Hadron-Hadron Scattering in the Nonrelativistic Quark Model

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Abstract. In this HADRON2001 contribution we summarize the status of our quark-model calculations of hadron-hadron scattering amplitudes in annihilation-free channels. The predictions are in reasonably good agreement with experimentally known S-wave meson-meson and meson-baryon phase shifts, and there are very recent indications that S-wave $\pi\omega$ scattering (extracted from FSIs in b_1 decay) may also be similar to our predictions. Finally, novel applications of this formalism to the dissociation cross sections of charmonia on light hadrons (relevant for QGP studies at RHIC) are discussed.

HADRON-HADRON SCATTERING

Introduction

Early models of strong interhadron forces were constructed by analogy with QED Feynman diagrams, and in the important NN problem it was assumed that these forces were dominated by t-channel meson exchange. At large distances, one-pion-exchange can indeed be confirmed in NN high partial waves. However at short distances one must have serious reservations about this type of model, since exchange of a ca. 1 GeV meson in t-channel implies a range of about 0.2 fm. Since this is much smaller than the extent of a typical hadron, the assumption of t-channel meson exchange appears rather dubious (see Maltman and Isgur [1] for a discussion). The success of meson-exchange models may simply be due to the many parameters available for fitting, and less trivially because it may prove difficult to distinguish short-distance QCD processes such as quark interchange from meson exchange, as these involve the same flavor flow.

Since QCD is a theory of quarks and gluons, and short-ranged scattering probes hadronic wavefunctions, it may be possible to describe hadron-hadron scattering in terms of explicit quark model wavefunctions and interquark forces taken from QCD and hadron spectroscopy. This approach has a long history in the NN problem, and has been applied to many hadronic reactions with much success and the occasional interesting failure. The technique most often used is the resonating-group method, although other variational or nonperturbative methods have also been applied. In this contribution we discuss our results from much simpler Born-order calculations of scattering amplitudes in the quark model, which are more straightfoward to evaluate and are also in reasonable agreement with experimental S-wave scattering amplitudes.

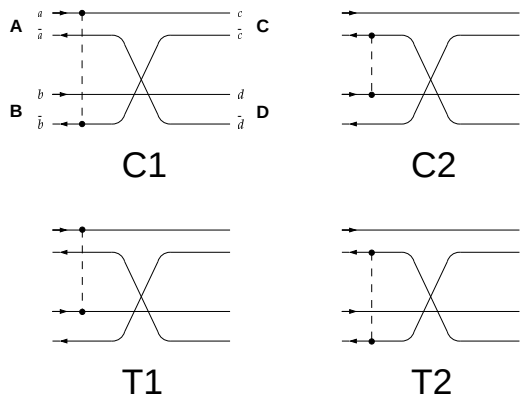

FIGURE 1. The four quark Born diagrams for meson-meson scattering.

Quark Born Diagrams

The usual quark model interaction has $\lambda \cdot \lambda$ color dependence, and in consequence a single interaction between quarks in different hadrons transforms the incident hadronic clusters from color singlets to color octets. Although this makes direct (no quark exchange) scattering zero at Born order, this modified state does have overlap with final color singlet hadrons, provided that we allow quark interchange. We refer to the resulting diagrams as “quark Born diagrams". The four quark Born diagrams one finds for the scattering of two $q\bar{q}$ mesons through this mechanism are shown in Fig.1. (We label these diagrams according to type; if the interacting constituents scatter into the same final hadron this is a “capture" diagram, and if not it is a “transfer" diagram.)

Each diagram has an associated spatial overlap integral, which is weighted by color, spin and flavor multiplicative matrix elements. Detailed evaluation of these diagrams was discussed in Refs.[2, 3], and the “Feynman rules" for the hadron-hadron T-matrices in our current notation are given in Ref.[4]. With Gaussian wavefunctions one may evaluate the T-matrices and phase shifts in closed form. As an example, the I=2 $\pi\pi$ S-wave phase shifts from standard quark model interactions (Ref.[4]) are give in Eq.(1).

