Spectral Effects in Quantum Teleportation

Quantum teleportation is analyzed in the context of multi-mode interference effects. The teleportation fidelity depends on the spectral relationship between the entangled photons and on the spectral overlap between the photons in the Bell-state measurement.

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Quantum Teleportation

Transfers quantum information between particles; particles 1 and 2 are brought together while particle 3 may be remote (Bennett et al., 1993).

Photon 1: unknown state

$$|\psi_1\rangle = a|h_1\rangle + b|v_1\rangle$$

Photon 2 and 3: entangled state

$$|\varphi_{23}^{(+)}\rangle = \left(|h_2, v_3\rangle + |v_2, h_3\rangle\right)/\sqrt{2}$$

Alice and Bob: A day at Long Beach

Composite system

$$|\Psi_{123}\rangle = |\psi_1\rangle|\varphi_{23}^{(+)}\rangle$$

Bell-state measurement (prob. 1/4)

$$\langle\varphi_{12}^{(+)}|\Psi_{123}\rangle \to a|h_3\rangle - b|v_3\rangle$$

Spectral Effects in Entanglement Generation

Broad bandwidth pumping of type-II SPDC in nonlinear crystals yields correlations in the joint spectral amplitude (Grice et al., 1997)

$$|\varphi_{23}\rangle = \frac{1}{\sqrt{2}} \int d\omega_2 \int d\omega_3 \left[f(\omega_2,\omega_3) |h_2(\omega_2), v_3(\omega_3)\rangle + g(\omega_2,\omega_3) |v_2(\omega_2), h_3(\omega_3)\rangle \right]$$

Spectra correlate with polarization

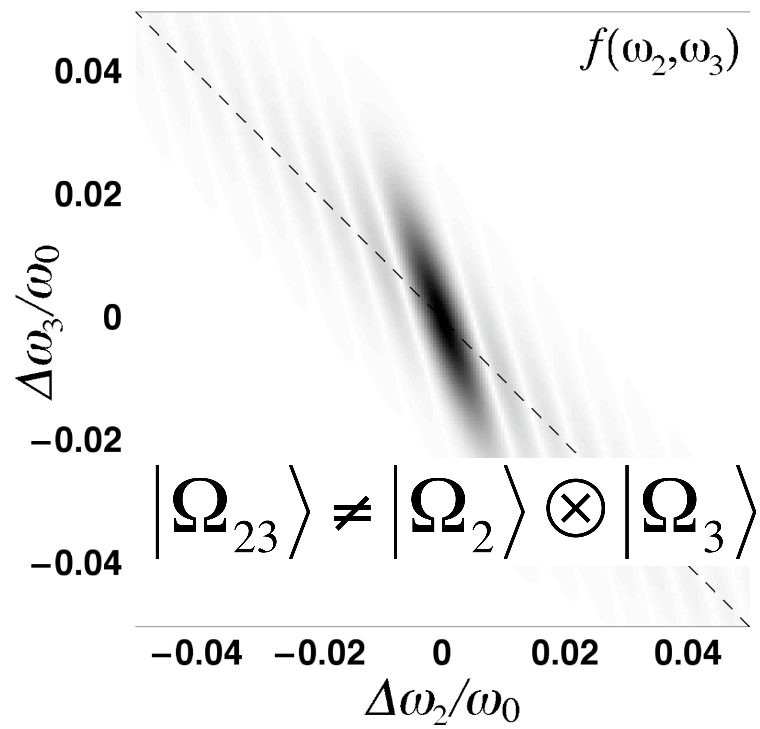
$$f(\omega_2,\omega_3) = g(\omega_3,\omega_2)$$

$$|\varphi_{23}\rangle \neq |\Omega_{23}\rangle \otimes \left(|h_2, v_3\rangle + |v_2, h_3\rangle \right) / \sqrt{2}$$

