Robust Control of Realistic Quantum Gates

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Abstract: We consider a generic NOT gate in contact with the environment and design and implement a control scheme to eliminate decoherence. Simulations show efficiency and robustness under a broad range of realistic conditions.

Background. Quantum computing is one of the most promising areas to have emerged in science in the last decade [1 and references therein]. If realizable, quantum computers would solve large quantum system simulations and (specific classes of) hard computational problems much more efficiently than classical (digital) universal computers. Quantum computation relies on the existence, persistence, and special features of *entangled states* [1], which are the exclusively quantum and highly counterintuitive result of the linear superposition principle in quantum mechanics. According to this principle, electrons, photons, neutrons, and even atoms and molecules, may be in more than one state at the same time. Formally, entanglement means that, in general, the wave function of a many particle system may not be factored into a product of single particle wave functions. As a result, (i) the whole system shares quantum correlations that cannot be described or prescribed in a classical probabilistic framework and (ii) these quantum correlations may – in principle – persist at large distances, with strange and powerful epistemological and practical consequences, amongst which teleportation and quantum computation stand out. The main obstacle that prevents the observation and exploitation of entanglement at the macroscopic scale and the realization of a practical size, room-temperature quantum computer is the difficulty to generate and especially maintain *quantum correlations* for a large number of entangled particles. Indeed, due to uncontrolled and/or unaccounted for interactions with the environment, entangled pairs lose their entanglement (i.e. the information about the quantum phases) and become a (random) statistical population. This effect, called *decoherence* [1, 2], is the major stumbling block on the road to achieving (sufficiently) fault tolerant quantum gates and eventually building a useful, scalable quantum computer. Several control schemes have been proposed to eliminate or reduce decoherence, but usually they are either theoretical proposals or very carefully monitored implementations in a few atoms, at extreme conditions, and for very short times. Optimal control [3] appeals through its general and systematic approach, but is both impractical, since calculations cannot be realized on line, and unnecessary, since for the envisaged applications, ability is important, rather than cost. Passive stabilization relies on decoherence free subspaces [4], which – due to special symmetry properties – are dynamically decoupled from the environment; however these subspaces are difficult to realize experimentally. Finally, a quantum (as opposed to the obviously impossible classical) feedback scheme has been recently proposed, but only for closed systems [5]. Here we report the design and implementation in a realistically simulated NOT gate of a control approach that has common features with the open-loop and bang-bang schemes [6].

Modeling the NOT Gate. Since a universal quantum computer can be assembled from a sufficiently large number of either NOT and CNOT gates or Toffoli gates [1], it seems reasonable to start by fighting decoherence at the gate level first and, if successful, try to scale up the procedure. NOT gates have been realized in various two-level systems [1], although implementations in three- or multi-level systems can be envisaged and sometimes turn out to be desirable [3]. Here we consider the NOT gate as a generic two-level system in interaction with its environment (bath), described by a bosonic gas [7,8]. The complete Hamiltonian is $H = H_S + H_B + H_I$, where H_S is a 2×2 matrix describing the transition between the two levels of the NOT gate, H_B is the Hamiltonian of the bath, described by an infinite sum over the free bosonic modes, and H_I is the interaction Hamiltonian. Here we take $H_I = as_x + bs_z$; for this model, the inclusion of s_y in the interaction Hamiltonian is not necessary, since it yields essentially the same result as s_x. The evolution of the whole system under the complete Hamiltonian H is, of course, unitary, but following the evolution of the two-level system (NOT gate) alone means that the environment is "traced out" (i.e. ignored). This operation is accompanied by a certain loss of information about the two-level system [7,8] and manifests itself as adiabatic and/or dissipation-induced decoherence. These two processes are produced by different interactions (which correspond to setting $a = 0, b = 0$, respectively) and take place at different time scales. Typically, the adiabatic decoherence sets in very quickly, modifies only the *non-diagonal* elements of the density matrix, and leads to loss of information about the relative phases; the dissipation-induced decoherence sets in at a much slower rate, is accompanied by loss of energy (particles), and results in an alteration of both moduli and phases of *all* the elements of the density matrix. In either case, the unitary character of the quantum evolution is lost and has to be restored, since any quantum computation is nothing but a sequence of unitary transformations. Moreover, this restoration has to be achieved as robustly, efficiently, and economically as possible.

