Analysis of depth-sensing indentation tests with a Knoop indenter

L. Riester,
ORNL, USA

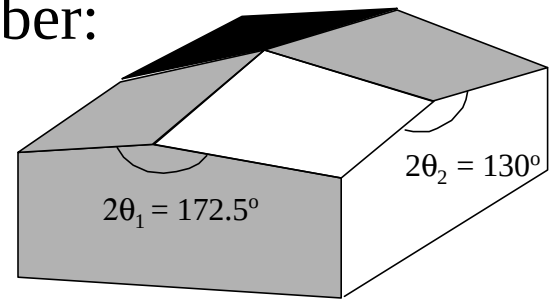
T.J. Bell, **A.C. Fischer-Cripps**
CSIRO Division of Telecommunications and Industrial Physics, Australia

Your speaker today

The “Knoop” indenter

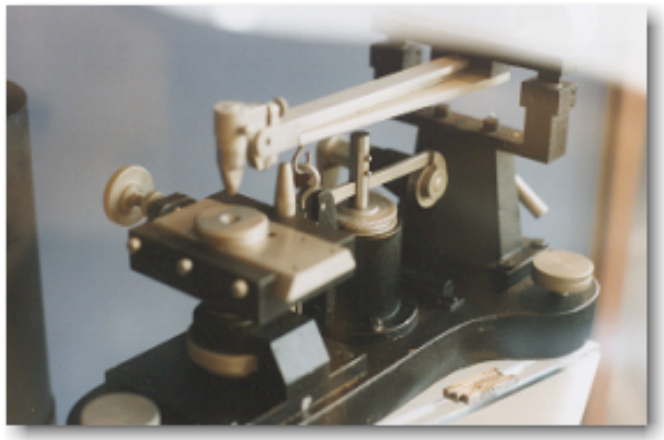
The Knoop hardness number:

$$KHN = \frac{P}{A}$$

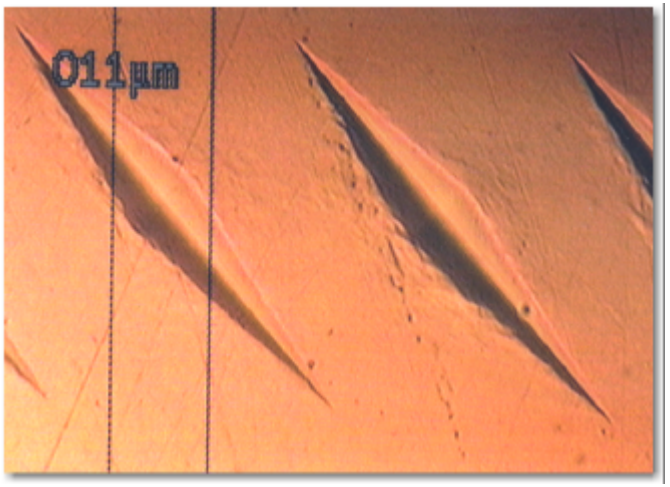


$$= \frac{d^2}{2}\left[\cot\theta_1 \tan\theta_2\right]$$

$$= 0.070275 d^2$$



Knoop’s original hardness testing apparatus (1939) on display at NIST, Gaithersburg MD

