

## APPLICATIONS OF SUBCRITICAL SOURCE-DRIVEN NOISE ANALYSIS MEASUREMENTS

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Subcritical measurements have typically been used at critical experiment facilities as a means to monitor the approach to the critical condition. However, subcritical measurement methods have many more applications than simply to serve as indicators for the approach to critical. One of the more fundamental uses involves using them as benchmark measurements. Monte Carlo codes have been developed that can directly simulate a variety of subcritical measurements in the same manner in which the measurements are performed. The Monte Carlo codes can be used to interpret the measurements to determine the subcritical neutron multiplication factor essentially independent of the cross section data set.

The source-driven noise analysis measurement<sup>1</sup> is one subcritical measurement method that has several advantages as compared to other subcritical measurement techniques and has been applied to benchmark measurements and for the determination of the subcriticality of fissile configurations. The use of subcritical measurements as a means to validate Monte Carlo codes and nuclear data substantially increases the reactivity and neutron spectrum regimes over which the codes and data are validated. In fact, subcritical benchmark measurements are the key to determining whether the code bias (the difference between measured and calculated reactivity indices) is a function of the degree of subcriticality. Subcritical measurements can be used to validate Monte Carlo codes and nuclear data through direct calculation of the measured quantities. Alternatively, the Monte Carlo codes and nuclear data can be validated from the  $k_{eff}$  value determined from the measurements. Interpretation of measurements requires a model to relate what is measured to the neutron multiplication factor. In the past, the subcritical measurements were interpreted using equations developed from point kinetics models for the time-dependent behavior of neutrons in the subcritical configuration. This limited the application of the measurement to situations in which point kinetics was applicable. The most general model to relate the measured quantities to  $k_{eff}$  would involve the use of the generalized stochastic model developed by Munoz-Cobos et al.<sup>2</sup> Although an analytical solution of subcritical measurements in terms of the stochastic model is not practical, the Monte Carlo method provides a means to simulate the subcritical measurements and to also calculate the neutron multiplication factor. In fact, the same Monte Carlo code and nuclear data can be used for both calculations.

The Monte Carlo code MCNP-DSP<sup>3</sup> was developed to simulate a variety of subcritical measurements. The Monte Carlo codes are used to interpret the measurement by performing a calculation of the measured parameters and a separate eigenvalue calculation. For example, a comparison of measured and calculated values of a particular spectral ratio from the noise measurement can be used to obtain the “experimental”  $k_{eff}$ . If the measured and calculated values of the spectral ratio are in agreement, then the bias in the spectral ratio is zero. The bias in the spectral ratio is defined as the difference between

measured and calculated values of the spectral ratio ( $R_m-R_c$ ) where  $R_m$  is the measured value and  $R_c$  is the calculated value. First order perturbation theory can be used to obtain an expression that can be used to determine the “experimental”  $k_{eff}$  and the bias in the  $k_{eff}$ . The low-frequency value of the spectral ratio has been shown in numerous experiments to be linear with reactivity over a wide range of values of  $k_{eff}$  with the values of  $k_{eff}$  being interpreted using point kinetics models. Given the linear dependence of the spectral ratio with  $k_{eff}$ , the bias in the spectral ratio varies linearly as the bias in  $k_{eff}$ , ( $k_m-k_c$ ). To determine the “experimental”  $k_{eff}$  value and its bias, the Monte Carlo models are slightly perturbed and new values of the spectral ratio ( $R_p$ ) and  $k_{eff}$  ( $k_p$ ) are obtained. If the linear dependence is valid, then the perturbation calculations can be used to obtain the “experimental”  $k_{eff}$  and its bias using the following linear relationship:

$$\frac{R_m - R_c}{k_m - k_c} = \frac{R_p - R_c}{k_p - k_c} \quad (1)$$

This methodology simply uses a linear interpolation or extrapolation between the standard and perturbed values of the spectral ratio and  $k_{eff}$  to determine the “experimental”  $k_{eff}$ . Using this relationship, the value of  $k_m$  can be determined along with its bias  $k_m-k_c$ . Propagation of error is used to obtain the uncertainty in  $k_m$  and its bias. Even if the measured spectral ratio value and the calculated value agree, the perturbation analysis is performed. This is required to equate the uncertainty in the measured spectral ratio to the uncertainty in the inferred  $k_{eff}$  value.