$$\delta_0^{I=2\ \pi\pi} = \begin{cases} kE_\pi \frac{\alpha_s}{m^2} \left(-\frac{1}{3^2}\,\frac{1}{x}\left(1-e^{-2x}\right) - \frac{2^4}{3^{7/2}}\,e^{-4x/3}\right) & \text{OGE } S\cdot S \\ kE_\pi \frac{\alpha_s}{\beta^2} \left(-\frac{2}{3^2}\frac{1}{x}\left(\mathrm{f}_{1,\frac{1}{2}}(-2x) - e^{-2x}\right) - \frac{2^3}{3^{5/2}}\,\mathrm{f}_{1,\frac{3}{2}}(-2x/3)\,e^{-4x/3}\right) & \text{color Cou.} \\ kE_\pi \frac{b}{\beta^4} \left(\frac{1}{3^2}\frac{1}{x}\left(\mathrm{f}_{2,\frac{1}{2}}(-2x) - e^{-2x}\right) + \frac{1}{3^{1/2}}\,\mathrm{f}_{2,\frac{3}{2}}(-2x/3)\,e^{-4x/3}\right) & \text{lin. conft.} \end{cases} \tag{1}$$

where $x = \vec{A}^2/4\beta^2$, $\mathrm{f}_{a,c}(x)$ is an abbreviation for the confluent hypergeometric function ${}_1\mathrm{F}_1(a;c;x)$, $|\vec{A}|$ is the pion momentum in the c.m. frame (we assume a relativistic dispersion relation), $\beta = 0.4$ GeV is the standard quark model $q\bar{q}$ wavefunction width parameter, and a conventional quark model parameter set of $\alpha_s = 0.6$, $m_q = 0.33$ GeV and $b = 0.18\ \mathrm{GeV}^2$ is used to give the curves in Fig.2. The total Born-order S-wave phase shift is the sum of these three contributions. We have confirmed that these results are quite similar to the variational results of Weinstein and Isgur [5], who used essentially

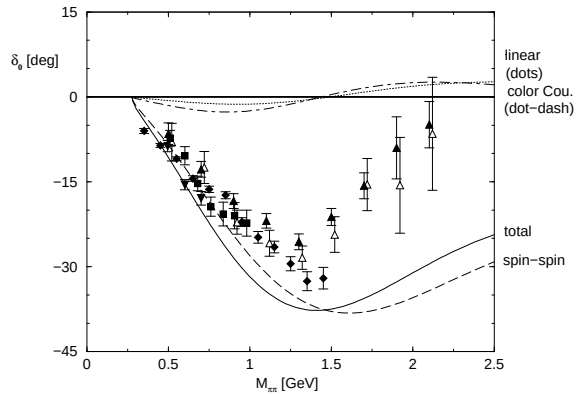

FIGURE 2. I=2 $\pi\pi$ experimental S-wave phase shifts versus Eq.(1), from Ref.[4].

the same interactions but included contributions beyond Born order.

Application of this approach to the scattering of other hadron pairs is straightforward, one need only enumerate the complete set of Born-order scattering diagrams, and evaluate these given a set of external hadron wavefunctions. We have applied this method to S-wave scattering of a wide range of annihilation-free channels, specifically I=3/2 Kπ [6], I=0,1 KN [7], I=0,1 BB [8] (compared to LGT data), and the NN repulsive cores [9], with generally reasonable results. These references consider many additional cases for which we do not have data at present.

More sensitive tests of the hadron scattering mechanism are possible if we consider higher partial waves. Here there is evidence of very interesting physics, for example in the large NN spin-orbit force (referred to by Isgur as the “Holy Grail" of quark-model scattering calculations) and the similarly large KN spin-orbit force. The KN spin-orbit force is apparently not well explained as elastic scattering with quark model forces [10]. (N.Black, unpublished, finds very similar results to this reference.) This discrepancy with experiment may be due to the large inelasticities known experimentally to be present in KN scattering.

We recently considered light vector-pseudoscalar meson scattering as a model spin-orbit problem for the quark Born diagram formalism, and derived the complete set of phase shifts in all partial waves given Gaussian wavefunctions and standard quark model forces [4]. We found that quark-model spin-orbit effects can indeed be quite large, for example in P-wave I=2 $\rho\pi$ elastic scattering we found a phase shift splitting $\delta(^3\mathrm{P}_2) - \delta(^3\mathrm{P}_0)$ that peaked at about 40°. Although one might suppose vector-pseudoscalar scattering to be experimentally inaccessible, it actually can be measured as a final state interaction in multiamplitude decays. In $b_1 \to \omega\pi$ in particular the S and D amplitudes have FSI phases of $e^{i\delta_S}$ and $e^{i\delta_D}$, so the S-D cross term in the $\omega\pi$ angular distribution is suppressed by $\cos(\delta_S - \delta_D)$ relative to the $|S|^2$ and $|D|^2$ terms. Our prediction is that

$$\delta_S(\omega\pi) - \delta_D(\omega\pi) = -14^\circ \tag{2}$$

at the b_1 mass. The E852 Collaboration has used this unusual FSI technique to extract this relative phase, and finds a consistent result of $\delta_S(\omega\pi) - \delta_D(\omega\pi) \approx -19^\circ(4^\circ)(8^\circ)$.