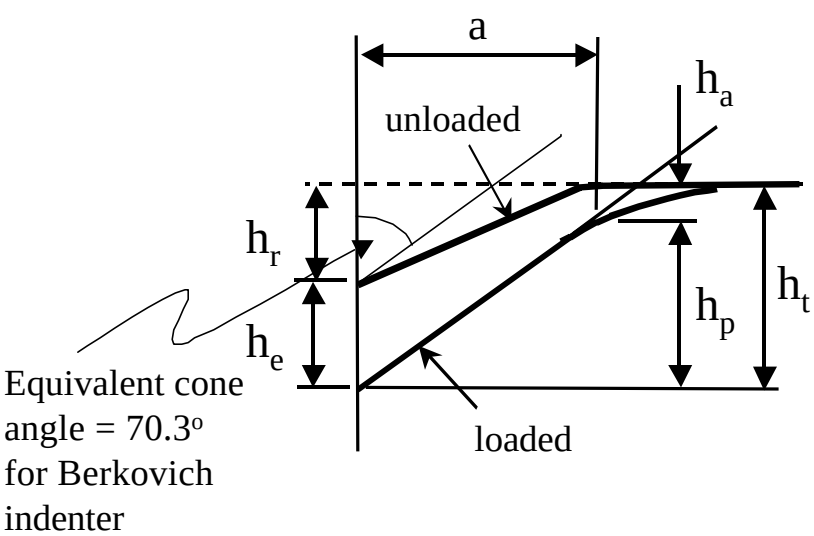


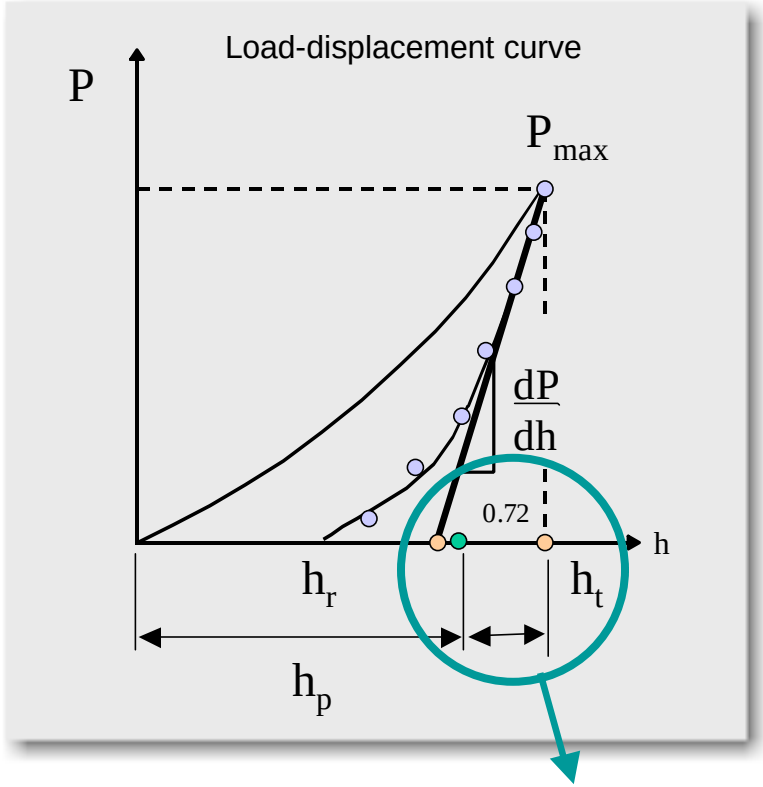
Some nice residual impressions in a steel hardness block made with a Knoop indenter.

The "Oliver and Pharr" method

Berkovich indenter

Based upon Sneddon's elastic solution for the loading of a cone of half angle α into a half space. $P = \frac{\pi}{2} a^2 E^* \cot\alpha$





Hardness

$$H = \frac{P}{A}$$

$$= 3\sqrt{3} h_p^2 \tan^2 65.3$$

$$= 24.5 h_p^2$$

Modulus

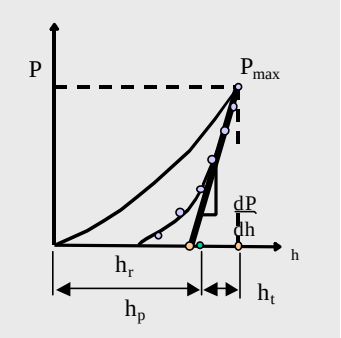
$$E^* = \frac{dP}{dh} \frac{\sqrt{\pi}}{2\sqrt{A}}$$

$$h_p = h_t - \left[\frac{2(\pi - 2)}{\pi}\right] \frac{P_{max}}{dP/dh}$$

Intercept factor =0.72

The "Oliver and Pharr" method

Any indenter



Hardness

$$H = \frac{P}{A}$$

Modulus

$$E^* = \frac{dP}{dh}\frac{\sqrt{\pi}}{2\sqrt{A}}$$

$$h_p = h_t - [\quad]\frac{P_{max}}{dP/dh}$$

Indenter	Area	Intercept factor
Sphere	$A \approx \pi 2Rh_p$	0.75
Berkovich	$A = 3\sqrt{3}h_p^2 \tan^2\alpha$	0.72 (0.75)
Vickers	$A = 4h_p^2 \tan^2\alpha$	0.72
Knoop	$A = 2h_p^2 \tan\alpha_1 \tan\alpha_2$	0.72
Cube Corner	$A = 3\sqrt{3}h_p^2 \tan^2\alpha$	0.72
Cone	$A = \pi h_p^2 \tan^2\alpha$	0.72

