Passive Coincidence Technique to Determine the Shape of Plutonium Objects using Second Order Statistics

L. G. Chiang

June 26, 2001

Abstract

The Nuclear Materials Identification System (NMIS) detector-detector cross correlation signatures collected from passive measurements for plutonium assemblies are highly sensitive to its position with respect to the detectors. This position sensitivity coupled with an optimum configuration of four detectors about the assembly can theoretically be used to extract the shape distribution of the ^{240}Pu in the assembly. This process is currently being simulated at the Oak Ridge National Laboratory by using four detectors arranged in a tetrahedron and ^{252}Cf sources as surrogates for the ^{240}Pu.

1 Methodology

Unfolding the shape of an object is based upon the principle of superposition of the detector-detector cross correlations. Each spontaneous fission site in the object, in this case ^{240}Pu, acts as a point source that emits gamma and neutron radiations. These radiations are measured by detectors sensitive to both. The detection events in each detector pair are correlated in time to form the detector-detector cross correlation[2]. This detector-detector cross correlation is position sensitive and thus can be used to determine the location of the fission site. Also, since detector counts in the cross correlation signature are additive, the detector-detector cross correlations for the object are a linear summation of the individual detector-detector correlations from each point source in the object. Since ^{240}Pu is distributed throughout the object, identification of the location of the fission sites reveals the shape of the object.

This model of the object as a collection of point sources whose cross correlations can be linearly added, only applies if there is no multiplication and no self-attenuation in the material. These caveats limit the possible distribution of to non-multiplying and non-attenuating shapes such as a discrete point, ring or line sources, and thin shells.

This linear response of the detector-detector cross correlations permits the unfolding of the object's shape in the following manner. A set of at least four detectors are placed around the object such that the detector set provides a three dimensional view of the object to be measured. A volume of interest is defined which should totally encompass the object to be measured. Next, a response matrix R, containing the detector-detector cross correlations for each point source location in the volume of interest is either analytically generated or empirically measured. In the first case, the probabilities of the radiation pairs can be used to generate all the detector-detector cross correlations for each point source location. In the latter case, a point source whose fission spectrum closely resembles the ^{240}Pu isotope is placed at each grid point in the volume of interest and all the detector-detector cross correlations are measured. All the point source responses, i.e., detector-detector cross correlations, are saved in the response matrix, R. Once the response matrix is generated, the object is placed in the volume of interest and all of the detector cross correlations are measured. This set of measurements is the vector B. The shape vector, X is found by performing a nonnegative least squares (NNLS) algorithm on the the system $RX = B$.

2 Position Sensitivity

The detector-detector cross correlations are position sensitive. That is, the features of the cross correlation change in a predictable manner as a point source is moved between two detectors. The cross correlation measures the difference in arrival times of the radiations emitted by the point source. Since the source emits both gammas and neutrons and since the detectors are sensitive to both radiations, the possible features in the cross correlation signature are the gamma-gamma, neutron-neutron, gamma-neutron, and neutron-gamma distributions. The radiation particle pairs are ordered. The first member of the pair is the particle detected in the first detector and the second member is the radiation particle detected in the second detector of the detector pair.

The measured cross correlation signature contains all four features although not all the features are equally discernable as shown in Figure 1. For instance, the gamma-gamma peak is the most prominent peak for several reasons. First, all the gammas, regardless of their energy, travel at a fixed speed and arrive at the detectors within nanoseconds of each other due to the short separation distances between detectors. Second, the ratio of fission gammas to neutrons is roughly 4:1 so this radiation pair has the highest probability. Finally, the detector thresholds for gammas is relatively low compared to the neutron thresholds. The gamma-gamma peak will appear at the time lag corresponding to the difference in fly-out times for a gamma from the source to each detector.

The next most prominent features are the neutron-gamma and gamma-neutron distributions. The neutron-gamma distribution lies to the left of the gamma-gamma peak and occurs when the source neutrons reach the second detector in the detector pair before the source gammas reach the first detector. The gamma-neutron distribution which lies to the right of the gamma-gamma peak occurs when source gammas reach the first detector before the source neutrons reach the second detector in the detector pair.

Lastly, the neutron-neutron distribution is obscured as it has the smallest amplitude due to having the lowest probability of occurrence and being the most broadly distributed. It lies underneath the gamma-gamma peak and extends past it in both the positive and negative time lag direction until it reaches the time lag corresponding to the neutron threshold energy.

