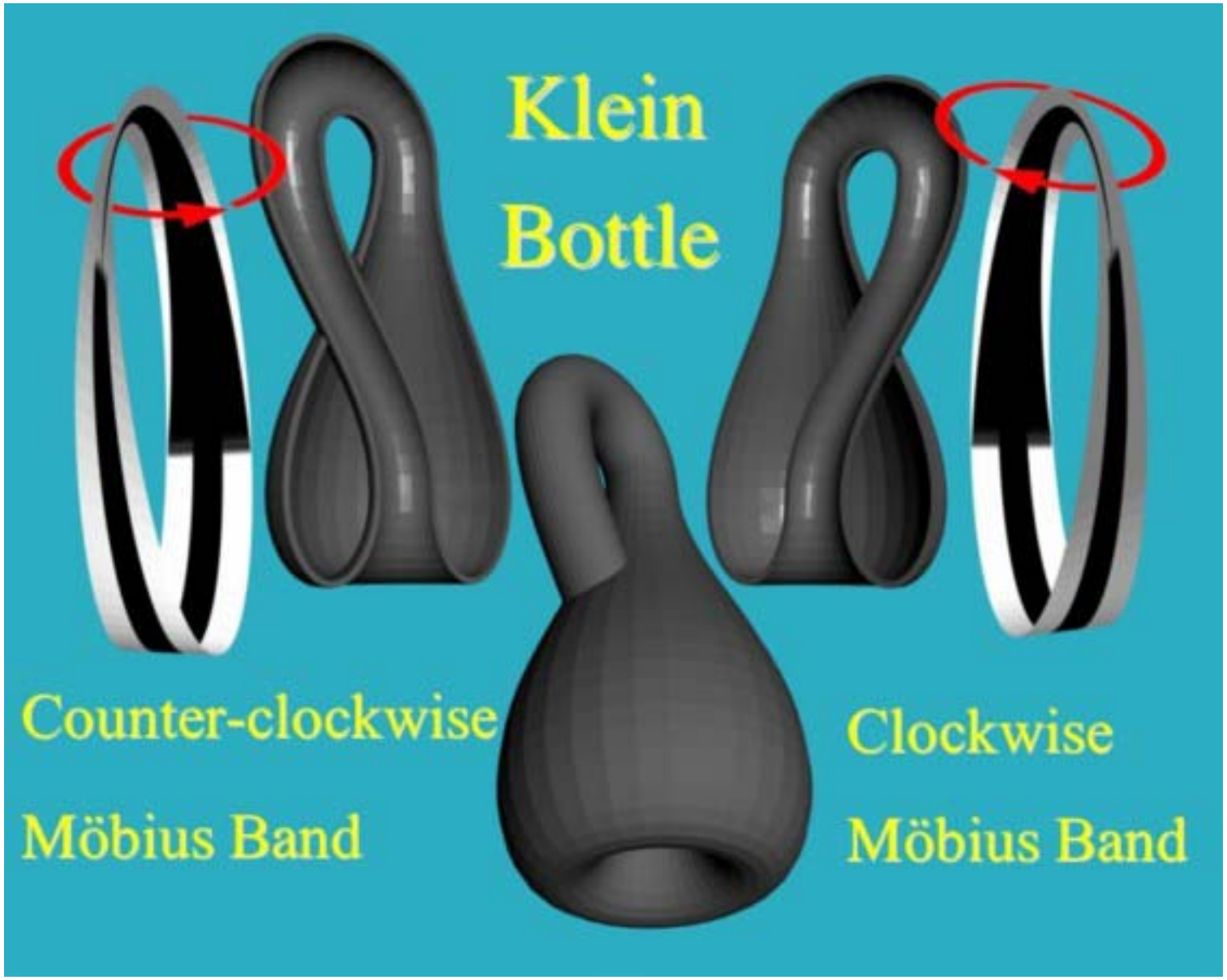
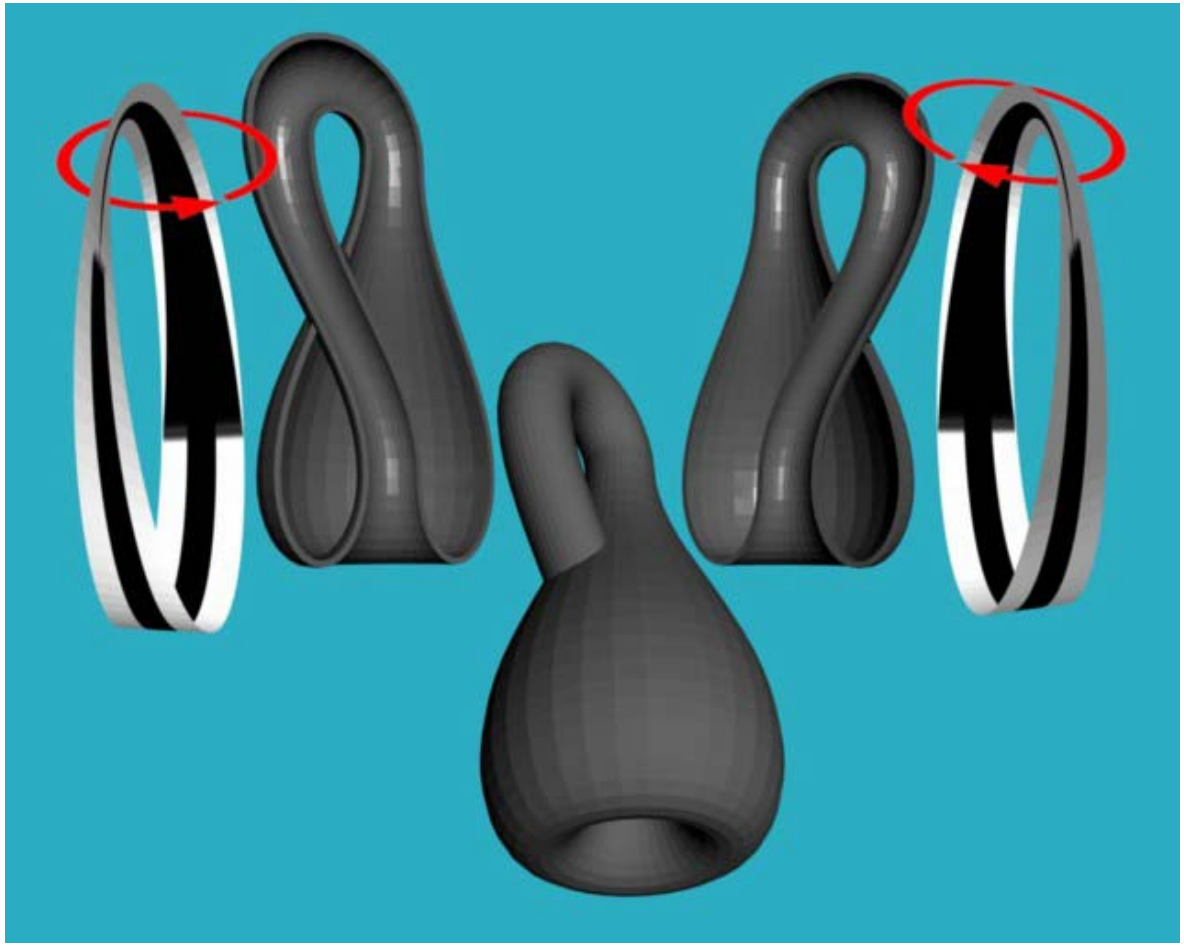
