Nonlinear Coupling Control Laws for an Overhead Crane System*

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Abstract

In this paper, we consider the regulation control problem for a two-degree-of-freedom (2-DOF), underactuated overhead crane system. Inspired by recently designed passivity-based controllers for underactuated systems, we design several controllers that asymptotically regulate the gantry position and payload position. Specifically, utilizing LaSalle's Invariance Set Theorem, we first illustrate how a simple proportional-derivative (PD) controller can be utilized to asymptotically regulate the overhead crane system. Motivated by the desire to achieve improved transient performance, we then design two nonlinear controllers that increase the coupling between the gantry position and the payload position.

1 Introduction

The operation of overhead crane systems in many industrial settings is achieved by relying on the skill of experienced crane operators. Unfortunately, precise payload positioning (*i.e.*, the operator using only visual feedback to position the payload) is difficult due to the fact that the payload is free to swing in a pendulum-like motion. Furthermore, the payload swings can result in several performance and safety concerns including: i) damage to the payload (*e.g.*, spillage or breakage), ii) damage to the surrounding environment or personnel, and iii) large internal forces that can result in reduced payload carrying capacity or premature failure of stressed parts. Motivated by the practical desire to achieve fast and reliable response with reduced cost and high precision positioning, several researchers have developed controllers for the overhead crane systems.

Most of the early research for overhead crane systems targeted simplified crane models. Specifically, in [10], a control design was developed for a reduced-order model of the crane system that incorporated the actuator dynamics. In [21], Yu *et al.* utilized singular perturbation techniques to design a nonlinear feedback controller which consists of a slow portion to make the payload follow a desired trajectory with a fast portion that was superimposed to eliminate oscillations and sway; however, an approximate linearized model of the system was utilized to facilitate the construction of the error systems. In [11], Noakes *et al.* discussed some hardware and software modifications that enable existing overhead crane systems to be utilized for developing swing-free algorithms. In [13] and [14], Sakawa *et al.* developed control algorithms to transfer the payload to a desired position with minimal payload swing for rotary crane systems. In [20], Yashida *et al.* proposed a saturating control law based on a guaranteed cost control method, however, the proposed controller was designed for a simplified linearized version of the system dynamics. An approximate crane model was also utilized by Martindale *et al.* in [9] to develop exact model knowledge and adaptive controllers. An adaptive controller was also designed in [3] using a modal decomposition technique. In [4], Chung and Hauser designed a nonlinear controller for regulating the swinging energy of the payload. For a survey of other control designs that are based on linearized models of overhead crane systems, including research that eliminates the rigid cable assumption, see [2], [12], and the references within.

One of the limiting factors in most of the above control designs is that the system nonlinearities were excluded from the closed-loop error system and control development. To overcome this drawback, some control researchers have targeted the development of controllers that account for the nonlinear dynamics of the crane system and similar underactuated control problems (*e.g.*, inverted pendulum, ball and beam, *etc.*). Specifically, in [16] and [18], Teel employed saturation functions to achieve a global asymptotic (and local exponential) stability result. In [17], Teel also utilized saturation functions to develop an output feedback controller which achieves a robust, semi-global stability result for the ball-and-beam control problem. In [2], Burg *et al.* adopted the ball-and-beam solution given in [16] and [18] to address the overhead crane problem. That is, Burg *et al.* transformed the nonlinear crane dynamics into a set of new dynamic equations resembling the ball-and-beam problem and then employed a controller with saturation functions to guarantee asymptotic positioning from a large set of initial conditions.

Recently, some researchers have utilized a passivity-based approach to address the control of underactuated systems such as the crane. Specifically, [8] and [6] proposed passivity-based controllers for the inverted pendulum and the pendubot (*i.e.*, an inverted pendulum-like robot with an unactuated second link). These exciting new results are based on the paradigm of driving the underactuated system to a homoclinic orbit using an energy-based nonlinear controller and then switching to a linear controller to stabilize the system around its unstable equilibrium point. Using similar stability analysis techniques, Collado *et al.* proposed a proportional-derivative (PD)

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controller for the overhead crane problem. In [7], Kiss *et al.* developed a PD controller for a vertical crane winch system that only requires the measurement of the winch angle and its derivative rather than a cable angle measurement.