This methodology has been successfully applied to benchmark source-driven noise measurements with uranyl nitrate solutions. The analysis of these measurements have been documented in the Handbook of the International Criticality Safety Benchmark Evaluation Project.<sup>4</sup> In the analysis of the uranyl nitrate solution experiments, a variety of perturbations were used to interpret  $k_{eff}$  and a variety of nuclear data sets used in the interpretations. The results demonstrated that the interpreted  $k_{eff}$  was independent of the perturbation method and had a very slight dependence on the nuclear data set. The interpreted (experimental)  $k_{eff}$  values for each different cross section data set are presented in Table I along with the measured and calculated spectral ratio values. As can be seen from this table, the calculated spectral ratio values depend on the cross section data set used for the calculations; however, the interpreted  $k_{eff}$  values do not depend significantly on the cross section data. The interpreted  $k_{eff}$  values differ slightly depending on which cross section data set was used. A general trend is observed that the interpreted  $k_{eff}$  values obtained using the ENDF/B-V cross section data are consistently higher than the interpreted values obtained from the ENDF/B-VI and JENDL-3.2 cross section data sets. The largest difference between the interpreted  $k_{eff}$  values among the results produced using the three different cross section data sets is approximately 0.003  $\Delta k$ . Hence, the perturbation methodology appears to be rather robust because although the computed spectral ratio values may be dependent on the cross section data set the interpreted  $k_{eff}$  values are only a slight dependence on the cross section data set.

Table I. Measured and calculated spectral ratio values and interpreted  $k_{eff}$  values.

Measured and calculated spectral ratio values.				
Solution Height (cm)	Benchmark Spectral Ratio ( $R_m$ ) ( $10^{-3}$ )	ENDF/B-V Spectral Ratio ( $R_c$ ) ( $10^{-3}$ )	ENDF/B-VI Spectral Ratio ( $R_c$ ) ( $10^{-3}$ )	JENDL-3.2 Spectral Ratio ( $R_c$ ) ( $10^{-3}$ )
30.48	$98.0 \pm 12.0$	$119.4 \pm 0.1$	$120.0 \pm 0.1$	$102.0 \pm 0.1$
27.94	$139.0 \pm 9.7$	$163.7 \pm 0.1$	$164.9 \pm 0.2$	$149.1 \pm 0.1$
25.40	$187.7 \pm 10.6$	$213.3 \pm 0.1$	$218.0 \pm 0.1$	$200.0 \pm 0.1$
22.86	$248.0 \pm 9.7$	$268.3 \pm 0.2$	$274.4 \pm 0.2$	$256.1 \pm 0.3$
20.32	$303.5 \pm 9.7$	$334.2 \pm 0.4$	$339.4 \pm 0.6$	$324.8 \pm 0.5$
Interpreted experimental $k_{eff}$ values for ENDF/B-V, ENDF/B-VI, and JENDL-3.2 cross section data sets for the high-enriched uranyl nitrate solution experiments.				
Solution Height (cm)	ENDF/B-V "Experimental" $k_{eff}$	ENDF/B-VI "Experimental" $k_{eff}$	JENDL-3.2 "Experimental" $k_{eff}$	
30.48	$0.9631 \pm 0.0052$	$0.9602 \pm 0.0052$	$0.9610 \pm 0.0050$	
27.94	$0.9448 \pm 0.0045$	$0.9430 \pm 0.0045$	$0.9432 \pm 0.0041$	
25.40	$0.9229 \pm 0.0052$	$0.9215 \pm 0.0050$	$0.9204 \pm 0.0056$	
22.86	$0.8889 \pm 0.0051$	$0.8888 \pm 0.0048$	$0.8883 \pm 0.0055$	
20.32	$0.8582 \pm 0.0055$	$0.8580 \pm 0.0061$	$0.8571 \pm 0.0052$	

## REFERENCES

1. V. K. Paré and J. T. Mihalcz, "Reactivity from Power Spectral Density Measurements with Californium-252," *Nucl. Sci. Eng.* **56**, 213 (1975).
2. J. L. Muñoz-Cobo and G. Verdu, "Neutron Stochastic Transport Theory with Delayed Neutrons," *Annals Nuclear Energy* **14**, 7,327 (1987).
3. T. E. Valentine, "MCNP-DSP Users Manual", ORNL/TM-13334-R2, Oak Ridge Nat. Lab, October 2000.
4. "International Handbook of Evaluated Criticality Safety Benchmark Experiments," NEA/NSC/DOC (95) 03, Nuclear Energy Agency, Organization for Economic Cooperation and Development (Sep. 2000).

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