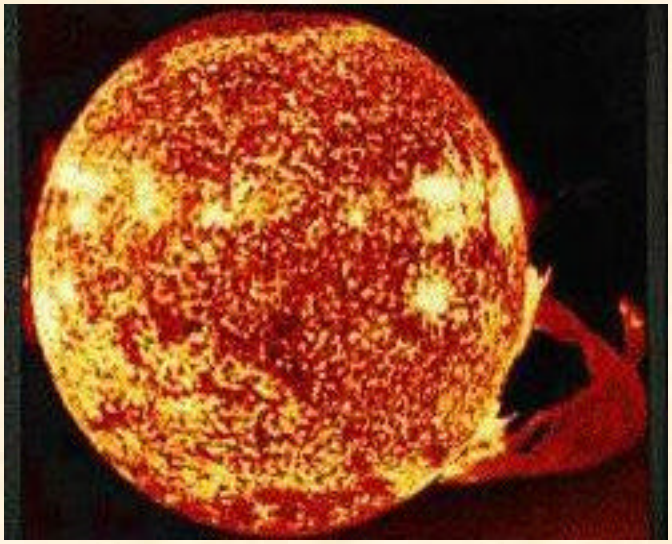
Traveling fronts in a model of turbulence-shear flow interaction in plasmas



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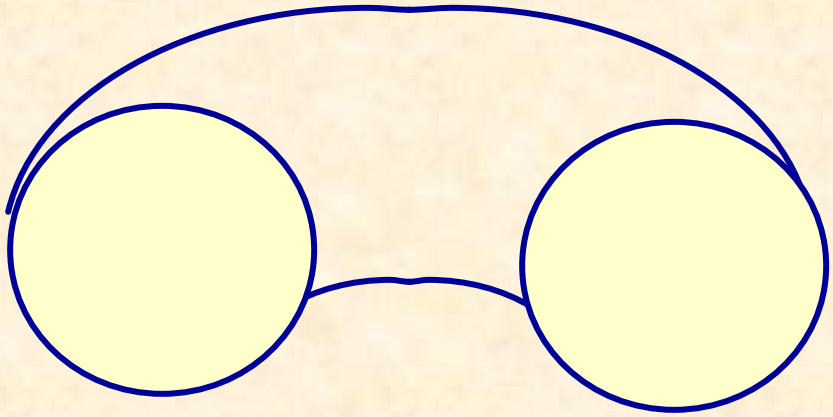
Abstract

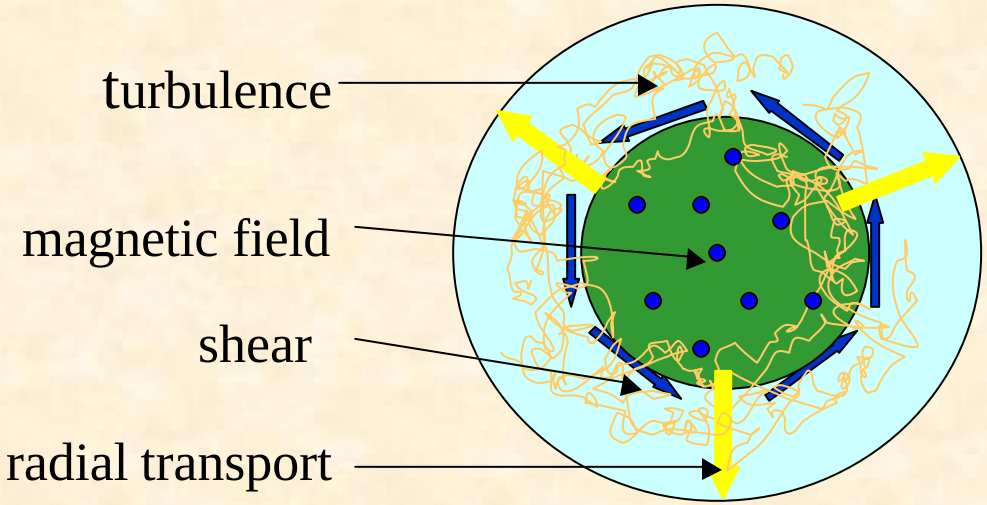
- Fusion Energy is the power source of the sun and the stars. Here at ORNL/FED and around the world, our problem is to understand and harness this energy process for the benefit of humankind. ORNL conducts R&D and design studies in fusion energy, plasma science, and plasma-based technologies. Researchers in the Fusion Energy Division have pioneered the application of massively parallel computer and associated programming techniques to fusion calculations and participate in the Numerical Tokamak Turbulence Project (NTTP). Plasma turbulence is a dominant form of particle and energy losses in most fusion experiments. In magnetically confined plasmas there is evidence of regions of improved confinement. These regions are usually associated with shear flows, and an important problem is to understand how such flows emerge and how they interact with the turbulence. To address this problem we consider a predator-prey type model of turbulence-shear flow interaction, where the shear is the predator and the turbulence is the prey. We solved the partial differential equations of the model using a C++ code that takes hours to run on a Macintosh. In particular, we found front solutions and traveling waves. By combining the time and distance variables in these partial differential equations into a single variable, traveling solutions can be found by solving a set of ordinary differential equations. These ordinary differential equations can be solved in a few seconds using MATLAB or Mathematica. Results from the ordinary differential equations match the partial differential equations results.