In this paper, we first utilize similar stability analysis techniques as those given in [8] to prove that a PD controller can asymptotically regulate the nonlinear crane dynamics. Motivated by the desire to achieve increased transient performance, we then develop two nonlinear controllers. The first controller follows a similar design as that outlined in [8]. Specifically, additional nonlinear terms are injected into the controller by squaring the energy in the Lyapunov function and adding an additional gantry squared velocity term in the Lyapunov function. The second controller is constructed by utilizing a standard energy term in the Lyapunov function in addition to a gantry kinetic energy-like term. When compared to the standard PD controller, both of these controllers contain additional nonlinear terms which tend to increase the coupling between the gantry position and the payload position. The increased coupling between the gantry position and the payload position is a desirable characteristic that provides for improved transient response (*e.g.*, reduced overshoot and faster settling time).

The paper is organized as follows. In Section 2, we present the dynamic model of the overhead crane system, and in Section 3, we rewrite the open-loop system in a more convenient form. In Section 4, we develop the PD controller and the two nonlinear controllers and examine the stability of the controllers through a Lyapunov-like stability analysis. Concluding remarks are given in Section 5.

2 Dynamic Model

The dynamic model for a 2-DOF overhead crane system (see Figure 1) is assumed to have the following form [5]

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) = u \tag{1}$$

where $q(t) \in \mathbb{R}^2$ is defined as follows

$$q = [x(t) \quad \theta(t)]^T \tag{2}$$

where $x(t) \in \mathbb{R}^1$ denotes the gantry position, $\theta(t) \in \mathbb{R}^1$ denotes the payload angle with respect to the vertical, and $M(q) \in \mathbb{R}^{2\times 2}$, $V_m(q,\dot{q}) \in \mathbb{R}^{2\times 2}$, $G(q) \in \mathbb{R}^2$, and $u(t) \in \mathbb{R}^2$ are defined as follows

$$\begin{gathered} M(q) = \begin{bmatrix} m_c + m_p & -m_p L\cos\theta \\ -m_p L\cos\theta & m_p L^2 \end{bmatrix}, \\ V_m(q,\dot{q}) = \begin{bmatrix} 0 & m_p L \sin\theta\dot{\theta} \\ 0 & 0 \end{bmatrix}, \\ G(q) = \begin{bmatrix} 0 & m_p g L\sin\theta \end{bmatrix}^T, \quad u(t) = \begin{bmatrix} F & 0 \end{bmatrix}^T, \end{gathered} \tag{3}$$

where $m_c, m_p \in \mathbb{R}^1$ represent the gantry mass and the payload mass, respectively, $L \in \mathbb{R}^1$ represents the length of the rod to the payload, $g \in \mathbb{R}^1$ represents the gravity coefficient, and $F(t) \in \mathbb{R}^1$ represents the control force input acting on the gantry (see Figure 1). Based on the structure of $M(q)$ and $V_m(q,\dot{q})$ given in (3), it is straightforward to show that the following skew-symmetric relationship holds

$$\xi^T\left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \qquad \forall\, \xi \in \mathbb{R}^2 \tag{4}$$

where $\dot{M}(q)$ represents the time derivative of $M(q)$. In a similar manner as [2] and [9], we assume that the dynamic model given in (1) and (3) have the following characteristics.

Assumption 1: The payload and the gantry are connected by a mass-less, rigid link.

Assumption 2: The angular position and velocity of the payload and the rectilinear position and velocity of the gantry are measurable.

Assumption 3: The payload mass is concentrated at a point and the value of this mass is exactly known; moreover, the gantry mass and the length of the connecting rod are exactly known.

Assumption 4: The hinged joint that connects the payload link to the gantry is frictionless.

Assumption 5: The angular position of the payload mass is restricted according to following inequality

$$-\pi < \theta(t) < \pi \tag{5}$$

where $\theta(t)$ is measured from the vertical position (see Figure 1).