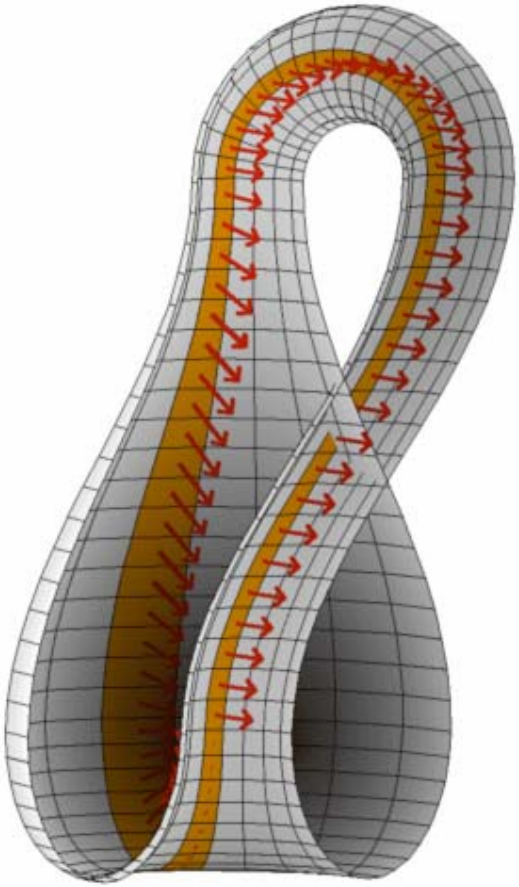


