Amplitude Dropout in Coupled Lasers

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Equations of Motion for Nd:YAG Laser Array

$$\dot{E}_j = (G_j - a - id_j)E_j + k(E_{j+1} + E_{j-1}) + E_e(t)$$

$$\dot{N}_j = g(p - (1 + |E_j|^2 G_j))$$

E_j complex field of laser j

G_j population inversion of laser j

E_e external injection field

p pumping into lasers

δ_j detuning

γ^{-1} population decay time

κ coupling coefficient (considered as complex)

α loss coefficient

Intensity-Phase Model

Substitute

$$E_e(t) = \sqrt{I_e}$$

$$E_j(t) = \sqrt{I_j}(t)\exp(i\phi_j(t))$$

To obtain

$$\dot{I}_j = 2(G_j - \alpha)I_j + 2k\sqrt{I_1 I_2}\cos(\phi_2 - \phi_1) + 2\sqrt{I_e I_j}\cos\phi_j$$

$$\dot{\phi}_j = \delta_j + (-1)^j k\frac{\sqrt{I_1 I_2}}{I_j}\sin(\phi_1 - \phi_2) - \sqrt{\frac{I_e}{I_j}}\sin\phi_j$$

$$\dot{G}_j = \gamma(p - G_j - G_j I_j)$$

Phase Model

We assume that intensities and gains of both lasers do not change significantly in time (basically, stays constants). This will allow to analyze the phase model:

$$\dot{f}_1 = d_1 + k(\sin(f_1 - f_2)) - A_e \sin f_1$$

$$\dot{f}_2 = d_2 + k(\sin(f_2 - f_1)) - A_e \sin f_2$$

$$A_e \equiv I_j / I_e$$

For more detailed information on transition to the phase model: A. I. Khibnik, Y. Braiman, T. A. B. Kennedy, and K. Wiesenfeld, Physica D **111**, 295 (1998).
A. Khibnik, Y. Braiman, V. Protopopescu, T. A. B. Kennedy, and K. Wiesenfeld, Phys. Rev. A **62**, 063815 (2001).

Stability Analysis of the Phase Model

$$\dot{f}_j = d_j - |k|\{\sin(f_{j+1} - f_j) + \sin(f_{j-1} - f_j)\} - A_e \sin f_j$$

$$d_j = 0 \Rightarrow f_j^0 = 0 \quad \text{is a solution}$$

Stability ?

$$f_j(t) = f_j^0 + df_j(t)$$

$$d\dot{f}_j = -|k|[df_{j+1} - 2df_j + df_{j-1}] - A_e df_j$$

$$df_j = x_{jm} \exp(l_m t) \qquad \Rightarrow l_m = -A_e + 4|k|\sin^2\frac{mp}{N}, \quad m = 0,1,\ldots N-1$$

In-phase solution stable for $\boxed{A_e > 4|k|}$

Y. Braiman, T. A. B. Kennedy, K. Wiesenfeld, and A. I. Khibnik, Phys. Rev. A **52**, 1500, (1995).

Two Coupled Lasers

$$\dot{f}_1 = d_1 + k\,(\sin(f_1 - f_2)) - A_e \sin f_1$$

$$\dot{f}_2 = d_2 + k\,(\sin(f_2 - f_1)) - A_e \sin f_2$$

Fixed Point Solutions

$$d_1 + k\,(\sin(f_1 - f_2)) - A_e \sin f_1 = 0$$

$$d_2 + k\,(\sin(f_2 - f_1)) - A_e \sin f_2 = 0$$

Injection Tuning

$$d_1 + d_2 \simeq 0$$

Analysis of the Phase Model

$$\sin f_1 + \sin f_2 = 0$$

$$d_1 - d_2 + 2k(\sin(f_2 - f_1)) - A_e(\sin f_2 - \sin f_1) = 0$$

The first equation in (1.3) implies that either a): $f_2 - f_1 = (2m+1)p$ or b): $f_1 + f_2 = 2pm$, where m is an integer. Solutions of class (a) imply $\sin(f_2 - f_1) = 0$, yielding $\sin f_1 = d_1 / A_e$, $\sin f_2 = d_2 / A_e$ and $\sin(f_1 - f_2) = \sin(\sin^{-1}(d_1 / A_e) - \sin^{-1}(d_2 / A_e)) \neq 0$, i.e. inconsistency. Hence, the only possibility is the class (b) of solutions which, in turn, can be divided in two sub-classes: m even and m odd. For m even, the second equation in becomes:

$$f(f) \equiv -d - 2k \sin f - 2A_e \sin \frac{f}{2} = 0$$

$$(f \equiv f_2 - f_1)$$

Nonmonotonicity Transition Point

$$f(f) = -d - 2k\sin f_c - A_c \sin\frac{f_c}{2} = 0$$

$$f'(f) = -2k\cos f_c - A_c \cos\frac{f_c}{2} = 0$$

Substitute $\tan(f_c/2) \equiv z$:

$$z = (-q/2 + \sqrt{D})^{1/3} + (-q/2 - \sqrt{D})^{1/3}$$

$$D = (p/3)^3 + (q/2)^2$$

$$p = -d^2/48k^2 \text{ and } q = d/4k + d^3/864k^3$$

$$A_c = -2k\,\frac{1 - z^2}{\sqrt{1 + z^2}}.$$

Another Solution

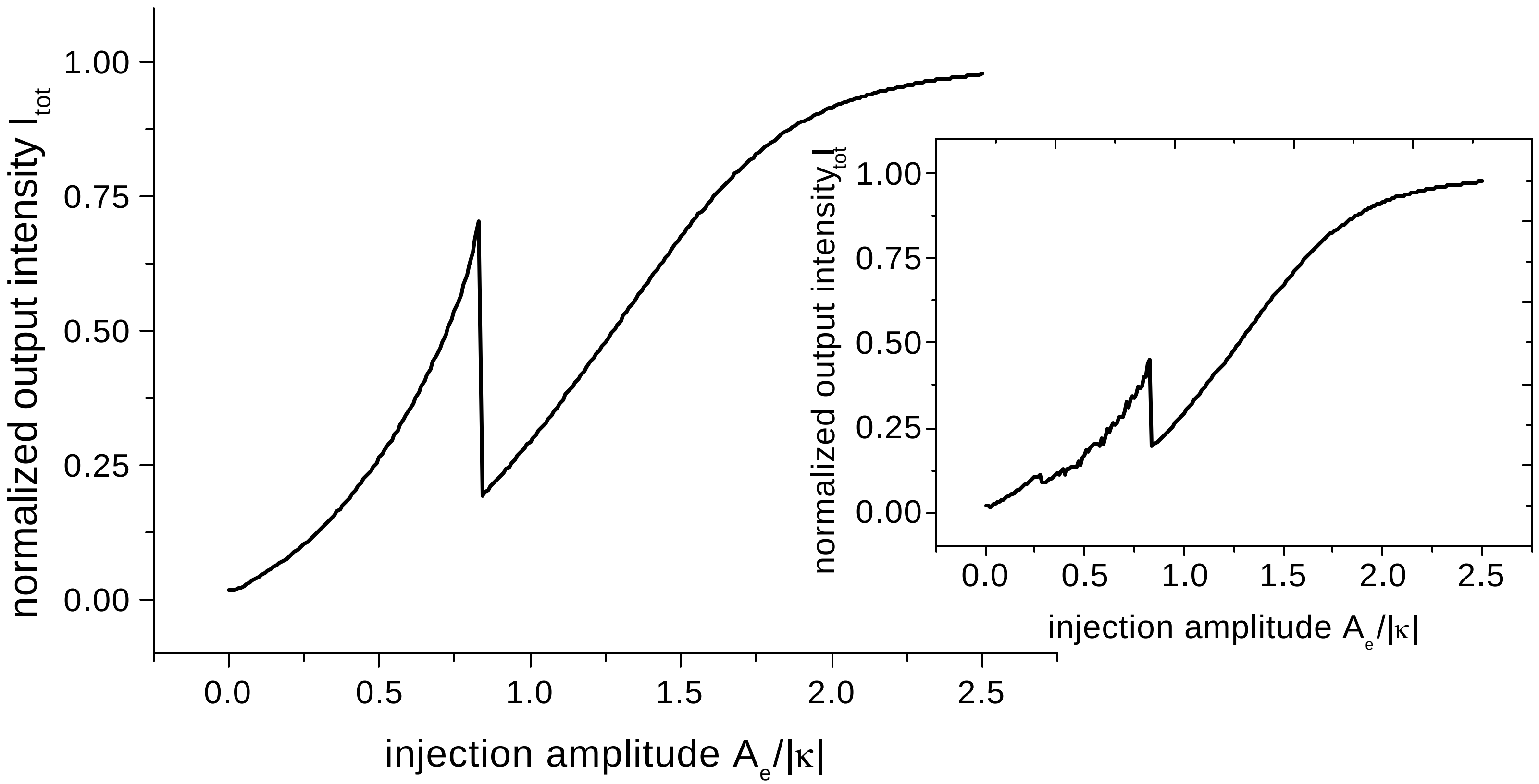
For m odd:

$$g(f) \equiv -d - 2k \sin f - 2A_e \sin\frac{f}{2} = 0$$

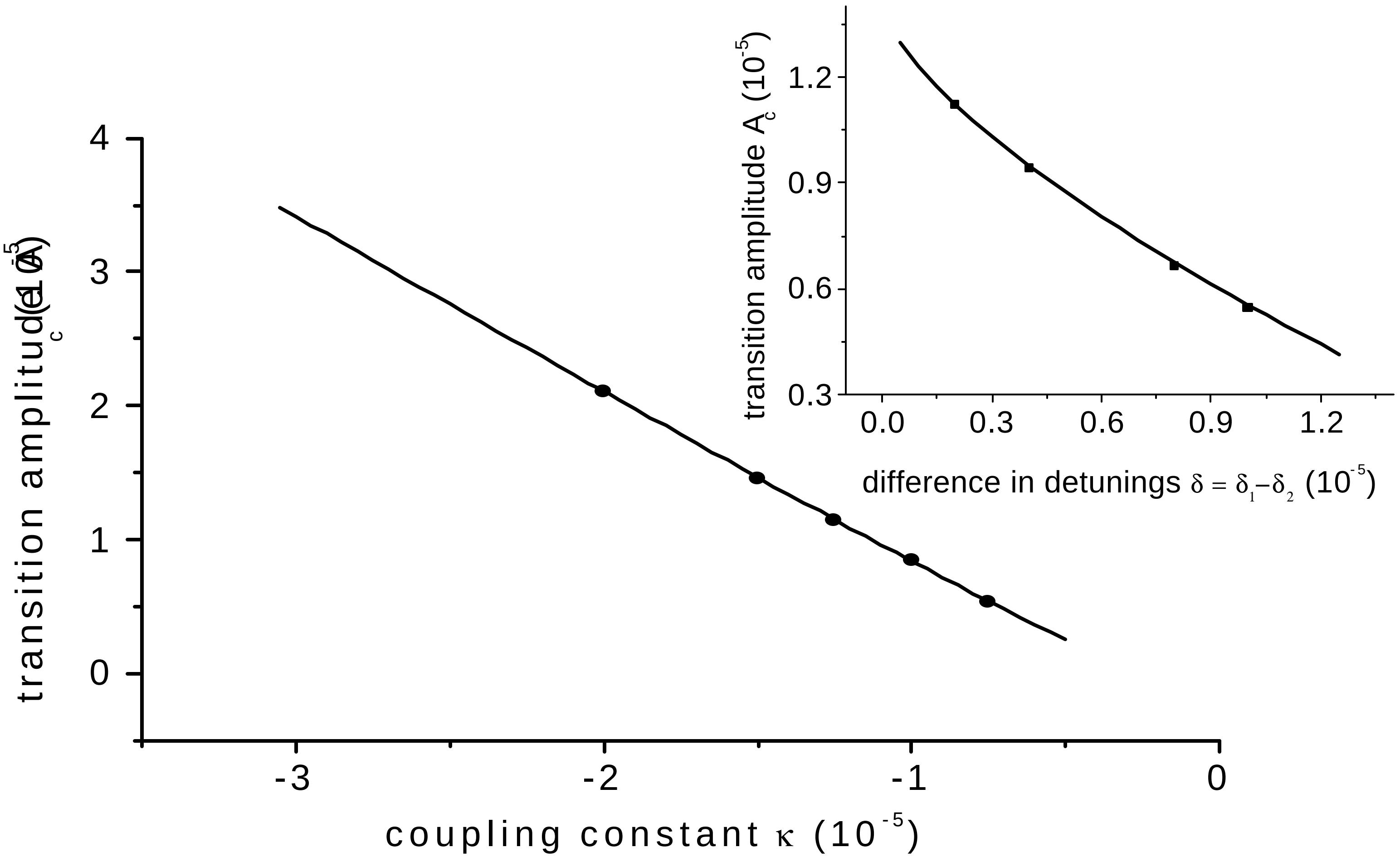
Two solutions: one close to ϕ≈0 (unstable), and another close to ϕ≈π (stable). This is low intensity solution, since:

$$I_{tot}/I_0 = 4\cos^2[(f_1 - f_2)/2] \equiv 4\cos^2 f/2$$

Nonmonotonicity



Comparison of the Analysis with Numerical Simulations



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