Local one-dimensional ICRF full-wave solutions valid to all orders in $k_\perp\rho$

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Introduction

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High harmonic ion cyclotron resonances are important for understanding ICRF experiments on:
- NSTX: RF heating with high harmonic fast waves
- TFTR and PBX-M: RF flow drive with high harmonic IBW

A realistic theoretic treatment of these cases requires full-wave models that go beyond the usual second order finite Larmor radius expansion used in most ICRF wave analysis codes. Possibilities include:
- Non-local (integral) codes:
  Valid to all orders in $k_\perp\rho$ and ρ/L
  require long computing times

This Paper
- Local conductivity (lowest order in ρ/L):
  assume $\rho/L << 1$ but still retain all orders in $k_\perp\rho$
  NSTX (T_i = 5 keV): $\rho/L \approx 0.0825$, $k_\perp\rho = 10$
  TFTR (T_i = 4 keV): $\rho/L \approx 0.0037$, $k_\perp\rho = 6$
  requires less computing time than integral codes

Spectral solution to the wave equation

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We apply a spectral method to solve the vector wave equation:

$$-\nabla\times\nabla\times\vec{E}+\frac{\omega^2}{c^2}\left(\vec{E}+\frac{i}{\omega\epsilon_0}\vec{J_p}\right)=-i\omega\mu_0\vec{J}_{ext}$$

The electric field and plasma current can be expanded in Fourier harmonics as:

$$\vec{E}(x)=\sum_{k_n}\vec{E}_{k_n}\,e^{ik_nx}$$

$$\vec{J_p}(x)=\sum_{k_n}\tilde{\sigma}(x,k_n)\cdot\vec{E}_{k_n}(\rho)\,e^{ik_nx}$$

where $k_n = 2\pi n/d$ *and* $0<x<d$. We assume the well-known (Stix) expression for the "local" plasma conductivity (lowest order in ρ/L). However, a first order correction in ρ/L could be included by retaining terms in $\sigma(x, k)$ that are proportional to gradients in the equilibrium quantities.

Local plasma conductivity ($\rho/L << 1$)

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To lowest order in ρ/L, the conductivity for an isotropic Maxwellian is (Stix):

$$\tilde{\sigma}(x,k_n)=\frac{\epsilon_0\omega}{i}\sum_{l=-\infty}^{\infty}\frac{\omega_p^2}{\omega^2}\,\zeta_0\;e^{-\Gamma_n}\,\tilde{T}_l$$

where

$$\tilde{T}_l=\begin{pmatrix}\frac{l^2I_l}{\Gamma_n}Z_l, & -il(I_l-I_l')Z_l, & -\sqrt{\frac{\Gamma_n}{2}}\frac{lI_l}{\Gamma_n}Z_l'\\ il(I_l-I_l')Z_l, & \left(\frac{l^2}{\Gamma_n}I_l+2\Gamma_nI_l-2\Gamma_nI_l'\right)Z_l, & -i\sqrt{\frac{\Gamma_n}{2}}(I_l-I_l')Z_l'\\ -\sqrt{\frac{\Gamma_n}{2}}\frac{lI_l}{\Gamma_n}Z_l', & i\sqrt{\frac{\Gamma_n}{2}}(I_l-I_l')Z_l', & -I_l\,\zeta_l\,Z_l'\end{pmatrix}$$

I_l is the modified Bessel function with argument Γ_n: $I_l=I_l(\Gamma_n)$

ζ_l is the argument of the plasma dispersion function: $Z_l=Z(\zeta_l)$

where: $\Gamma_n=\frac{1}{2}\left(\frac{k_nv_{th}}{\Omega}\right)^2$, $\quad\zeta_l=\frac{\omega-l\Omega}{|k_\parallel|\,v_{th}}$, $\quad Z'(\zeta_l)=-2[1+\zeta_lZ(\zeta_l)]$

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Compare spectral and second-order finite difference solutions for two-ion-hybrid case