So for a Knoop indenter, all we have to do is use the Oliver and Pharr method with:

$$A = 2h_p^2[\tan\theta_1 \tan\theta_2]$$

EASY!

The “Oliver and Pharr” method

Knoop indenter analysis #1

IT DIDN’T WORK!

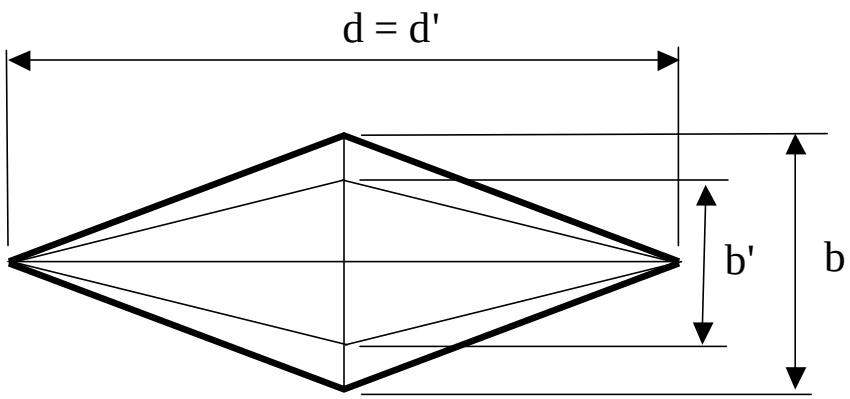
Material	E	Book value
Fused silica	86.6 GPa	70 GPa
Steel	242 GPa	210 GPa
Alumina	457 GPa	380 GPa

Berkovich indenter? No problem, E and H as expected.
Knoop indenter? the modulus and hardness were both over-estimated – Why?

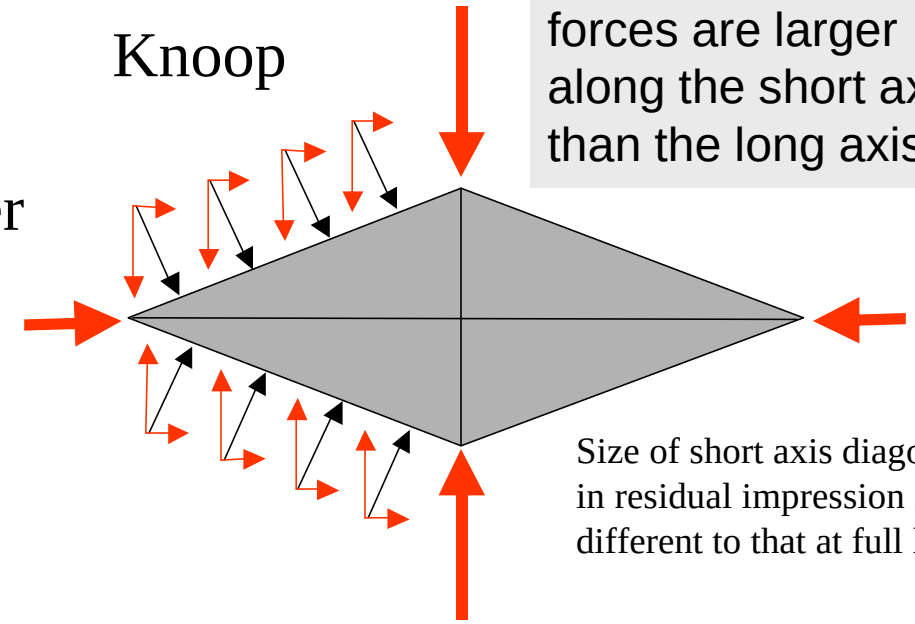
Knoop indenter

Why is it different?

