Eliminating frequency and space-time entanglement in multi-photon states

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Abstract: Visibility in interference experiments involving photon pairs from separate down-conversion events is reduced when these pairs are frequency-entangled. We present a method for eliminating the entanglement at the source, making pre-detection spectral filters unnecessary.

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Experimentalists in the field of quantum optics have performed numerous interesting experiments utilizing pairs of photons generated in the process of spontaneous parametric down-conversion. Although it was the two-photon state that was of interest in most of these experiments, a number of recent proposals require states of greater than two photons. In the few experiments that have been performed to date, these multi-photon states have been "constructed" from pairs of photons emitted in separate down-conversion events. Generating multi-photon states in this way exploits many of the properties of the constituent pairs and it simplifies matters experimentally because it reduces the number of sources required.

However, this method does present a problem: since energy must be conserved in the down-conversion process, the emitted pairs are entangled in frequency. This entanglement makes it possible to identify members of the multi-photon state as siblings, thus providing information that may render interfering pathways distinguishable. Two recent experiments[1,2], for example, relied on Bell state measurements in which it was impossible to determine the origins of the detected photons. This was accomplished by placing narrow spectral filters in front of the detectors to minimize the effect of the entanglement. However, this method of removing the distinguishing spectral information results in a greatly diminished count rate. We propose an alternate solution to this problem—one that does not require pre-detection filtering. We show that by adjusting parameters such as crystal length, crystal material, pump bandwidth, and center pump wavelength, the frequency entanglement is eliminated at the down-conversion source. With no need for spectral filters, counting rates would improve significantly.

The two-photon state produced in the process of spontaneous parametric down-conversion is $|\psi\rangle = \left(\frac{1}{2\pi}\right)^2 \iint d\omega_s d\omega_i\, f(\omega_s,\omega_i)\hat{a}_s^\dagger(\omega_s)\hat{a}_i^\dagger(\omega_i)|vac\rangle$. In general, this state is entangled in frequency, although if the probability amplitude can be written as $f(\omega_s,\omega_i) = f_s(\omega_s) f_i(\omega_i)$, then the state becomes $|\psi\rangle = \left(\frac{1}{2\pi}\right)^2 \int d\omega_s f_s(\omega_s)\hat{a}_s^\dagger(\omega_s)\int d\omega_i f_i(\omega_i)\hat{a}_i^\dagger(\omega_i)|vac\rangle$, which is a direct product state. That is, the degree of frequency entanglement is zero if the probability amplitude is factorable. In addition, the degree of *space-time* entanglement is zero when this condition is satisfied. This follows directly from the relationship between the spectral and temporal representations of the two-photon state. In the temporal domain, the state is $|\psi\rangle = \iint dt_s dt_i g(t_s,t_i)\hat{a}_s^\dagger(t_s)\hat{a}_i^\dagger(t_i)|vac\rangle$, where $\hat{a}_{s,i}(\omega)$ and $f(\omega_s,\omega_i)$ are the Fourier and double Fourier transforms of $\hat{a}_{s,i}(t)$ and $g(t_s,t_i)$, respectively. Thus, if there exist functions $f_s(\omega_s)$ and $f_i(\omega_i)$ such that $f(\omega_s,\omega_i) = f_s(\omega_s) f_i(\omega_i)$, then there also exist $g_s(t_s)$ and $g_i(t_i)$ such that $g(t_s,t_i) = g_s(t_s) g_i(t_i)$.

In order to determine how this factorability condition is related to experimental parameters, we examine the two-photon state in more detail. The general expression for the two-photon amplitude is $f(\omega_s,\omega_i)=\alpha(\omega_s+\omega_i)\phi(\omega_s,\omega_i)$, where $\alpha(\omega_s+\omega_i)$ is a function describing the envelope of the pump field and $\phi(\omega_s,\omega_i)$ is a phase-matching function[3]. A plot of $|f(\omega_s,\omega_i)|^2$ is shown in Fig. 1(a) for type-II down-conversion in Beta Barium Borate (BBO) with a 400 nm pump. It is evident in this plot that $f(\omega_s,\omega_i)$ is not factorable, since the range of idler frequencies depends somewhat upon the signal frequency.

In general, it is not possible to express the two-photon amplitude as a product of functions of the respective frequencies. However, if $\phi(\omega_s,\omega_i)$ and $\alpha(\omega_s+\omega_i)$ are both approximated by Gaussians, and if only the lowest order terms in power series expansions of the wave numbers are retained, then $f(\omega_s,\omega_i)$ is factorable (and the state is not frequency-entangled) as long as

$$\frac{1}{\sigma^2}+\gamma L^2\left(k_p'-k_s'\right)\left(k_p'-k_i'\right)=0. \qquad (1)$$

In this expression, σ is the $1/e$ width of $\alpha(\omega_s+\omega_i)$, L is the crystal length, $\gamma=0.04822$, and the primes indicate derivatives with respect to frequency.

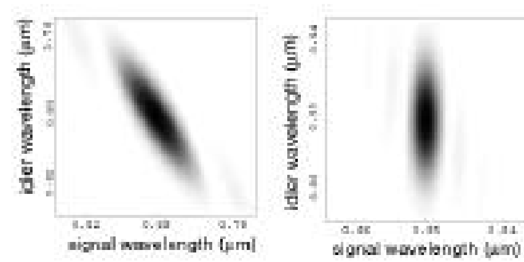

Fig. 1. The square modulus of the two-photon probability amplitude in type-II (a) BBO pumped at 400 nm; and (b) ADP pumped at 425 nm. As an aid to the reader, the axes have been converted from frequency to wavelength.

Through the parameters σ and L, the expression in Eq. (1) imposes a constraint on the relative widths of the pump envelope and phase-matching functions. The equation can not be satisfied at all, though, unless $\left(k_p'-k_s'\right)\left(k_p'-k_i'\right)<0$. Since the derivatives are the reciprocals of the respective group velocities, this inequality will be satisfied only if the group velocity of the pump lies between the group velocities of the signal and idler fields. This group velocity mismatch can be realized in the degenerate case only in a type-II configuration and does not occur in any of the recently reported experimental schemes. However, it can be achieved in Ammonium Dihydrogen Phosphate (ADP) as long as the down-converted fields lie within the range 0.82 µm < λ < 1.64 µm. The plot in Fig. 1(b) shows $|f(\omega_s,\omega_i)|^2$ for type-II ADP with the pump centered at 425 nm and with the pump bandwidth and crystal length chosen so as to satisfy Eq. (1). Note that the range of idler frequencies is independent of the signal frequency.

When experimental constraints (*e.g.*, available pump wavelengths) make it impossible to realize the group velocity mismatch, the two-photon state will retain some frequency-entanglement. It is possible, nonetheless, to reduce the *degree* of entanglement. That is, the degree of entanglement may be minimized with respect to the remaining parameters—crystal length and pump bandwidth. Of course, the minimum degree of entanglement attainable via this method is lower for some crystal materials than for others. This research was supported by the Engineering Research Program of the US Department of Energy, Office of Basic Energy Sciences under contract No. DE-AC05-00OR22725 with UT-Battelle, LLC.

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