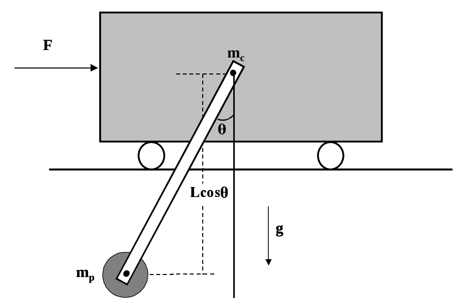

Figure 1: Schematic for the Overhead Crane System

Remark 1 *Note that the model given by (1) could be modified to include other dynamic effects associated with the gantry dynamics (e.g., gantry friction); however, these additional dynamic effects were not included in the model since these effects can be directly cancelled by the controller.*

3 Open-Loop System Development

To express (1) in a form that facilitates the subsequent control development and stability analysis, we premultiply both sides of (1) by $M^{-1}(q)$ to obtain the following expression

$$\ddot{q} = M^{-1}\left(u - V_m\dot{q} - G\right) \tag{6}$$

where $M^{-1}(q) \in \mathbb{R}^{2\times 2}$ is defined as follows

$$M^{-1} = \frac{1}{\det(M)}\begin{bmatrix} m_p L^2 & m_p L\cos\theta \\ m_p L\cos\theta & m_c + m_p \end{bmatrix} \tag{7}$$

and $\det(M)$ denotes the determinant of $M(q)$ which is explicitly defined as follows

$$\det(M) = \left(m_c + m_p\sin^2\theta\right) m_p L^2. \tag{8}$$

After substituting (3) and (7) into (6) and preforming some algebraic manipulation, we obtain the following dynamics for each degree of freedom

$$\ddot{x} = \frac{F}{m(\theta)} - \frac{\varsigma(\theta,\dot{\theta})}{m(\theta)} \tag{9}$$

$$\ddot{\theta} = \frac{1}{Lm(\theta)}\left(\left(F - m_p L \sin\theta\dot{\theta}^2\right)\cos\theta - (m_c + m_p)g\sin\theta\right) \tag{10}$$

where the auxiliary terms $m(\theta)$, $\varsigma(\theta,\dot{\theta}) \in \mathbb{R}^1$ are defined as follows

$$m(\theta) = m_c + m_p \sin^2\theta > 0 \tag{11}$$

$$\varsigma(\theta,\dot{\theta}) = m_p \sin\theta\left(L\dot{\theta}^2 + g\cos\theta\right). \tag{12}$$

After utilizing (9) and performing some additional algebraic manipulation, the dynamics for $\theta(t)$ given in (10) can also be written in the following convenient form

$$\ddot{\theta} = \frac{1}{L}\cos\theta\ddot{x} - \frac{g}{L}\sin\theta. \tag{13}$$

To facilitate the subsequent Lyapunov-based control design, we develop an expression for the energy of the overhead system, denoted by $E(q,\dot{q}) \in \mathbb{R}^1$, as given below

$$E(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + m_p g L(1-\cos\theta) \geq 0. \tag{14}$$

After taking the time derivative of (14), substituting (1) for $M(q)\ddot{q}(t)$, and canceling common terms, we obtain the following expression for the time derivative of the energy

$$\dot{E} = \dot{x}F \tag{15}$$

where (3) and (4) were utilized.

4 Control Development

Our control objective is to regulate the gantry position of the overhead crane to a constant desired position denoted by $x_d \in \mathbb{R}^1$ while also regulating the payload angle to zero. To quantify the objective of regulating the gantry position to a constant desired position, we define a gantry position error signal $e(t) \in \mathbb{R}^1$ as follows

$$e(t) = x - x_d. \tag{16}$$

In the subsequent control development, we will design a proportional-derivative control law and two nonlinear controllers to achieve the stated control objective.

4.1 Proportional-Derivative Control Law

Based on the subsequent stability analysis, we design the following proportional-derivative (PD) control law

$$F = \frac{-k_d\dot{x} - k_p e}{k_E} \tag{17}$$

where k_d, k_E, $k_p \in \mathbb{R}^1$ are positive constant control gains.