[Image by Stewart Dickson] Research sponsored by Computer Science and Mathematics Division of Oak Ridge National Laboratory, managed by UT-Battelle, LLC for the U.S. DOE under Contract No. DE-AC05-00OR22725

The Klein bottle can be formed from two Moebius bands twisted in opposite directions and joined at their edge. Note that the surface of the Moebius band is non-orientable. [E.g., there Is no uniform covering of surface with normal vectors -- there is discontinuity.]

[Note that the edge of the Klein bottle halves can be traced in a single, closed loop.]
[Please see the physical models of the Klein bottle and its two halves, below.]

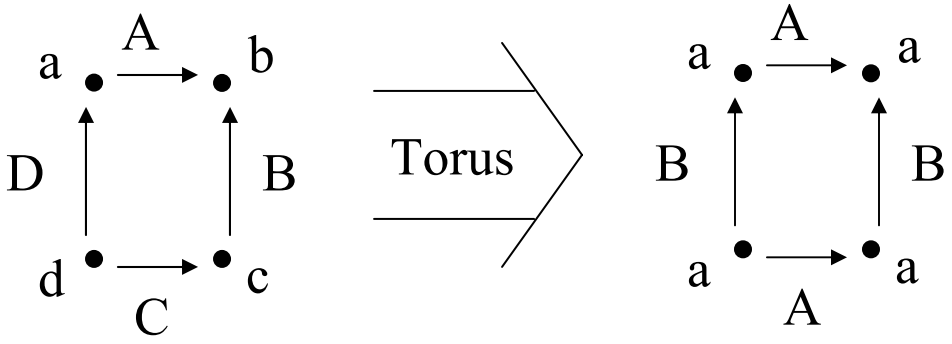


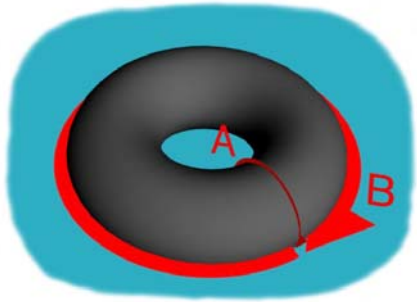


[Image by: Konrad Polthier,
Technische Universitaet Berlin]

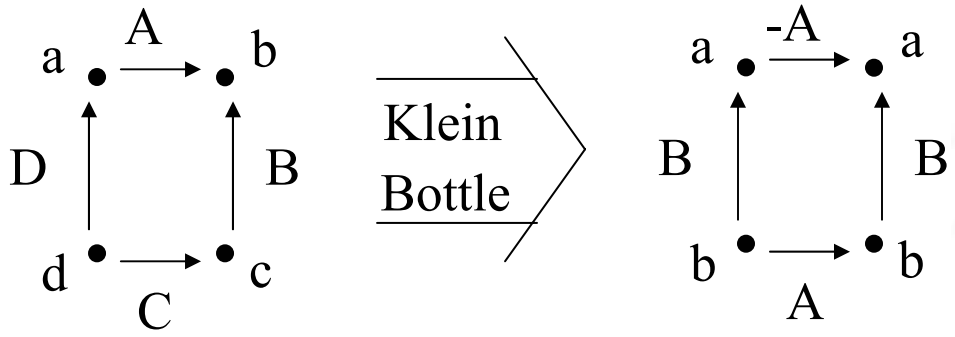
Topology of the Klein Bottle

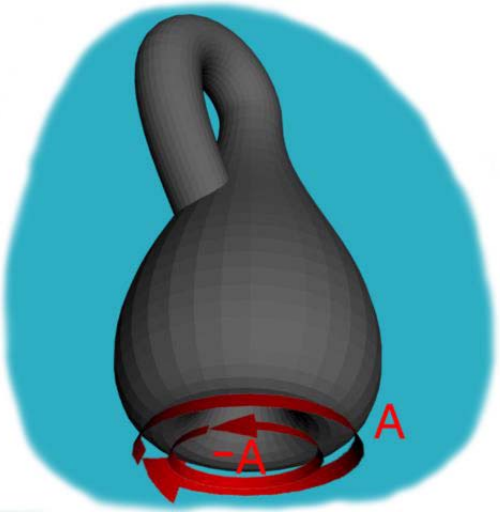
The Klein bottle is formed
by associating opposite edges of its domain