The position, breadth and height of each peak depends on the line-of-sight (LOS) distances of the point source to each detector in the detector pair. The relative position of the four features change as the source position changes. Note that in Figure 1 the gamma-gamma peak no longer appears at time lag zero since the source was closer to one of the detectors. A gamma-gamma peak at positive time lags indicates that the gammas take longer to travel to the second detector than the first so the source must be closer to the first detector. Conversely, a negative time lag indicates that the source is closer to the second detector. The position of the neutron-neutron distribution must also change in the same direction as the gamma-gamma peak although it is not apparent due to its small amplitude. The asymmetric placement of the source with respect to the detectors also leads to asymmetry in the position and shape of the gamma-neutron and neutron-gamma distributions. The neutron-gamma distribution to the left of time lag zero has a larger amplitude and is narrower than the gamma-neutron distribution.

Another key observation is that the movement of the gamma-gamma peak is less sensitive than the gamma-neutron and neutron-gamma distributions to the change in source position. This is to be expected as the source position must change more than 33 cm for the gamma-gamma peak to shift 1 nanosecond. Neutrons travel at much lesser speeds though. For example, the most probable neutron energy is approximately 2-MeV and travels roughly 2 cm/ns. Theoretically, the source

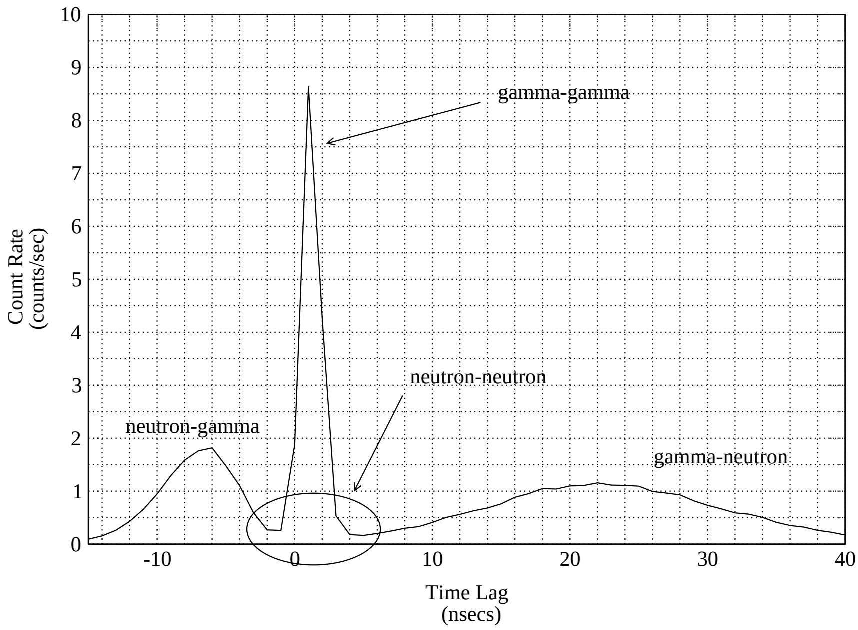

Figure 1: Cross correlation with point source located asymmetrically between two detectors.

position needs to only change 2 cm before the gamma-neutron or neutron-gamma peak shifts by 1 ns. Therefore, the neutrons just above the detector threshold energy of 1-MeV will be most sensitive to source position as they have the slowest speeds.

3 Spatial Ambiguity

Each pair of detectors measures one detector-detector cross correlation which provides one view of the object. If sufficient numbers of pairs of detectors are used then a unique reconstruction of the shape of the object is possible. The question then is, how many detector pairs are required to uniquely define a point in space?

If one starts with a single pair of detectors, as shown in Figure 2 one can see that a point in any position on the ring gives rise to the same detector-detector cross correlation. This occurs because only the line-of-sight distance from the point source to each detector, R_1 and R_2, determine the cross correlation signature. Since every point on the ring has the same set of distances to each detector pair, the ring forms a set of ambiguous points. That is, any point source on the ring will have the same detector-detector cross correlation. Note that in this configuration, there is only one detector-detector cross correlation, R_{12}.

Now, add another detector such that the three detectors form a triangle. There are now three possible detector-detector cross correlations, R_{12}, R_{13} and R_{23}. But for now, just consider detector pairs $1-2$ and $1-3$. As shown in Figure 3, there are now two ambiguous points formed by the intersection of the two rings of ambiguity.