Theorem 1 *The controller given in (17) ensures asymptotic regulation of the overhead crane system in the sense that*

$$\lim_{t\to\infty}\begin{pmatrix} x(t) & \dot{x}(t) & \theta(t) & \dot{\theta}(t)\end{pmatrix} = \begin{pmatrix} x_d & 0 & 0 & 0\end{pmatrix} \tag{18}$$

where x_d represents the constant desired gantry position.

Proof: To prove Theorem 1, we define the following non-negative function

$$V_1 = k_E E + \frac{1}{2}k_p e^2 \tag{19}$$

where $E(q,\dot{q})$ and $e(t)$ were defined in (14) and (16), respectively. After taking the time derivative of (19) and substituting (15) and the time derivative of (16) into the resulting expression for $\dot{E}(q,\dot{q})$ and $\dot{e}(t)$, respectively, we obtain the following expression

$$\dot{V}_1 = \dot{x}\left(k_E F + k_p e\right). \tag{20}$$

After substituting (17) into (20) for $F(t)$, we obtain the following expression

$$\dot{V}_1 = -k_d\dot{x}^2 \tag{21}$$

which implies that the origin of the closed-loop system is stable in the sense of Lyapunov [15] and that $x(t)$, $\dot{x}(t)$, $\dot{\theta}(t)$, $F(t) \in \mathcal{L}_\infty$.

In a similar manner as [8], we will employ LaSalle's invariance theorem to prove (18). To this end, we define Γ as the set of all points where

$$\dot{V}_1 = 0. \tag{22}$$

In the set Γ, it is clear from (21) and (22) that

$$\dot{x}(t) = 0. \tag{23}$$

From (22) and (23), we can conclude that $x(t)$ and $V_1(t)$ are constant, and that

$$\ddot{x}(t) = 0. \tag{24}$$

Furthermore, from (15), (16), and (23), it is clear that

$$\dot{E}(q,\dot{q}) = \dot{e}(t) = 0. \tag{25}$$

Based on (25), it is clear that $E(q,\dot{q})$ and $e(t)$ are constant and from (17), (22), and (23), we can also prove that $F(t)$ is constant. To complete the proof, we use similar arguments as those given in [8] to examine the cases when $\dot{\theta}(t) = 0$ and when $\dot{\theta}(t) \neq 0$.

Case 1. $\dot{\theta}(t) = 0$

Based on the assumption that $\dot{\theta}(t) = 0$, we can conclude that

$$\ddot{\theta}(t) = 0. \tag{26}$$

Given (23) through (26), it is straightforward from (13) to see that

$$\varsigma(\theta,\dot{\theta}) = 0 \quad \text{and} \quad \sin(\theta) = 0 \tag{27}$$

where $\varsigma(\theta,\dot{\theta})$ was defined in (12), and hence, from Assumption 5, it is also clear that

$$\theta(t) = 0. \tag{28}$$

From (23) through (26) and the assumption that $\dot{\theta}(t) = 0$, we can utilize (1) and (3) to show that

$$F(t) = 0. \tag{29}$$

Given (23), (28), and (29), we can utilize (16) and (17) to prove (18).

Case 2. $\dot{\theta}(t) \neq 0$

In this case, we prove that the assumption $\dot{\theta}(t) \neq 0$ leads to a contradiction. To this end, we multiply (9) by $m(\theta)$, take its time derivative, then substitute the time derivative of (12) for $\varsigma(\theta, \dot{\theta}, \ddot{\theta})$, and substitute (13) into the resulting expression for $\ddot{\theta}(t)$ to obtain the following expression

$$\begin{aligned} \dot{F} &= \dot{m}(\theta)\ddot{x} + m(\theta)x^{(3)} + m_p\dot{\theta}\left(L\dot{\theta}^2\cos\theta + g\cos^2\theta\right) \\ &\quad + m_p\dot{\theta}\left(2\sin\theta\left(\cos\theta\ddot{x} - g\sin\theta\right) - g\sin^2\theta\right) \end{aligned} \tag{30}$$

where the notation $(\cdot)^{(i)}$ represents the $i-th$ time derivative of a signal. After dividing (30) by $m_p\dot{\theta}(t)$ and utilizing the fact that

$$\cos^2\theta = 1 - \sin^2\theta, \tag{31}$$

we obtain the following expression

$$L\cos\theta\dot{\theta}^2 + g - 4g\sin^2(\theta) = S_1 \tag{32}$$

where the auxiliary signal $S_1(t) \in \mathbb{R}^1$ is defined as

$$S_1 = \frac{\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}}{m_p\dot{\theta}} - 2\sin\theta\cos\theta\ddot{x}. \tag{33}$$