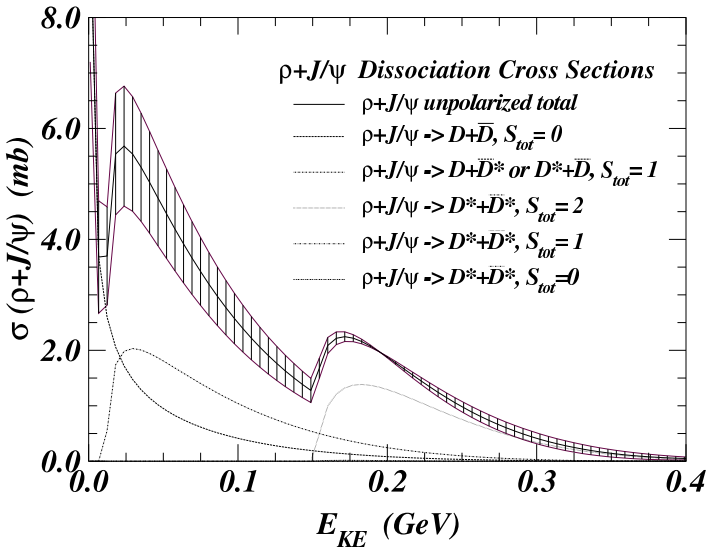

FIGURE 3. The $\rho + J/\psi$ dissociation cross section predicted by quark Born diagrams [12].

($\delta_D - \delta_S = 0.327 \pm 0.061 \pm 0.143$ radians is the tentative, as yet unpublished E852 result; private communication from M.Nozar.)

A New Application: Charmonium Dissociation at RHIC

Recently a novel class of hadronic reactions has attracted the attention of physicists searching for evidence of quark gluon plasma formation in heavy ion collisions. One signature proposed as an indicator of QGP formation is a suppression of the production rate of charmonium bound states such as the J/ψ, since the QGP is expected to screen the linear potential that would normally encourage a $c\bar{c}$ pair produced in the collision to remain bound [11].

If the charmonia that are formed in the collisions can penetrate the cloud of "comoving" light hadrons also produced in the collision, they can be detected through characteristic decays such as $J/\psi \to \ell^+\ell^-$, and this will be a “clean" experiment. Alternatively, if inelastic charmonium + light hadron cross sections into open-charm final states are sufficiently large, this more conventional $c\bar{c}$ dissociation process may imitate the expected QGP signal and will complicate the interpretation of the experiment. Of course these charmonium + light hadron dissociation cross sections are not at all well known at low energies, and estimates of the scale of these cross sections assuming different theoretical scattering mechanisms cover many orders of magnitude. Here we may have an exciting opportunity to establish the preferred hadron-hadron scattering mechanism in a new regime of QCD.

We have carried out a series of calculations of these charmonium + light meson dissociation cross sections in the constituent interchange model, using the approach described above. We use a standard Coulomb + linear + smeared hyperfine Hamiltonian to determine wavefunctions and scattering amplitudes, which are evaluated using numerical

techniques. Our results are that these cross sections at leading order are dissociation to open charm rather than elastic scattering (which is obvious from the flavor flow in Fig.1), and the low-energy cross sections are typically ca. 1 mb in scale [12, 13]. (Earlier work by Martins, Blaschke and Quack [14] using the same scattering formalism found somewhat larger cross sections, due to their assumption of a color-independent confining interaction.) Interestingly, $\rho + J/\psi$ cross sections are much larger than $\pi + J/\psi$, in part because $\rho + J/\psi \to$ DD is exothermic and hence diverges as we approach threshold. This process may lead to considerable suppression of the initial J/ψ population, and will require careful consideration in applications of this charmonium-suppression idea to QGP searches.

In future we plan to extend these calculations to a wide range of initial and final states, so that total cross sections (summed over all accessible final states) can be evauated, and more complicated reactions such as N+J/ψ inelastic scattering can be treated. Many other fascinating questions, such as the possibility that charmed mesons might bind to nucleons and nuclei, can also be considered through the application of this model of low-energy interhadron forces.

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