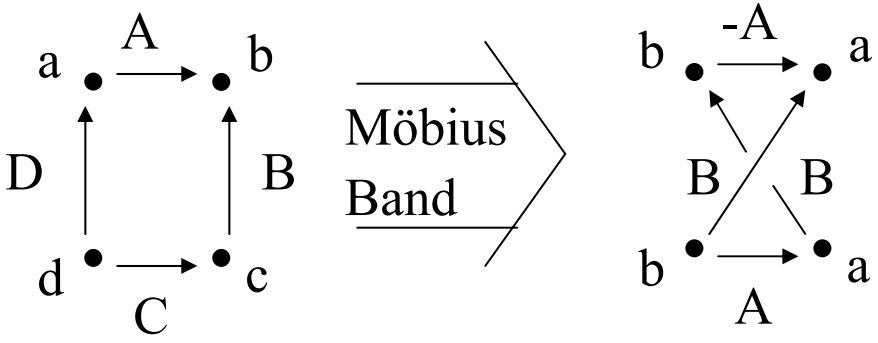


There is a change of sign at edge A

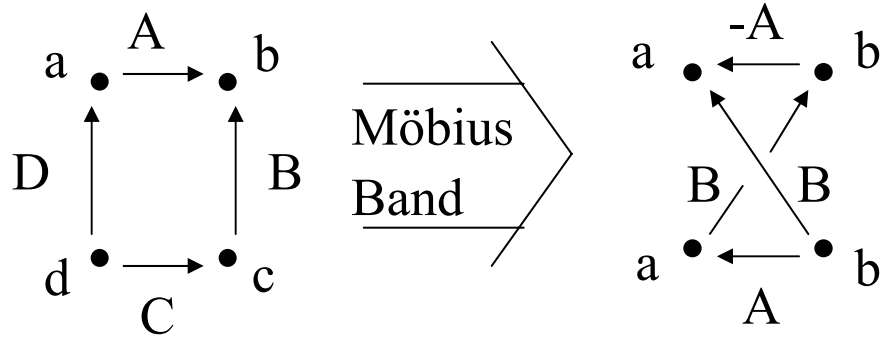




"Right-Handed"

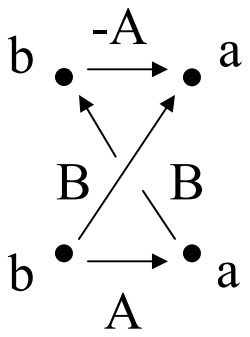


"Left-Handed"

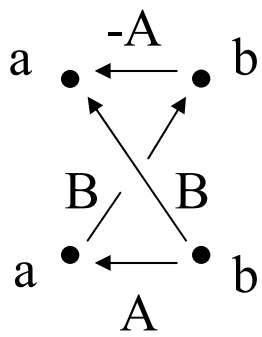


PROPOSITION:

"Right-Handed" + "Left-Handed" = Klein Bottle



+



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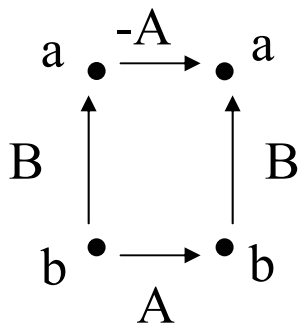
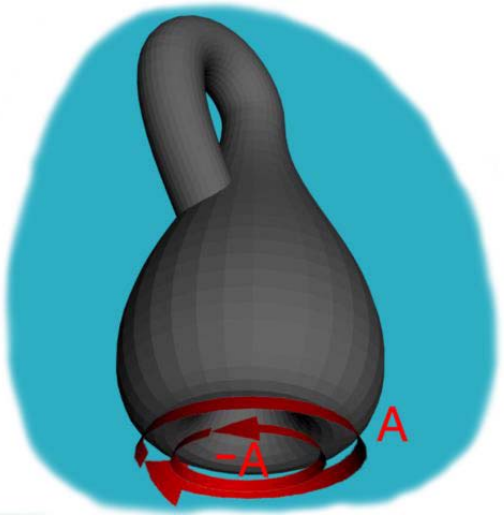




$$\left\{\begin{array}{c}((2.5+1.5\cos v)\cos u,\ (2.5+1.5\cos v)\sin u,\ -2.5\sin v)\\ ((2.5+1.5\cos v)\cos u,\ (2.5+1.5\cos v)\sin u,\ 3v)\\ (2-2\cos v+\sin u,\ \cos u,\ 3v)\\ (2+(2+\cos u)\cos v,\ \sin u,\ 3\pi+(2+\cos u)\sin v)\end{array}\right\}_{u_0}^{2\pi};{}_{v_0}^{\pi}.$$