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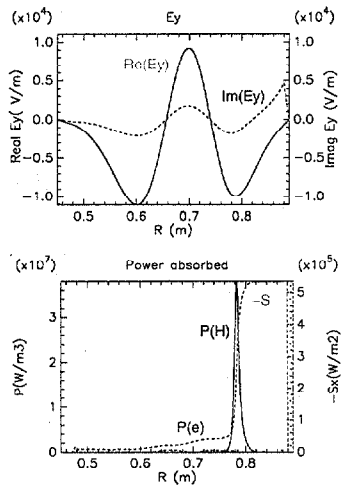
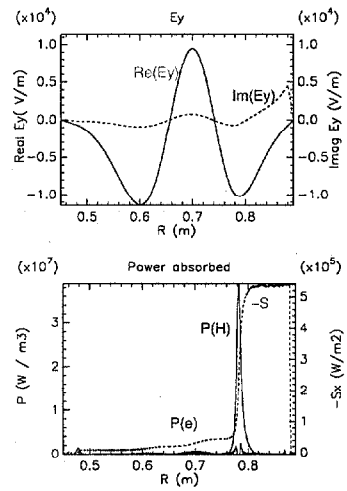
Finite difference (ORION1D)



Spectral (AORSA)



Compare the driven flow for two-ion-hybrid case

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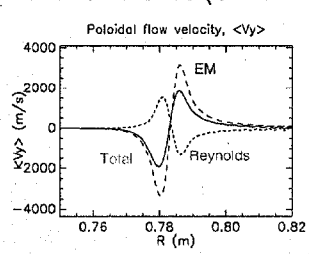
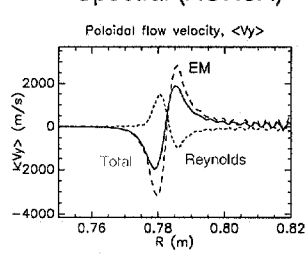
Finite difference (ORION1D)



Spectral (AORSA)



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High harmonic fast waves in NSTX: T_i = 1 keV

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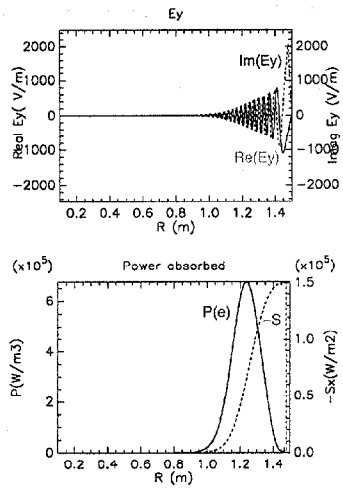
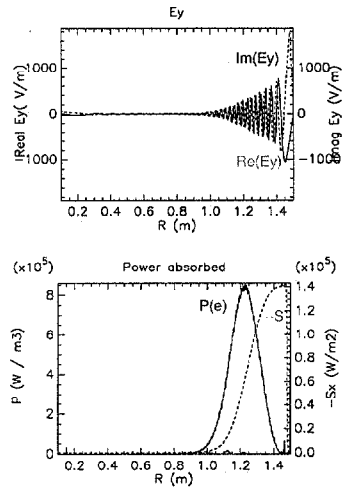
Finite difference (ORION1D)



Spectral (AORSA)



High harmonic fast waves in NSTX: T_i = 5 keV

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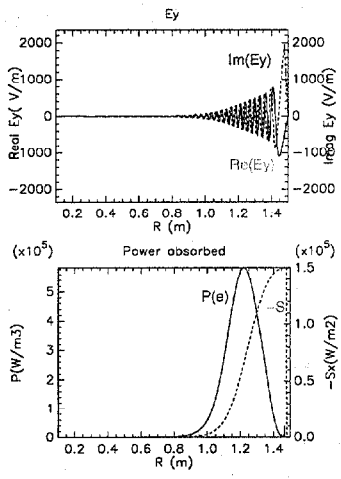
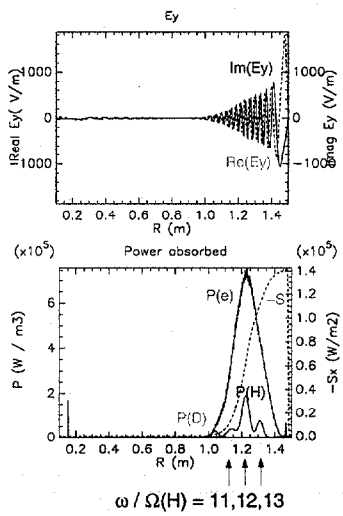
Finite difference (ORION1D)



