Simultaneous Teleportation of the Spectral and Polarization States of a Photon

Travis S. Humble, Ryan S. Bennink, and Warren P. Grice, Oak Ridge National Laboratory, Oak Ridge, TN

Synopsis

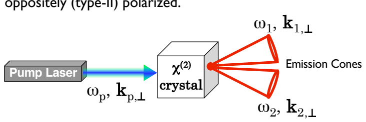
Photonic implementations of quantum teleportation transfer the quantum state of one photon onto another. Recent experimental demonstrations have teleported states encoded into either the polarization or field-quadrature degrees of freedom.

An outstanding question is how to simultaneously teleport quantum information encoded in multiple photonic degrees of freedom. The capability to teleport information carried by multiple degrees of freedom could support techniques for manipulating qubits, e.g., embedded Bell-state analysis, or help overcome experimental inefficiencies, e.g., losses due to spectral filtering.

We report how the spectral and polarization states of a single photon can be simultaneously teleported by using a Bell-state analyzer based on optical sum-frequency generation.

Broad Bandwidth SPDC

A pair of spectrally entangled photons can be generated by spontaneous parametric down conversion. In SPDC, a nonlinear medium induces a high frequency pump photon to decay into a pair of lower frequency photons. Depending on the type of SPDC, the photons may be similarly (type-I) or oppositely (type-II) polarized.



The emission cones in type-II SPDC.

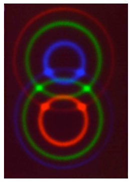
The SPDC process conserves energy and momentum and generates a joint spectral amplitude equal to the pump spectrum $A(\omega)$ times the phase-matching function $\Phi(\omega, \omega')$.

$$\omega_1 + \omega_2 = \omega_p \qquad \mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} = \mathbf{k}_{p,\perp}$$

$$f(\omega_1,\omega_2) = A(\omega_1 + \omega_2)\Phi(\omega_1,\omega_2)$$

For type-II SPDC, the emission cones of the photons can be overlapped by adjusting the phase-matching angle. In the far field, this overlap images as intersecting rings.

At positions where the cones overlap, the two spectrally multimode photons exist in a polarization-entangled state.



$$|\varphi_{12}\rangle = \frac{1}{\sqrt{2}}\int d\omega_1 \int d\omega_2 \Big[f(\omega_1,\omega_2)\big|h_1(\omega_1),v_2(\omega_2)\big\rangle + g(\omega_1,\omega_2)\big|v_1(\omega_1),h_2(\omega_2)\big\rangle \Big]$$

Depending on experimental conditions, the joint spectral amplitudes f and g can be either equal (type-I) or related by permutation of the frequency arguments (type-II).

Spectral Entanglement

If the joint spectral amplitude cannot be factored into a product of single-photon spectral amplitudes u and v, then the photon pair is spectrally entangled.

$$f(\omega_1,\omega_2) \neq u(\omega_1)v(\omega_2)$$

Spectral entanglement for a photon pair can be quantified with respect to the Schmidt decomposition of the joint spectral amplitude.

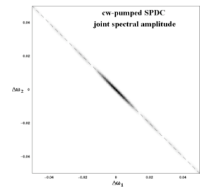
$$f(\omega_1,\omega_2) = \sum_{n=0}^{\infty} \lambda_n^{1/2} u_n(\omega_1) v_n(\omega_2)$$

The Schmidt coefficient λ_n measures the relative amplitude of the nth pair of Schmidt modes, u_n and v_n, while either the Schmidt number K or the von Neumann entropy S quantifies the degree of spectral entanglement.

$$K = 1\Big/\sum_{n=0}^{\infty}\lambda_n^2 \qquad S = -\sum_{n=0}^{\infty}\lambda_n \log_2 \lambda_n$$

In cw-pumped SPDC, the pump spectrum $A(\omega)$ is sharply peaked and the joint spectral amplitude exhibits strong correlation between the frequencies of the down-converted photons modulated by the phase-matching function.

Neglecting the effects of the phase-matching function, and approximating the latter by a constant, all the Schmidt coefficients are unity and the Schmidt modes represent delta distributions.



$$\lambda_n \cong 1$$

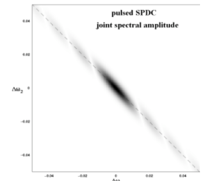
$$u_n(\omega) = v_n(\omega) \cong \delta(\omega)$$

K is ill defined.

$$S \to \infty$$

The joint spectral amplitude (density) and Schmidt decomposition for a 1 mm bulk BBO crystal pumped by a 405-nm pump pulse with a 0.3 nm bandwidth.

Approximating the effects of the phase-matching function by a Gaussian envelope, the Schmidt coefficients depend on the spectral entanglement through the frequency cross correlation ρ, and the Schmidt modes are n^{th}-order Hermite functions [1].



$$\lambda_n = \mathrm{sech}^2\varphi \tanh^{2n}\varphi$$

$$\rho = \tanh 2\varphi$$

$$K = \cosh 2\varphi$$

$$S = \cosh^2\varphi \log_2 \cosh^2\varphi - \sinh^2\varphi \log_2 \sinh^2$$

The joint spectral amplitude (density) and Schmidt decomposition for a 1 mm bulk BBO crystal pumped by a 405-nm pump pulse with a 1.0 nm bandwidth.