To continue the analysis, we take the time derivative of (32), substitute (13) into the resulting expression for $\ddot{\theta}(t)$, and divide by $-\dot{\theta}(t)$ to obtain the following expression

$$L\sin\theta\dot{\theta}^2 + 10g\sin\theta\cos\theta = -\frac{\dot{S}_1}{\dot{\theta}} + 2\cos^2\theta\ddot{x} \tag{34}$$

where $\dot{S}_1(t)$ is given by the following expression

$$\begin{aligned} \dot{S}_1 &= \frac{\ddot{F} - \ddot{m}(\theta)\ddot{x} - 2\dot{m}(\theta)x^{(3)} - m(\theta)x^{(4)}}{m_p\dot{\theta}} \\ &\quad - \frac{\ddot{\theta}\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^2} \\ &\quad - 2\sin\theta\cos\theta x^{(3)} + 2\dot{\theta}\ddot{x}\left(\sin^2\theta - \cos^2\theta\right). \end{aligned} \tag{35}$$

After taking the time derivative of (34), dividing by $\dot{\theta}(t)$, substituting (13) into the resulting expression for the occurrence of $\ddot{\theta}(t)$ in the left side, and then utilizing (31), we obtain the following expression

$$L\cos\theta\dot{\theta}^2 + 22g\cos^2\theta - 12g = \frac{S_2}{\dot{\theta}} - 2\sin\theta\cos\theta\ddot{x} \tag{36}$$

where the auxiliary signal $S_2(t) \in \mathbb{R}^1$ is defined as follows

$$S_2 = \frac{-\left(\ddot{S}_1\dot{\theta} - \dot{S}_1\ddot{\theta}\right)}{\dot{\theta}^2} + 2\cos^2\theta x^{(3)} - 4\sin\theta\cos\theta\ddot{x}\dot{\theta} \tag{37}$$

and $\ddot{S}_1(t)$ is given by the expression below

$$\begin{aligned} \ddot{S}_1 &= \frac{F^{(3)} - m^{(3)}(\theta)\ddot{x} - 3\ddot{m}(\theta)x^{(3)} - 3\dot{m}(\theta)x^{(4)}}{m_p\dot{\theta}} \\ &\quad - \frac{m(\theta)x^{(5)}}{m_p\dot{\theta}} - 2\frac{\ddot{\theta}\left(\ddot{F} - \ddot{m}(\theta)\ddot{x} - 2\dot{m}(\theta)x^{(3)}\right)}{m_p\dot{\theta}^2} \\ &\quad + 2\frac{\ddot{\theta}m(\theta)x^{(4)}}{m_p\dot{\theta}^2} - 2\sin\theta\cos\theta x^{(4)} \\ &\quad - \frac{\theta^{(3)}\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^2} \\ &\quad + \frac{2\ddot{\theta}^2\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^3} \\ &\quad + \left(2x^{(3)}\dot{\theta} + 2\ddot{\theta}\ddot{x} + 2\dot{\theta}x^{(3)}\right)\left(\sin^2\theta - \cos^2\theta\right) \\ &\quad + 8\dot{\theta}^2\ddot{x}\sin\theta\cos\theta. \end{aligned} \tag{38}$$