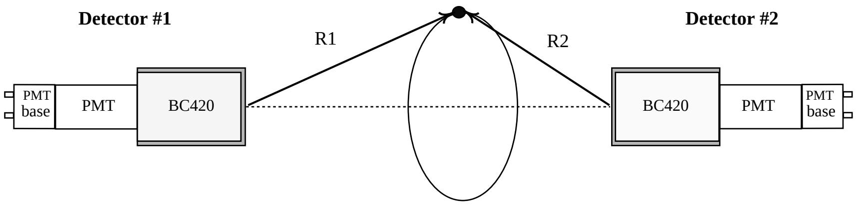

Figure 2: One detector pair and ring of ambiguous points.

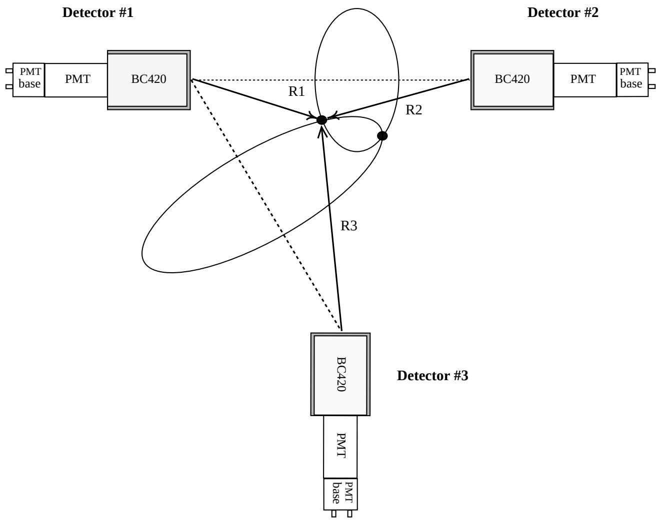

Figure 3: Two detector pairs and two ambiguous points.

Now add the third detector-detector cross correlation formed by detector pair $2-3$. There are now three rings of ambiguity, one for each detector pair, and they intersect at a single point as shown in Figure 4.

4 Generating a Response Matrix

The response matrix is an $m \times n$ matrix whose dimensions correspond to the number of cross correlations and the number of grid points in the volume of interest. The number of columns, n, corresponds to the number of grid points and the number of rows, m, is an integer multiple of the number of time bins in each cross correlation.

As pointed out in Section 3, a minimum of three cross correlations are required to uniquely define a point in three dimensions. This requires three detectors. By adding a fourth detector, one increases the number of unique views of the object from one to four. A simple configuration of four detectors is a regular tetrahedron in which all six sides are of equal length. If the detectors are numbered 1 to 4, the four unique views correspond to the four faces of the tetrahedron (1-2-3, 1-2-4, 2-3-4, and 1-3-4) and the cross correlations correspond to the six detector pairs (R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, and R_{34}).

The minimum requirements would be four response matrices, each composed of three cross correlations. Notice that this configuration requires that the linear system $RX = B$ be solved four

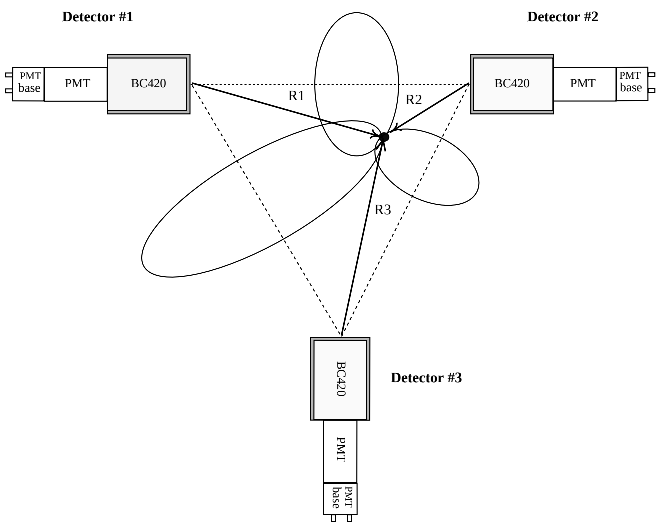

Figure 4: Three detector pairs and no ambiguous points.

times and that each cross correlation appears in two of the response matrices. For example, R_{12} would appear in the response matrices for detector set 1-2-3 and 1-2-4. In addition, this approach

Table 1: Composition of Response matrix.