Spectral (AORSA)



$\omega / \Omega(H) = 11,12,13$

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High harmonic IBW launch in TFTR

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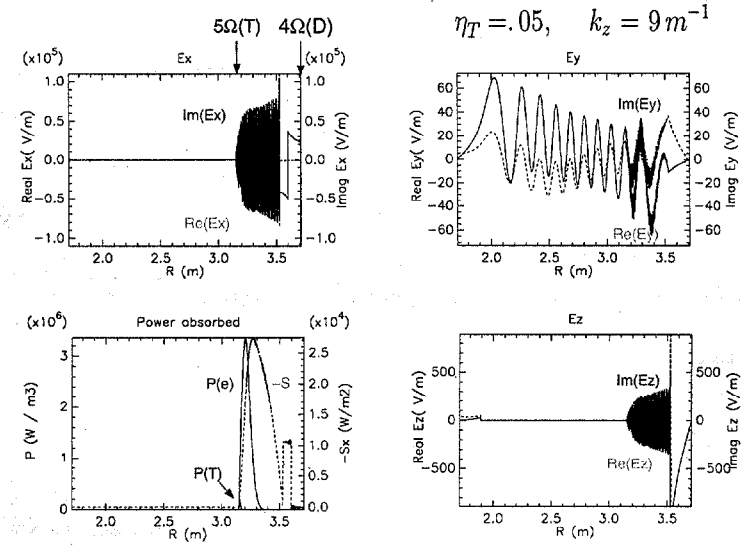
$\eta_T = .05, \quad k_z = 9\,m^{-1}$



High harmonic IBW launch in TFTR

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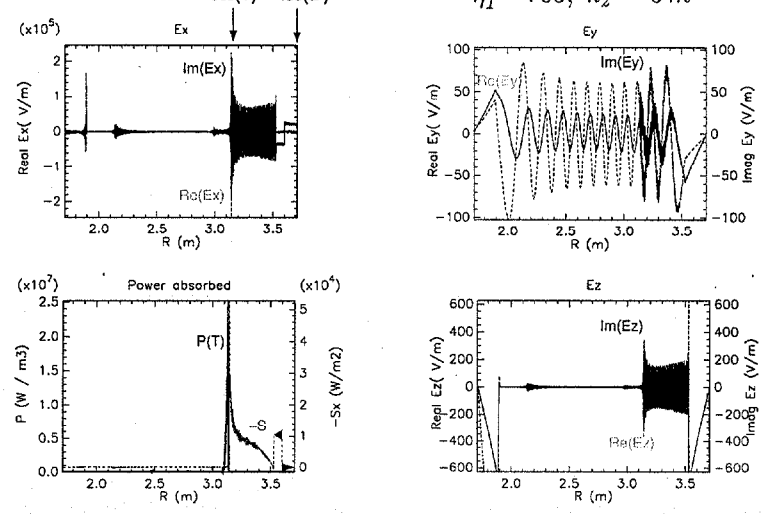
$\eta_T = .05, \; k_z = 5\,m^{-1}$



Summary

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We have applied a spectral method to solve the 1-D wave equation:
- Avoids the second order gyroradius expansion
- Requires less computing time than a full integral code

We use the well-known "local" plasma conductivity ($\rho/L << 1$)
- a first order correction in ρ/L could be included by retaining terms in σ that are proportional to gradients in equilibrium quantities

Results:
- Spectral method and finite difference solutions agree when $\omega/\Omega_i \leq 2$
- NSTX: the agreement with second order solution is good for $T_i \leq 1$ keV; some differences are seen for $T_i = 5$ keV.
- TFTR: directly launched 4th harmonic IBW is absorbed at the fifth harmonic of tritium for k_z small (cut-off for k_z large)
- Run times: vary between 2 and 100 minutes depending on how many Fourier modes and ion harmonics are required

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