Magnetically confined plasmas

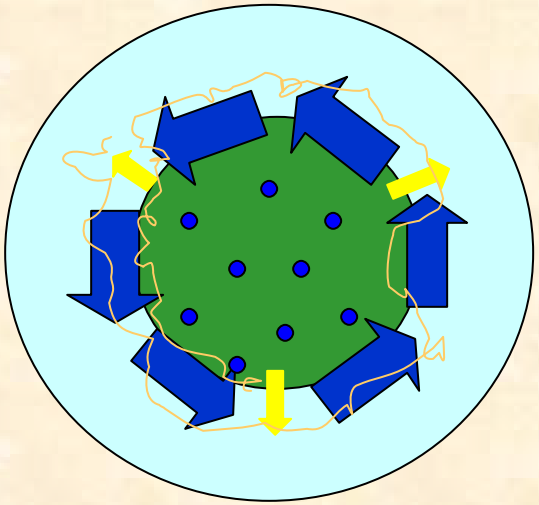
A tokamak is doughnut shaped device in which high temperature plasma is confined by strong magnetic fields. The goal of these devices is to achieve controlled thermonuclear fusion.



Low confinement



High confinement



A simplified predator-prey model

Prey

$$\P_t E = E - E^2 - s^2 E + (D_A/2)\P_x^2 E^2$$

Rate of change of turbulence | growth | saturation | suppression | diffusion

Predator

$$\P_t s = -m s + a s E + D_A \P_x^2 [E s]$$

Rate of change of shear | damping | production | diffusion

We solve this set of partial differential equations using a C++ code.

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A model for traveling front solutions

New variables

$$q_1 = E$$

$$q_2 = E\,s$$

Traveling front variable

$$t = x - ct$$

We solve this set of ordinary differential equations using MATLAB and MATHEMATICA to get traveling front solutions.

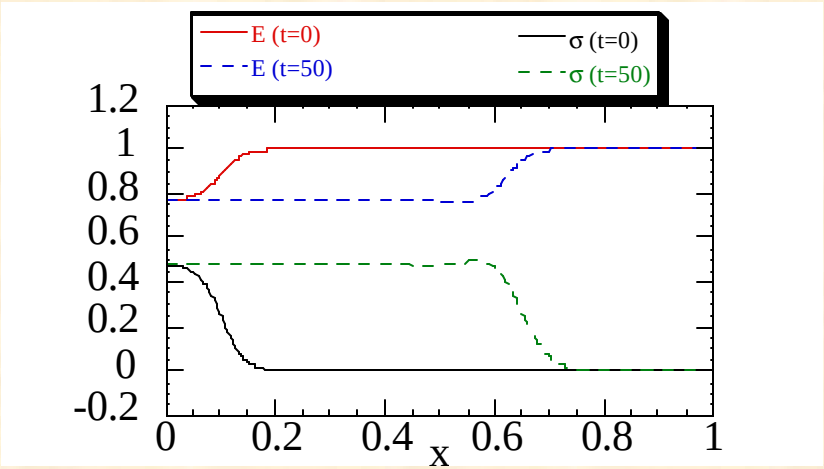
$$\frac{d\,P_1}{d\,t} = -\frac{P_1}{q_1}\left(l + P_1\right) + q_1 + \left(\frac{q_2}{q_1}\right)^2 - 1$$

