zeta_2\int_0^t e^{-\zeta_1(t-\xi)}\|\varepsilon(\xi)\|\,d\xi. \tag{69}$$

After utilizing Holder's inequality [26], (69) can be rewritten as follows

$$\begin{aligned}|\psi(t)| &\le |\psi(0)|e^{-\zeta_1 t}+\zeta_2\sqrt{\int_0^t e^{-\zeta_1(t-\xi)}d\xi} \\ &\quad \cdot\sqrt{\int_0^t e^{-\zeta_1(t-\xi)}\|\varepsilon(\xi)\|^2 d\xi} \\ &\le |\psi(0)|e^{-\zeta_1 t}+\frac{\zeta_2}{\sqrt{\zeta_1}}\sqrt{\int_0^t e^{-\zeta_1(t-\xi)}\|\varepsilon(\xi)\|^2 d\xi}.\end{aligned} \tag{70}$$

After squaring (70) and integrating the resulting expression, we obtain the following expression

$$\int_0^t|\psi(\sigma)|^2 d\sigma \le \frac{|\psi(0)|^2}{\zeta_1}+\frac{2\zeta_2^2}{\zeta_1}\int_0^t\left(\int_0^\sigma e^{-\zeta_1(\sigma-\xi)}\|\varepsilon(\xi)\|^2 d\xi\right)d\sigma \tag{71}$$

where the fact that

$$\begin{aligned}&\left(|\psi(0)|e^{-\zeta_1 t}+\frac{\zeta_2}{\sqrt{\zeta_1}}\sqrt{\int_0^t e^{-\zeta_1(t-\xi)}\|\varepsilon(\xi)\|^2 d\xi}\right)^2 \\ &\le 2(\left|\psi^2(0)\right|e^{-2\zeta_1 t}+\frac{\zeta_2^2}{\zeta_1}\int_0^t e^{-\zeta_1(t-\xi)}\|\varepsilon(\xi)\|^2 d\xi)\end{aligned} \tag{72}$$

has been utilized. After reversing the order of integration in (71), the following expression is obtained

$$\begin{aligned}\int_0^t|\psi(\sigma)|^2 d\sigma &\le \frac{|\psi(0)|^2}{\zeta_1}+\frac{2\zeta_2^2}{\zeta_1}\int_0^t e^{\zeta_1\xi}\|\varepsilon(\xi)\|^2\left(\int_\xi^t e^{-\zeta_1\sigma}d\sigma\right)d\xi \\ &\le \frac{|\psi(0)|^2}{\zeta_1}+\frac{2\zeta_2^2}{\zeta_1}\int_0^t\|\varepsilon(\xi)\|^2\frac{1}{\zeta_1}d\xi.\end{aligned} \tag{73}$$

Based on the assumption that $\dot{\hat{\vartheta}}(u, \varepsilon)$ is designed to ensure that $\varepsilon(t) \in L_2$, we can utilize (73) to prove that

$$\|\psi\|_2 \le \frac{|\psi(0)|^2}{\zeta_1}+\frac{2\zeta_2^2}{\zeta_1^2}\|\varepsilon\|_2 < \infty \tag{74}$$

where $\|\cdot\|_2$ denotes the L_2 norm of a signal. From (74), we can conclude that $\psi(t) \in L_2$; hence, from (65), we can prove that $\chi(t) \in L_2$. Based on the results from Theorem 2, we can utilize (61) and (63) to prove that $A(t), \dot{\chi}(t) \in L_\infty$. After taking the time derivative of (62) and utilizing the fact that $\dot{\eta}(t)$, $\dot{\hat{M}}^*(u, \varepsilon)$, $\dot{\chi}(t) \in L_\infty$, we can conclude that $\dot{\varepsilon}(t) \in L_\infty$. Since $\chi(t)$, $\varepsilon(t) \in L_\infty \cap L_2$ and $\dot{\chi}(t)$, $\dot{\varepsilon}(t) \in L_\infty$, we can use Barbalat's Lemma [24] to prove that

$$\lim_{t\to\infty}\chi(t),\, \varepsilon(t) = 0 \tag{75}$$

and hence, from (62), we can prove that

$$\lim_{t\to\infty}\eta(t) = 0. \tag{76}$$

From (45), (48), and (76), we can now prove that

$$\lim_{t\to\infty}\tilde{z}(t),\, w(t) = 0. \tag{77}$$

Based on the result given in (77), we can now apply the triangle inequality to (24) and utilize (35) to prove that

$$\lim_{t\to\infty}\|z\| = \delta_d(t). \tag{78}$$

By utilizing (30), (77), and (78), the result given in (55) be be obtained from taking the inverse of the transformation given in (6). ■