Detectors	Cross correlations
1-2-3	R_{12}, R_{13}, R_{23}
1-2-4	R_{12}, R_{14}, R_{24}
2-3-4	R_{23}, R_{24}, R_{34}
1-3-4	R_{13}, R_{14}, R_{34}

leads to four solutions which defines a probability space for each grid point. In other words, a grid point, even if it is part of the true solution, may not be in all four solutions. Some type of weighting would have to be applied to determine if a grid point is part of the true solution.

In order to avoid having four possible solutions, one can place all six cross correlations into the same response matrix. Since there are 99 time lags in each cross correlations ($-49 \leq \tau \leq 49$), and six cross correlations, the response matrix will have dimensions of 594×125 as shown in Figure 5.

The number of grid points and the distribution of the grid points depend on the volume of interest. The volume of interest should be chosen such that it lies mostly inside the volume formed by the set of detectors and more importantly, provides a three-dimensional view of the object to be measured. Since the volume formed by four detectors is a tetrahedron, the volume of interest was chosen to lie inside this tetrahedron. It is possible to embed either a sphere or a cube inside this tetrahedron. For simplicity, a cube was chosen and thus a right-handed Cartesian coordinate system was adopted. The number of grid points was chosen to be 125 since the volume of interest was divided into a $5 \times 5 \times 5$ set of cubes that almost fits entirely inside the interior of the tetrahedron. The grid spacing was chosen to coincide with the expected spatial resolution of the measurement system.

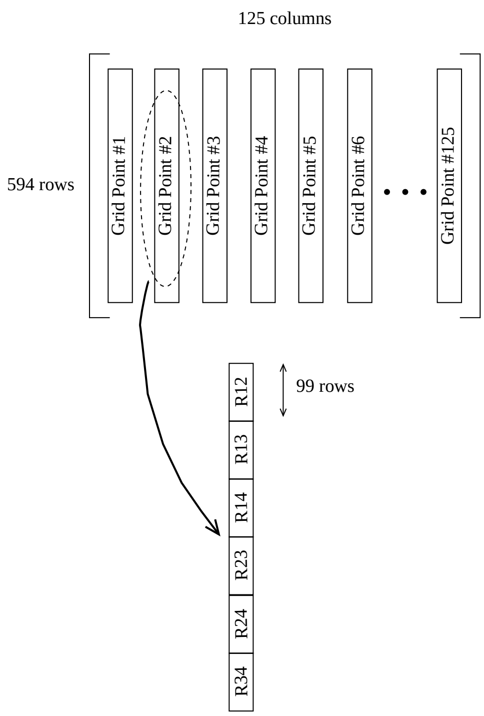

Figure 5: Structure of R-matrix

5 Solving $RX = B$ with NNLS

The linear system $RX = B$ can be solved using algorithms developed for constrained least squares problems. The solution vector, X, also called the shape vector, cannot have negative values as a point source cannot have a negative source strength. Therefore, the values of X must be nonnegative and can be considered a special case of the bounded least squares (BLS) problem whose solution is found by minimizing for all values of X, the Euclidean norm of $AX - B$ such that $l \leq X \leq u$ where l is the lower bound and u is the upper bound. Mathematically, this set of problems is often stated as:

$$\min_{X} ||AX - B||_2, \text{ subject to } l \leq X \leq u \tag{1}$$

It has been proven that if the response matrix R has full column rank[1], which it does, then the BLS problem is a convex optimization problem which has a unique solution for any vector B. (Conversely, if the rank of R is less than full column rank, a solution still exists but it is not guaranteed to be unique.

Now that one knows that a unique solution exists, the question arises whether further restriction of the constraints will further simplify solving for X. The answer is yes. Since the lower bound, $l = 0$ one can further redefine the problem as a nonnegative least squares problem (NNLS),

$$\min_{X} ||AX - B||_2, \text{ subject to } X \geq 0 \tag{2}$$

The NNLS algorithm used in this thesis was developed and published by Charles L. Lawson and Richard J. Hanson in 1974[1]. The source code, written in FORTRAN 77, has been ported to many

[1] full column rank means that the rank of R equals n, the number of columns of R

computers and verified. The algorithm has been incorporated into several commercial numerical processing packages but the source code remains freely distributed from NETLIB via the Internet.

References

[1] *Solving Least Squares Problems.* SIAM, 1995.

[2] J. T. Mihalczo, J. A. Mullens, J. K. Mattingly, and T. E. Valentine. Physical description of Nuclear Materials Identification System (NMIS) signatures. *Nuclear Instruments and Methods*, 450:531–555, August 2000.