Finally, we take the time derivative of (36), divide it by $\dot{\theta}(t)$, and then substitute (13) into the resulting expression for the occurrence of $\ddot{\theta}(t)$ in the left side to obtain the following expression

$$L\sin\theta\dot{\theta}^2 + 46g\sin\theta\cos\theta = -\frac{S_3}{\dot{\theta}} + 2\cos^2\theta\ddot{x} \tag{39}$$

where the auxiliary signal $S_3(t) \in \mathbb{R}^1$ is defined as follows

$$S_3 = \frac{\dot{S}_2\dot{\theta} - S_2\ddot{\theta}}{\dot{\theta}^2} + 2\left(\sin^2\theta - \cos^2\theta\right)\ddot{x}\dot{\theta} - 2\sin\theta\cos\theta x^{(3)} \tag{40}$$

where $\dot{S}_2(t)$ and $S_1^{(3)}(t)$ are given by the following expressions

$$\begin{aligned} \dot{S}_2 &= -\frac{\left(S_1^{(3)}\dot{\theta} - \dot{S}_1\theta^{(3)}\right)}{\dot{\theta}^2} + \frac{2\ddot{\theta}\left(\ddot{S}_1\dot{\theta} - \dot{S}_1\ddot{\theta}\right)}{\dot{\theta}^3} \\ &\quad + 2\cos^2\theta x^{(4)} - 8\sin\theta\cos\theta x^{(3)}\dot{\theta} - 4\sin\theta\cos\theta\ddot{x}\ddot{\theta} \\ &\quad - 4\ddot{x}\dot{\theta}^2\left(\cos^2\theta - \sin^2\theta\right) \end{aligned} \tag{41}$$

$$\begin{aligned} S_1^{(3)} &= \frac{F^{(4)} - m^{(4)}(\theta)\ddot{x} - 4m^{(3)}(\theta)x^{(3)}}{m_p\dot{\theta}} \\ &\quad \frac{-6\ddot{m}(\theta)x^{(4)} - 4\dot{m}(\theta)x^{(5)} - m(\theta)x^{(6)}}{m_p\dot{\theta}} \\ &\quad - 3\ddot{\theta}\left(\frac{F^{(3)} - m^{(3)}(\theta)\ddot{x} - \ddot{m}(\theta)x^{(3)}}{m_p\dot{\theta}^2}\right) \\ &\quad - 3\ddot{\theta}\left(\frac{-2\ddot{m}(\theta)x^{(3)} - 3\dot{m}(\theta)x^{(4)} - m(\theta)x^{(5)}}{m_p\dot{\theta}^2}\right) \\ &\quad - \frac{3\theta^{(3)}\left(\ddot{F} - \ddot{m}(\theta)\ddot{x} - 2\dot{m}(\theta)x^{(3)} - m(\theta)x^{(4)}\right)}{m_p\dot{\theta}^2} \\ &\quad + \frac{6\ddot{\theta}^2\left(\ddot{F} - \ddot{m}(\theta)\ddot{x} - 2\dot{m}(\theta)x^{(3)} - m(\theta)x^{(4)}\right)}{m_p\dot{\theta}^3} \\ &\quad - \frac{\theta^{(4)}\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^2} \\ &\quad + \frac{6\ddot{\theta}\theta^{(3)}\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^3} \\ &\quad - \frac{6\ddot{\theta}^3\left(\dot{F} - \dot{m}(\theta)\ddot{x} - m(\theta)x^{(3)}\right)}{m_p\dot{\theta}^4} \\ &\quad + \left(-2x^{(5)} + 24x^{(3)}\dot{\theta}^2 + 24\dot{\theta}\ddot{\theta}\ddot{x}\right)\sin\theta\cos\theta \\ &\quad + \left(6\ddot{\theta}x^{(3)} + 6x^{(4)}\dot{\theta} + 2\theta^{(3)}\ddot{x} - 8\dot{\theta}^3\ddot{x}\right) \\ &\quad \left(\sin^2\theta - \cos^2\theta\right) \end{aligned} \tag{42}$$