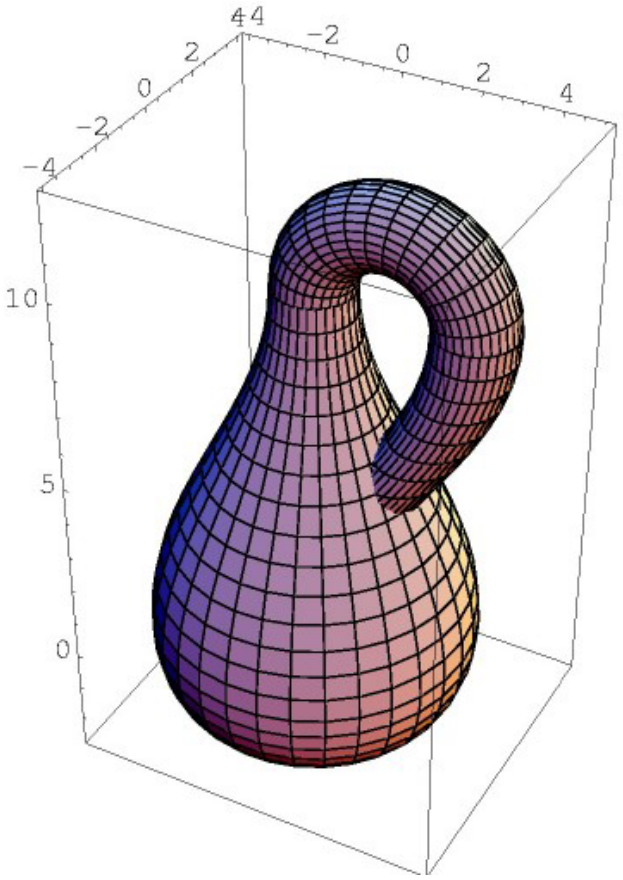
Here is the Mathematica code which generated this Klein bottle:

In[1]:=

```
(* File: klein_PC.m Klein bottle of Paul Chang, paul@math.ucla.edu *)
(* << Graphics/ParametricPlot3D.m *)

bot={(2.5+1.5 Cos[v]) Cos[u], (2.5+1.5 Cos[v]) Sin[u], -2.5 Sin[v]};
mid={(2.5+1.5 Cos[v]) Cos[u], (2.5+1.5 Cos[v]) Sin[u], 3v};
han={2-2 Cos[v]+Sin[u], Cos[u], 3v};
top={2+(2+Cos[u]) Cos[v], Sin[u], 3Pi + (2+Cos[u]) Sin[v]};

bottom=ParametricPlot3D[bot,{u,0,2Pi},{v,0,Pi},PlotPoints->{32, 16}];
middle=ParametricPlot3D[mid,{u,0,2Pi},{v,0,Pi},PlotPoints->{32, 16}];
topper=ParametricPlot3D[top,{u,0,2Pi},{v,0,Pi},PlotPoints->{32, 16}];
handle=ParametricPlot3D[han,{u,0,2Pi},{v,0,Pi},PlotPoints->{32, 16}];
all=Show[handle,topper,middle,bottom]
Display["klein_pc.ps",all]
```



The Klein bottle and two halves [congruent to Moebius bands twisted in opposite directions] manufactured via Stereolithography, material: DSM SOMOS 8120 photopolymer.
[Image by Stewart Dickson, Rapid Prototyping was done on a 3D Systems SLA-3500 Stereolithography Apparatus by the Rapid Prototyping and Manufacturing Institute Georgia Institute of Technology, Andrew Layton, Program Manager.

Sofware used to create these objects and images:

* Mathematica http://www.wolfram.com

* ThreeScript [Mathematica Standard Package]
http://support.wolfram.com/mathematica/kernel/Symbols/Graphics/ThreeScript/ThreeScript.html

* fromThreeScript [Program written by Stewart Dickson]
http://emsh.calarts.edu/~mathart/sw/objView/fromThreeScript.html

* thicken [Program written by Stewart Dickson]
http://emsh.calarts.edu/~mathart/sw/objView/thicken.html

* Maya [by Alias|Wavefront] http://www.aliaswavefront.com/en/news/home.shtml

* tostl, towaveobj [Program written by Stewart Dickson]
http://emsh.calarts.edu/~mathart/sw/SPD_software.html

* fromobj [Program written by Stewart Dickson]
http://emsh.calarts.edu/~mathart/sw/objView/fromobj.html