Open Loop Control of the NOT Gate. In this proof-of-principle implementation, we restrict ourselves to: (i) two limit cases, namely $a = 0, b = 0$; (ii) first order perturbation theory; and (iii) a simple form of the dispersion relation for the bath (the so-called Wigner-Weisskopf approximation). In the absence of interactions, the evolutions of the bath and two-level system are decoupled and unitary. In the presence of interaction, the evolutions of the two parts are coupled and by tracing out the bath, the "reduced" evolution of the two-level system becomes non-unitary. In principle, knowing the interaction, the departure from unitarity (i.e. the decoherence constants) can be calculated as follows. From the Schrödinger equation one derives an evolution equation for the complete evolution operator, $U(t) = \exp(itH)$. By passing to the interaction picture and tracing out the environment, one obtains the evolution equation for the "reduced" evolution operator of the two-level system, $U_s^{\sim}(t)$. Of course, the solution of this problem can be found only approximately, within various orders of perturbation theory. Once $U_s^{\sim}(t)$ is determined to the desired order, one can calculate the corresponding evolutions for the density matrix and/or modal amplitudes. The evolution of the modal amplitudes ruled by $U_s^{\sim}(t)$ is different from the unitary evolution, $U_S(t)$, one would obtain in the absence of interaction. To the first order of perturbation theory, the two evolutions can be written as two systems of two first order differential equations for the complex modal amplitudes: $dw_i / dt = i(V_0 + V_1)w_i - g_i w_i$, i = 1,2, for the open gate and $dc_i / dt = i V_0 c_i$, i = 1,2, for the isolated gate. By requiring the equality of the two evolutions, we find the explicit expression of the additional interaction (potential), V_1, that has to be applied on the system in order to restore unitarity. Formally, this potential is applied as an additional fourth term in the complete Hamiltonian. Practically, the control is calculated in turn to each modal amplitude and applied in the form of small pulses whose rate and strength depends on the decoherence constant. By its very nature, this term is not affecting the bath, and therefore does not produce an unwanted feedback into the control mechanism. We have applied this control to a NOT gate subjected to various types and degrees of decoherence. The decoherence constants calculated from the reduced evolutions as well as the controls' magnitude and repetition rate are consistent with the experimental values [1,2,7,8 and references therein]. Simulation results show the efficiency of the control for a broad range of conditions as well as its robustness with respect to unavoidable imperfections in the control set-up.

Discussion. The following comments are in order. *1*. The method we propose does not rely on the explicit (albeit approximate) Hamiltonian description of the quantum system of interest. Knowledge of decoherence constants is sufficient to design and implement the control algorithm. *2*. Many of the simplifying assumptions adopted here may be relaxed without fundamental modifications, at the expense though of lengthier calculations and a less transparent approach. *3*. We strongly believe that preventing decoherence by controlling it "*in situ*, as it happens" has – in the long run – a better chance of success than error correcting codes [7] that are marred by huge overheads and may themselves have to be corrected for errors. *4*. Our algorithm relies on the assumption that the bath and the interaction with the two-level system are stationary. However, since during the evolution the system's parameters may change in an unknown way, the open loop control is generally unpredictable. This is a problem though only if changes occur at time scales shorter than the computation time. Moreover, one can periodically probe the bath and adjust the control to account for possible changes. In principle, this could be achieved without interfering with the system. *5*. For quantum computing, elimination of decoherence is necessary, but not sufficient; concomitantly, one has to steer the dynamics of the gate to perform the desired calculation. Our control scheme can accommodate this requirement. *6*. Finally, with minimal, obvious modifications, this control scheme can be applied to multi-level NOT gates and four-level or multi-level CNOT gates.

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