Based on the facts that

$$F^{(i)} = 0, \qquad x^{(i)} = 0, \qquad i \geq 1, \tag{43}$$

it is straightforward from (33), (35), (37), (38), (40), (41), and (42) to show that (34) and (39) can be rewritten as follows, respectively

$$L\sin\theta\dot{\theta}^2 + 10g\sin\theta\cos\theta = 0 \tag{44}$$

$$L\sin\theta\dot{\theta}^2 + 46g\sin\theta\cos\theta = 0. \tag{45}$$

After comparing (44) to (45), it is clear that based on the assumption that $\dot{\theta}(t) \neq 0$, the following expression must be true

$$\sin\theta = 0 \tag{46}$$

for (44) and (45) to be valid. However, after differentiating (46), we can conclude that

$$\cos\theta = 0 \tag{47}$$

based on the assumption $\dot{\theta}(t) \neq 0$. Since (46) and (47) cannot hold simultaneously, the assumption that $\dot{\theta}(t) \neq 0$ must be invalid, and hence, the arguments given for case 1 can be utilized to prove (18). ■

4.2 The E^2 Coupling Control Law

Based on previous work presented in [8], we design the following E^2 coupling control law[2]

$$F = \frac{-k_d\dot{x} - k_p e + \dfrac{k_v\varsigma(\theta,\dot{\theta})}{m(\theta)}}{k_E E + \dfrac{k_v}{m(\theta)}} \tag{48}$$

where $m(\theta)$ was defined in (11), $\varsigma(\theta,\dot{\theta})$ was defined in (12), $E(q,\dot{q})$ was defined in (14), m_p is given in (3), and k_d, k_p, k_E, $k_v \in \mathbb{R}^1$ are positive constant control gains.

Theorem 2 *The controller given in (48) ensures asymptotic regulation of the overhead crane system in the sense that*

$$\lim_{t\to\infty} \begin{pmatrix} x(t) & \dot{x}(t) & \theta(t) & \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} x_d & 0 & 0 & 0 \end{pmatrix} \tag{49}$$

where x_d represents the constant desired gantry position.

Proof: To prove Theorem 2, we define the following non-negative function

$$V_2 = \frac{1}{2}k_E E^2 + \frac{1}{2}k_v\dot{x}^2 + \frac{1}{2}k_p e^2. \tag{50}$$

After taking the time derivative of (50) and substituting (9), (15), and the time derivative of (16), into the resulting expression for $\dot{E}(q,\dot{q})$, $\dot{x}(t)$, and $\dot{e}(t)$, respectively, we obtain the following expression

$$\dot{V}_2 = \dot{x}\left(\left(k_E E + \frac{k_v}{m(\theta)}\right)F - \frac{k_v\varsigma(\theta,\dot{\theta})}{m(\theta)} + k_p e\right). \tag{51}$$

After substituting (48) into (51) for $F(t)$, we obtain the following expression

$$\dot{V}_2 = -k_d\dot{x}^2 \tag{52}$$

which implies that the origin of the closed-loop system is stable in the sense of Lyapunov [15] and that $x(t)$, $\dot{x}(t)$, $\dot{\theta}(t)$, $F(t) \in \mathcal{L}_\infty$. In a similar manner as in the proof of Theorem 1, we define Γ as the set of all points where

$$\dot{V}_2 = 0. \tag{53}$$

In the set Γ, it is clear from (52) and (53) that

$$\dot{x}(t) = 0. \tag{54}$$

and hence, we can conclude that $x(t)$ and $V_2(t)$ are constant, and that

$$\ddot{x}(t) = 0. \tag{55}$$

Furthermore, from (15), (16), and (54), it is clear that

$$\dot{E}(q,\dot{q}) = \dot{e}(t) = 0. \tag{56}$$

Based on (56), it is clear that $E(q,\dot{q})$ and $e(t)$ are constant.

Similar to the proof of Theorem 1, we can now divide the rest of the analysis into two cases. For the case of $\dot{\theta}(t) = 0$, we can easily follow case 1 in the proof of Theorem 1 to prove the result given by (49). For the case of $\dot{\theta}(t) \neq 0$, we first note that we can utilize (9) to rewrite (48) in the following equivalent form[3]

$$F = \frac{-k_p e - k_d\dot{x} - k_v\ddot{x}}{k_E E}. \tag{57}$$

It is now clear from (54), (55), (56) and (57) that $F(t)$ is constant; hence, the result given in (49) can now be obtained by following the same analysis in the proof of Theorem 1 for case 2. ■

4.3 Gantry Kinetic Energy Coupling Control Law

To illustrate how additional controllers can also be derived, we design the following nonlinear coupling control law[4]

$$F = \frac{-k_d\dot{x} - k_p e + k_v\left(\varsigma(\theta,\dot{\theta}) - m_p\sin\theta\cos\theta\dot{\theta}\dot{x}\right)}{k_E + k_v} \tag{58}$$

where $\varsigma(\theta,\dot{\theta})$ was defined in (12), m_p is given in (3), and k_d, k_p, k_v, $k_E \in \mathbb{R}^1$ are positive constant control gains.