The impression in the surface of a specimen made by a Knoop indenter undergoes significant **elastic recovery** along the short axis.



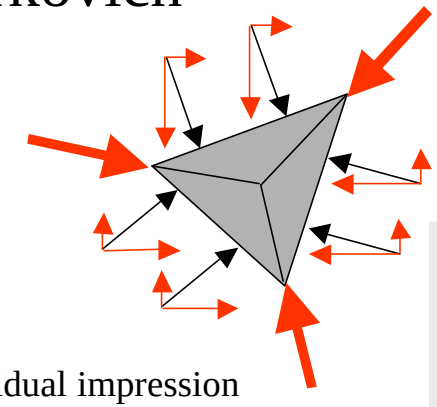
Knoop



Elastic restoring forces are larger along the short axis than the long axis

Size of short axis diagonal in residual impression is different to that at full load

Berkovich

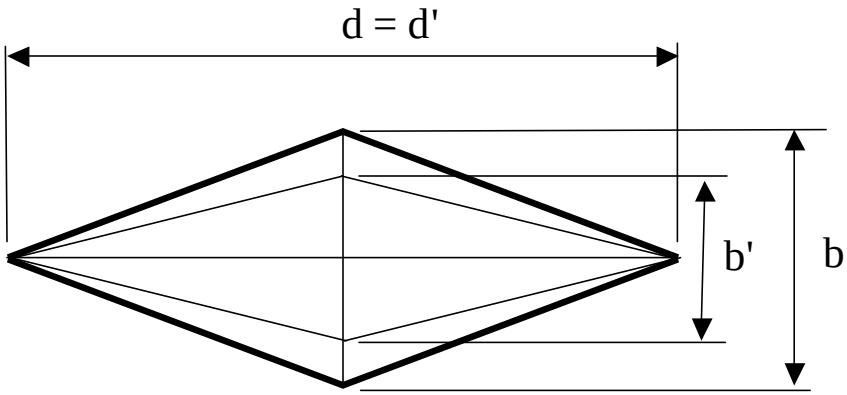


Elastic restoring forces are symmetric

Size of residual impression is same as that at full load

Knoop Indenter

Elastic Recovery



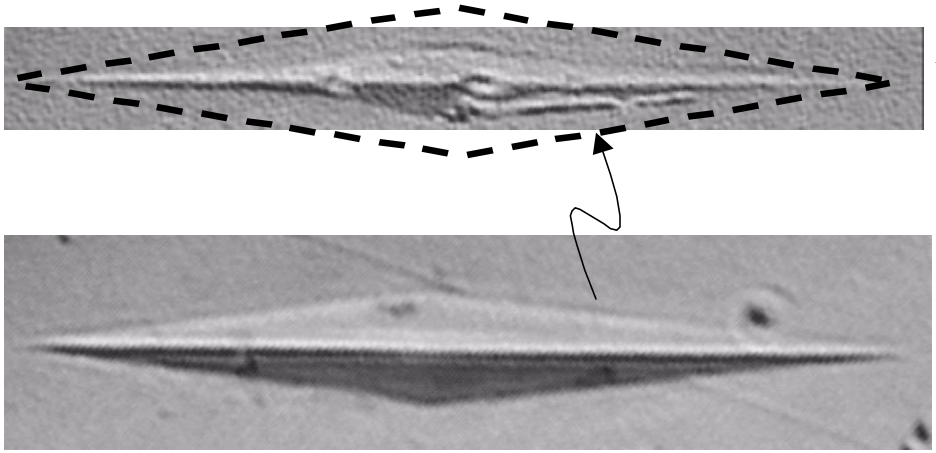
Marshall, Noma and Evans (1980)

$$\frac{b'}{d'} = \frac{b}{d} - \alpha \frac{H}{E}$$

d'≈ d

Dimensions of the indenter = 1/7.11

Geometry correction factor = 0.45



Ceramics: H/E high.

b'<>b large elastic recovery

Metals: H/E low.

b'≈ b little elastic recovery

D.B. Marshall, T. Noma, and A.G. Evans, "A Simple method for determining elastic-modulus-to-hardness ratios using Knoop indentation measurements", J.Amer.Ceram.Soc. **65**, C175 (1980).