Remark 4: The proof for Theorem 3 requires that $\dot{\hat{\vartheta}}(u,\varepsilon)$ be designed so that $\hat{\vartheta}(t) \in L_\infty$, $\hat{M}^*(q)$ is positive-definite, and $\varepsilon(t) \in L_2$. Typical parameter adaptation algorithms which ensure that $\hat{\vartheta}(t) \in L_\infty$ and $\varepsilon(t) \in L_2$ include the following gradient update law

$$\dot{\hat{\vartheta}} = \Gamma Y_f^T \varepsilon \tag{79}$$

where $\Gamma \in R^{p\times p}$ is a positive-definite gain matrix and the following least-squares estimator

$$\dot{\hat{\vartheta}} = \frac{PY_f^T\varepsilon}{1+\gamma_p tr\left\{Y_f P Y_f^T\right\}}, \quad \dot{P} = -\frac{PY_f^T Y_f P}{1+\gamma_p tr\left\{Y_f P Y_f^T\right\}} \tag{80}$$

where $P(t) \in R^{p\times p}$ is a time-varying symmetric matrix, $\gamma_p \in R$ is a nonnegative constant (if $\gamma_p = 0$ the standard unnormalized least-squares estimator is obtained), and $tr\{\cdot\}$ denotes the trace of a matrix (see [7] for further details). To ensure that $\hat{M}^*(q)$ is positive-definite, a standard projection algorithm can be incorporated in the design of (79) and (80) (see [1] and [15]).

Remark 5: Based on the fact that no restrictions were placed on the reference trajectory $v_r(t)$ with the exception that $v_r(t), \dot{v}_r(t) \in L_\infty$, it is straightforward to prove that the tracking control result given in (3) is also valid for the regulation problem (i.e., $v_r(t) = 0$).

VIII. Conclusions

We have developed an adaptive torque control input for wheeled mobile robots that can be utilized in a modular manner with parameter estimate update laws to solve the unified tracking and regulation problems. To achieve this result, we first leveraged off of our previous work in [9] to develop a differentiable kinematic controller that solves the unified tracking and regulation problems and facilitates integrator backstepping. Another motivation for the WMR kinematic control design is that a Lyapunov-based function can be constructed such that its time derivative is negative-definite, and hence, facilitates the modular adaptive control design and stability analysis (e.g., it is not clear how typical WMR kinematic controllers which exploit the use of extended Barbalat's Lemma to prove the stability result (such as [21]), can be utilized in conjunction with the modular adaptive control strategy). After developing the kinematic control design, we then leveraged off of the work of [6], [13] for robot manipulators, to develop a torque control input that was proven to yield update law modularity. That is, provided a prediction error-based update law ensures the parameter estimate vector is bounded, then all of the signals were proven to be bounded. An additional stability analysis was then provided to prove that if the adaptive update law was designed such that the prediction error is square-integrable and the estimated inertia matrix is positive-definite, the WMR tracking and regulation errors are asymptotically forced to a control term that can be made arbitrarily small. An advantage of the update law modularity is that the control designer is provided with additional flexibility in the design of the adaptive update law. That is, faster parameter convergence, and hence, potentially faster transient performance can be facilitated by various parameter update laws (e.g., least squares estimator). Future work will target experimental demonstration of the modular adaptive controller, and the development of a control scheme that ensures global asymptotic tracking and regulation control.

References

[1] M. M. Bridges, D. M. Dawson, and C. T. Abdallah, "Control of Rigid-Link Flexible-Joint Robots: A Survey of Backstepping Approaches", J. Robotic Sys., Vol. 12, No. 3, pp. 199-216, 1995.

[2] R. Brockett, "Asymptotic Stability and Feedback Stabilization", Differential Geometric Control Theory, (R. Brockett, R. Millman, and H. Sussmann Eds.), Birkhauser, Boston, 1983.

[3] C. Canudas de Wit, and O. Sordalen, "Exponential Stabilization of Mobile Robots with Nonholonomic Constraints", IEEE Trans. on Automatic Control, Vol. 37, No. 11, pp. 1791-1797, Nov. 1992.

[4] J. Coron and J. Pomet, "A Remark on the Design of Time-Varying Stabilizing Feedback Laws for Controllable Systems Without Drift", Proc. IFAC Symp. Nonlinear Control Systems Design (NOLCOS), pp. 413-417, 1992.

[5] J. J. Craig, P. Hsu, and S. Sastry, "Adaptive Control of Mechanical Manipulators", IEEE Conf. Rob. and Auto., pp. 190-195, 1986.