Theorem 3 *The controller given in (58) ensures asymptotic regulation of the overhead crane in the sense that*

$$\lim_{t\to\infty} \begin{pmatrix} x(t) & \dot{x}(t) & \theta(t) & \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} x_d & 0 & 0 & 0 \end{pmatrix} \tag{59}$$

where x_d represents the constant desired gantry position.

Proof: To prove Theorem 3, we define the following non-negative function

$$V_3 = k_E E + \frac{1}{2}k_v m(\theta)\dot{x}^2 + \frac{1}{2}k_p e^2. \tag{60}$$

After taking the time derivative of (60) and substituting (9), (15), and the time derivative of (16), into the resulting expression for $\dot{x}(t)$, $\dot{E}(q,\dot{q})$, and $\dot{e}(t)$, respectively, we obtain the following expression

$$\dot{V}_3 = \dot{x}\left(\left(k_E + k_v\right)F - k_v\varsigma(\theta,\dot{\theta}) + k_v m_p\sin\theta\cos\theta\dot{\theta}\dot{x} + k_p e\right). \tag{61}$$

[2] The control strategy is called an E^2 coupling control law because its structure is spawned from a squared energy term in the Lyapunov function and an additional gantry squared velocity term in the Lyapunov function.

[3] Since $\dot{\theta}(t) \neq 0$, we can use (14) to show that $E(q,\dot{q}) > 0$, and hence, the denominator of (57) does not go to zero for this case.

[4] The control strategy is called a gantry kinetic energy coupling control law because its structure is spawned from an additional gantry kinetic energy term in the Lyapunov function.

After substituting (58) into (61) for $F(t)$, we obtain the following expression

$$\dot{V}_3 = -k_d \dot{x}^2. \tag{62}$$

In a similar manner as in the proof of Theorem 1, we define Γ as the set of all points where

$$\dot{V}_3 = 0. \tag{63}$$

In the set Γ, it is clear from (62) and (63) that

$$\dot{x}(t) = 0. \tag{64}$$

and hence, we can conclude that $x(t)$ and $V_3(t)$ are constant, and that

$$\ddot{x}(t) = 0. \tag{65}$$

Furthermore, from (15), (16), and (64), it is clear that

$$\dot{E}(q, \dot{q}) = \dot{e}(t) = 0 \tag{66}$$

Based on (66), it is clear that $E(q, \dot{q})$ and $e(t)$ are constant.

Similar to the proof of Theorem 1, we can now divide the rest of the analysis into two cases. For the case of $\dot{\theta}(t) = 0$, we can easily follow case 1 in the proof of Theorem 1 to prove the result given by (59). For the case of $\dot{\theta}(t) \neq 0$, we first note that we can utilize (9) to rewrite (58) in the following equivalent form

$$F = \frac{-k_p e - k_d \dot{x} - k_v m(\theta)\ddot{x} - \frac{1}{2} k_v \dot{m}(\theta)\dot{x}}{k_E}. \tag{67}$$

It is now clear from (64), (65), (66) and (67) that $F(t)$ is constant; hence, the result given in (59) can now be obtained by following the same analysis in the proof of Theorem 1 for case 2. ■

5 Conclusion

In this paper, we have presented three controllers for an overhead crane system. By utilizing a Lyapunov-based stability analysis along with LaSalle's Invariance Theorem, we proved asymptotic regulation of the gantry position and payload position for a PD controller and two nonlinear controllers. Future work will focus on leveraging off of the current results to develop controllers for an overhead crane with a gantry that moves in a 2-DOF Cartesian plane (versus the 1-DOF motion allowed for the gantry in the present crane assembly).

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