The “Oliver and Pharr” method

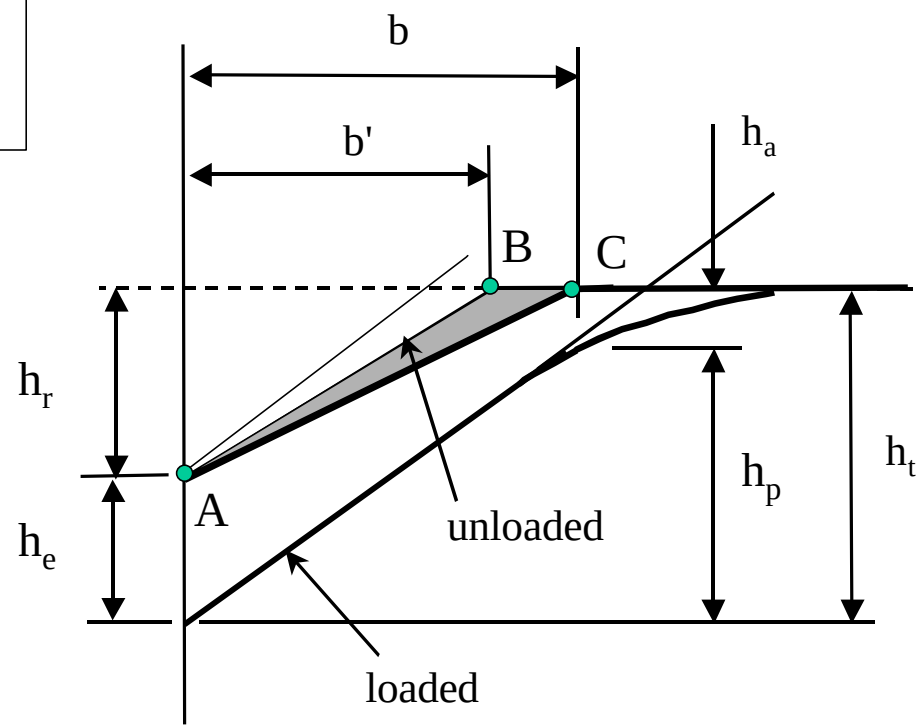
Knoop indenter analysis #2

For a Knoop indenter, we would expect that for a given indenter load, the depth of penetration would be less than that for an equivalent conical indenter.

WHY?

Because the indenter is now supported by material ABC which ordinarily would not be there.

The same effect could be had if we had no elastic recovery but a slightly larger value of θ_2 for the indenter.



The "Oliver and Pharr" method

Knoop indenter analysis #2

Assuming no elastic recovery along the long axis, then d=d' and so:

$$\frac{b}{b'} = 1 - \alpha \frac{d}{b}\frac{H}{E}$$

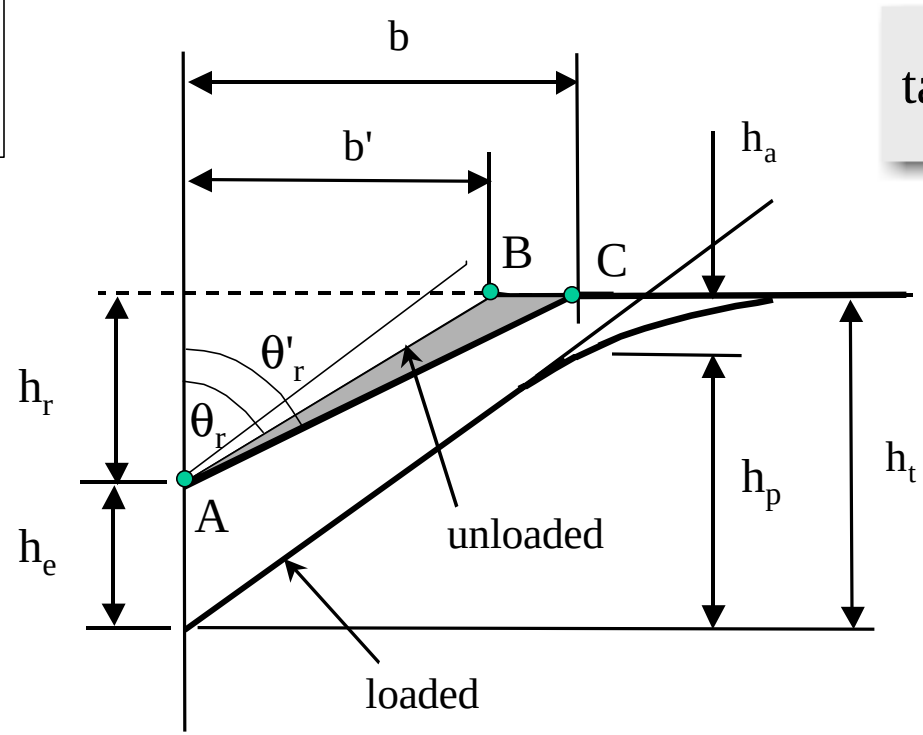
$$= 1 - (0.45)(7.11)\frac{H}{E}$$

From the geometry of the indentation, we can see that:

$$\frac{\tan\theta'_r}{\tan\theta_r} = \frac{b'}{b}$$

If we agree that the new angle θ'_2 for the Knoop indenter should be adjusted in the same proportion as the change in the angle of the residual impression, then:

$$\tan\theta'_2 = \frac{b'}{b}\tan\theta_2$$



Effective θ_2 angle for Knoop indenter

The "Oliver and Pharr" method

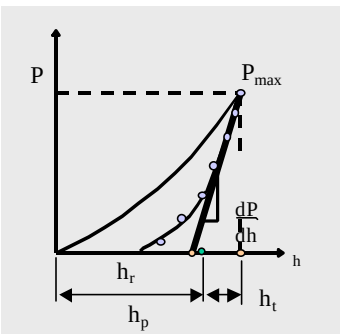
Knoop indenter analysis #2 – the results

Modulus

$$E^* = \frac{dP}{dh}\frac{\sqrt{\pi}}{2\sqrt{A}}$$

with

$$A = 2h_p^{\,2}\left[\tan\theta_1 \tan\theta'_2\right]$$



$$h_p = h_t - [0.72]\frac{P_{max}}{dP/dh}$$

Or 0.75

IT WORKED!

Material	E	Book value
Fused silica	67.8 GPa	70 GPa
Steel	213 GPa	210 GPa
Alumina	394GPa	380 GPa

OK, what about hardness H?

The "Oliver and Pharr" method

Hardness: An interesting problem…

IT WORKED – sort of…

Material	H GPa	Book value	Optical
Fused silica	5.5	7.6	6.5
Steel	8.5	9	7.4
Alumina	16	18	18.7

Hardness

$$H = \frac{P}{A}$$

Hardness is "**mean contact pressure**" but usually assumes a fully-developed plastic zone.

Now, the Oliver and Pharr method uses the elastic equations for the unloading of a "cone".

$$p_m = \frac{E^*}{2}\cot\alpha$$

The equivalent cone angle for a Berkovich indenter is 70.3°. For a Knoop indenter, it is 77.6°. This larger angle, together with the value for E for fused silica, results in a mean contact pressure (elastic contact) which is always less than the hardness of the material!

Analysis for a Knoop indenter

Summary

1. We have used the Oliver and Pharr method to determine hardness and modulus.

$$H = \frac{P}{A} \qquad E^* = \frac{dP}{dh}\frac{\sqrt{\pi}}{2\sqrt{A}}$$

$$A = 2h_p^{\,2}\left[\tan\theta_1 \tan\theta'_2\right]$$

$$h_p = h_t - [0.72]\frac{P_{max}}{dP/dh}$$

2. We have adjusted the angle of the Knoop indenter to account for elastic recovery along the short axis diagonal.

$$\tan\theta'_2 = \frac{b'}{b}\tan\theta_2$$

Need an initial estimate of the ratio E/H to start.

$$\frac{b}{b'} = 1 - (0.45)(7.11)\frac{H}{E}$$

3. Method appears to work OK for wide range of H/E. Need to be careful in calculating Hardness due to blunt effective cone angle for Knoop indenter.