[6] M. S. de Queiroz, D. M. Dawson, and M. Agarwal, "Adaptive Control of Robot Manipulators with Controller/Update Law Modularity", Automatica, Vol. 35, No. 8, pp. 1379-1390, 1999.

[7] M. S. de Queiroz, D. M. Dawson, S. Nagarkatti, and F. Zhang, Lyapunov-Based Control of Mechanical Systems, Boston, MA: Birkhäuser, 2000.

[8] W. E. Dixon, D. M. Dawson, E. Zergeroglu, and A. Behal, Nonlinear Control of Wheeled Mobile Robots, Vol. 262 Lecture Notes in Control and Information Sciences, Springer-Verlag, 2000.

[9] W. E. Dixon, D. M. Dawson, E. Zergeroglu, and F. Zhang, "Robust Tracking and Regulation Control for Mobile Robots", Int. J. of Robust and Nonlinear Control, Vol. 10, pp. 199-216, 2000.

[10] W. E. Dixon, D. M. Dawson, F. Zhang, and E. Zergeroglu, "Global Exponential Tracking Control of A Mobile Robot System via a PE Condition", IEEE Trans. on Systems, Man, and Cyber. -Part B: Cyber., Vol. 30, No. 1, pp. 129-142, 2000.

[11] W. Dong and W. Huo, "Adaptive Stabilization of Dynamic Nonholonomic Chained Systems with Uncertainty", Proc. of the 36th IEEE Conf. on Decision and Control, pp. 2362-2367, 1997.

[12] P. Kokotovic, "The Joy of Feedback: Nonlinear and Adaptive", IEEE Control Syst. Magazine, Vol. 12, pp.7-17, 1992.

[13] M. Krstic and P. V. Kokotovic, "Adaptive Nonlinear Design with Controller-Identifier Separation and Swapping", IEEE Trans. Automatic Control, Vol. 40, No. 3, pp. 426-440, 1995.

[14] R. L. Leal and C. Canudas de Wit, "Passivity Based Adaptive Control for Mechanical Manipulators Using LS-Type Estimation", IEEE Trans. Automatic Control, Vol. 35, No. 12, pp. 1363-1365, 1990.

[15] R. L. Leal and B. Brogliato, "Adaptive Control of Robot Manipulators with Flexible Joints", IEEE Trans. Automatic Control, Vol. 37, No. 2, pp. 174-181, 1992.

[16] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, Control of Robot Manipulators, New York: MacMillan Publishing Co, 1993.

[17] R. McCloskey and R. Murray, "Exponential Stabilization of Driftless Nonlinear Control Systems Using Homogeneous Feedback", IEEE Transactions on Automatic Control, Vol. 42, No. 5, pp. 614-628, May 1997.

[18] R. H. Middleton and C. G. Goodwin, "Adaptive Computed Torque Control for Rigid Link Manipulators", Syst. Control Lett., Vol. 10, pp. 9-16, 1988.

[19] J. Pomet, "Explicit Design of Time-Varying Stabilizing Control Laws For A Class of Controllable Systems Without Drift", Syst. Contr. Lett., Vol. 18, no. 2, pp. 147-158, 1992.

[20] N. Sadegh and R. Horowitz, "An Exponentially Stable Adaptive Control Law for Robot Manipulators", IEEE Trans. Robotics and Automation, Vol. 6, No. 4, pp. 491-496, 1990.

[21] C. Samson, "Velocity and Torque Feedback Control of a Nonholonomic Cart", Proc. International Workshop in Adaptive and Nonlinear Control: Issues in Robotics, Grenoble, France, 1990.

[22] C. Samson, "Control of Chained Systems Application to Path Following and Time-Varying Point-Stabilization of Mobile Robots", IEEE Trans. on Automatic Control, Vol. 40, No. 1, pp. 64-77, 1997.

[23] A. Teel, R. Murray, and C. Walsh, "Nonholonomic Control Systems: From Steering to Stabilization with Sinusoids", Int. Journal Control, Vol. 62, No. 4, pp. 849-870, 1995.

[24] J. J. E. Slotine and W. Li, Applied Nonlinear Control. Englewood Cliff, NJ: Prentice Hall, Inc, 1991.

[25] Y. Tang and M. A. Arteaga, "Adaptive Control of Robot Manipulators Based on Passivity", IEEE Trans. Automatic Control, Vol. 39, No. 9, pp. 1871-1875, 1994.

[26] M. Vidyasagar, Nonlinear Systems Analysis, Englewood Cliffs, NJ: Prentice Hall, Inc, 1978.

[27] E. Zergeroglu, W. E. Dixon, D. Haste, and D. M. Dawson, "Composite Adaptive Output Feedback Tracking Control of Robotic Manipulators", Robotica, Vol. 17, pp. 591-600, 1999.