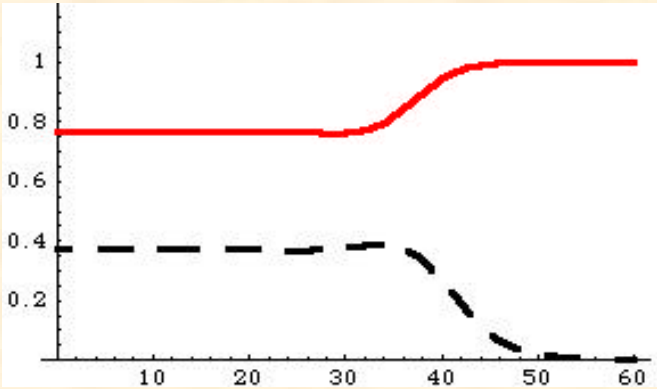
$$\frac{d\,P_2}{d\,t} = m\,\frac{q_2}{q_1} - a\;q_2 - l\left(\frac{q_1\,P_2 - q_2\,P_1}{q_1^{\,2}}\right)$$

$$\frac{d\,q_1}{d\,t} = P_1 \qquad \frac{d\,q_2}{d\,t} = P_2$$

Traveling front solutions

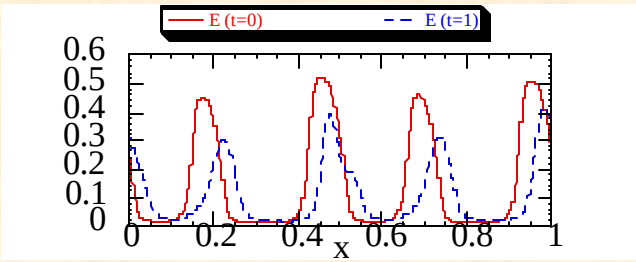
- The solution to the PDEs on the left is calculated as a function of both time and space.
- The solution to the ODEs on the right is calculated as a function of $\tau = x - ct$.
- Both methods find the same traveling front.

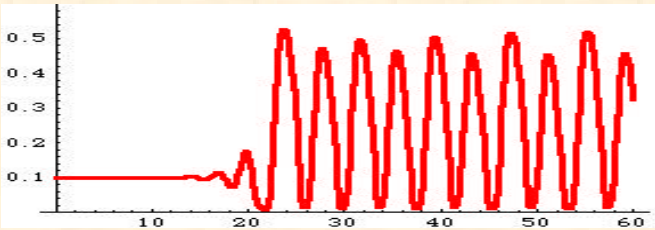


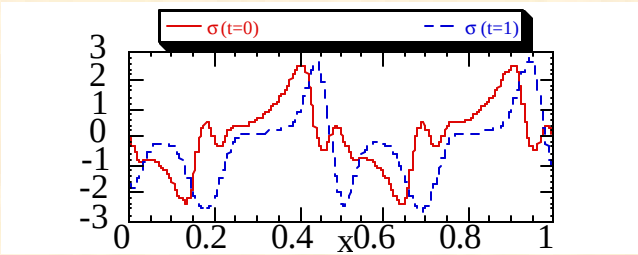


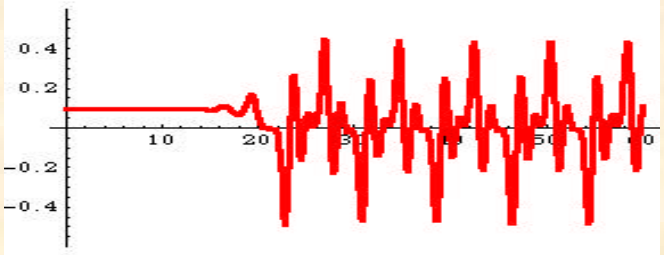
Traveling wave solutions

- The solution to the PDEs on the left is calculated as a function of both time and space.
- The solution to the ODEs on the right is calculated as a function of τ=x-ct .
- Both methods find the same traveling wave.









Summary

- The traveling solutions obtained with the ODEs are the same as the solutions to the PDEs
- The ODEs can be solved with MATLAB or Mathematica in seconds, while the PDEs can take an hour or more to solve.
- The parameter, λ, in the ODEs must be exact to many decimal places to find the traveling front solutions.

Disclaimer and Acknowledgements

Disclaimers

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