Polarization Entanglement

When the composite biphoton state factors into a product of spectral and polarization states, it belongs to a class of hyper-entangled states [2], i.e., states where entanglement exists simultaneously in multiple degrees of freedom.

$$|\varphi_{12}\rangle = |\Omega_{12}\rangle|\Psi_{12}^{(+)}\rangle$$

$$|\Omega_{12}\rangle = \int d\omega_1 \int d\omega_2 f(\omega_1,\omega_2)|\omega_1,\omega_2\rangle$$

$$|\Psi_{12}^{(+)}\rangle = \frac{1}{\sqrt{2}}\big(|h_1,v_2\rangle + |v_1,h_2\rangle\big)$$

Then the polarization entanglement is independent of the spectral entanglement and the polarization entanglement can be maximal (as it is above).

But when the composite biphoton state does not factor into a product of spectral and polarization states, the polarization entanglement depends on the spectral entanglement via the overlap of the two underlying joint (spectral) amplitudes [3].

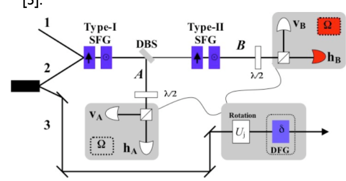
$$C_{12} = \left|\int d\omega_1 \int d\omega_2 f(\omega_1,\omega_2) g(\omega_1,\omega_2)^*\right|$$

As measured by the concurrence, the polarization entanglement is less than maximal when the joint amplitudes are distinguishable, although such states can optimize certain effects, e.g., the rotated joint spectral amplitudes produced in type-II SPDC optimize Hong-Ou-Mandel interference [4].

Quantum Teleportation

Quantum teleportation requires entanglement to establish a quantum communication channel. For teleporting the spectral and polarization states of a photon, quantum teleportation requires both spectral and polarization entanglement.

Kim et al. previously demonstrated quantum teleportation of a polarization state using a complete Bell-state measurement based on a series of type-I and type-II sum-frequency generation (SFG) events [5].



Schematic of a complete Bell-state measurement based on SFG, where dashed boxes represent additions needed for simultaneously teleporting the spectral and polarization state of photon 1 onto photon 3.

For Kim et al., photons 1 and 2 pass through a pair of orthogonally oriented type-I SFG crystals. Either crystal can induce up-conversion to photon A, which is then isolated using a dichroic beam splitter and measured in the diagonal basis. Similarly, photon B arises from one of two type-II SFG crystals and is then measured in the diagonal basis.

Each detection event uniquely signals one of the four polarization Bell states and the unitary transformation U_j needed to complete the protocol. This latter step is done by relaying the detection information to the location of photon 3.

Spectral and Polarization Teleportation

Spectral considerations underlay the SFG phase-matching requirements in Kim et al. but teleportation of the spectral amplitude was otherwise neglected. We modify the complete polarization-Bell-state analyzer by spectrally resolving the up-converted photon. We show that the spectral-polarization state of photon 1can then be teleported to photon 3.

Consider the initial photons to be in multimode analogs of the conventional polarization states.

$$|\psi_1\rangle = \int \big(\alpha(\omega_1)|h_1(\omega_1)\rangle + \beta(\omega_1)|v_1(\omega_1)\rangle\big) d\omega_1$$

$$|\varphi_{23}\rangle = \int d\omega_2 \int d\omega_3 f(\omega_2,\omega_3)\Big[\big|h_2(\omega_2),v_3(\omega_3)\big\rangle + \big|v_2(\omega_2),h_3(\omega_3)\big\rangle\Big]$$

Accounting for the four possible SFG events and polarization rotations applied to the up-converted photon prior to detection, we then determine the outcomes for the four possible detection events.

As an example, detection of a horizontal photon in path B at a frequency Ω projects photon 3 into a state that resembles the initial state of photon 1.

$$|\psi_3\rangle \propto \int d\omega \int d\omega' \Big[\alpha(\Omega-\omega) f(\omega,\omega')|h_3(\omega')\rangle + \beta(\Omega-\omega) f(\omega,\omega')|v_3(\omega')\rangle\Big]$$

The joint spectral amplitude serves to transfer the spectral information of photon 1 onto photon 3. For infinite spectral entanglement, λ_n=1, the joint spectral amplitude acts as an identify operator in the frequency domain.

$$|\psi_3\rangle = \int d\omega \Big[\alpha(\omega)|h_3(\omega+\delta)\rangle + \beta(\omega)|v_3(\omega+\delta)\rangle\Big]$$

$$\delta = \omega_{\mathrm{pump}} - \Omega_{\mathrm{detected}}$$

Recovering the complete quantum state requires a shift $-\delta$ in the spectral state. The latter can be achieved using difference frequency generation (DFG), making perfect teleportation possible, in principle.

More general, the fidelity for the cw-pump limit depends explicitly on the phase-matching function.

$$F = \int d\omega \int d\omega' \Big[\alpha(\omega)\Phi(\omega,\omega'+\delta)\alpha(\omega')^* + \beta(\omega)\Phi(\omega,\omega'+\delta)\beta(\omega')^*\Big]$$

In summary, we have shown how the composite spectral-polarization state of a single photon can be teleported using a combination of sum- and difference-frequency generation, an effort that supports the possibility of teleporting the full quantum state of a single photon [6].

References

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