Spectra correlate with path

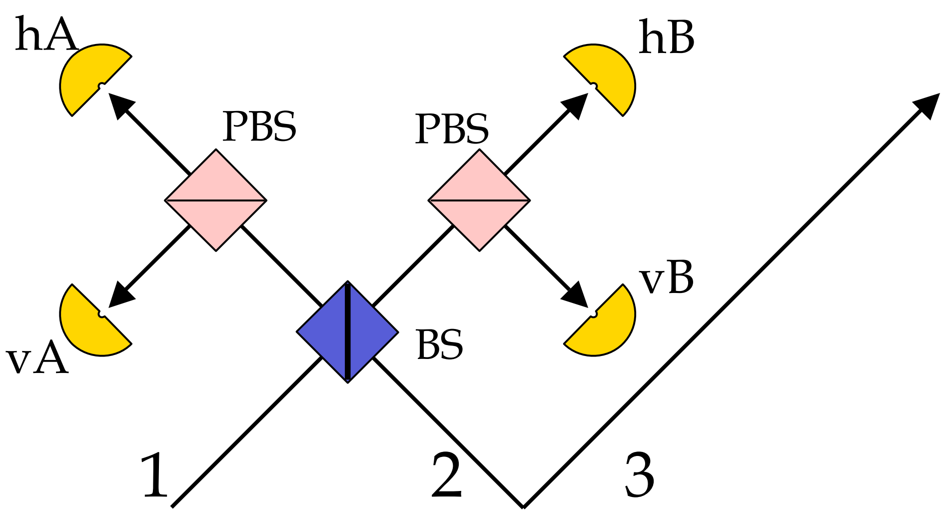
$$f(\omega_2,\omega_3) = g(\omega_2,\omega_3)$$

$$|\varphi_{23}\rangle = |\Omega_{23}\rangle \otimes \left(|h_2, v_3\rangle + |v_2, h_3\rangle \right) / \sqrt{2}$$



Spectral Effects in Interference

The (partial) Bell-state measurement employing beam splitters and detectors needs interference (Braunstein and Mann, 1995)



$$h_A(\omega) = \frac{1}{\sqrt{2}}\left(h_2(\omega) + i h_1(\omega)\right)$$

$$h_B(\omega) = \frac{1}{\sqrt{2}}\left(h_2(\omega) - i h_1(\omega)\right)$$

and similarly for $v_{A,B}$

Coincidence detection signals BSM, e.g. hA + vA

Single mode projection

$$\Pi = \left| h_A, v_A \right\rangle \left\langle h_A, v_A \right|$$

Density matrix of photon 3

$$\rho_3 = \mathrm{Tr}_{12}\left[\Pi \rho_{123}\right] \rightarrow \begin{pmatrix} |a|^2 & ab^* \\ a^* b & |b|^2 \end{pmatrix}$$

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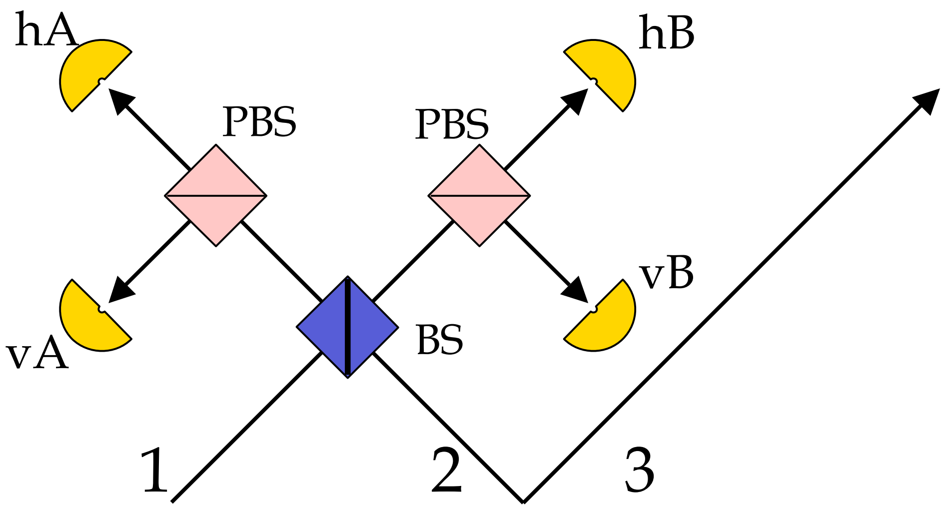

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and similarly for $v_{A,B}$

Coincidence detection signals BSM, e.g. hA + vA

Multi-mode projection

$$\Pi = \int d\omega \int d\omega' \left| h_A(\omega), v_A(\omega') \right\rangle \left\langle h_A(\omega), v_A(\omega') \right| \qquad \rho_3 = \mathrm{Tr}_{12}\left[\Pi \rho_{123}\right]$$

Spectral Effects on Teleportation Fidelity

Overlaps between the joint spectral amplitudes and the spectrum of photon 1 determine the fidelity of teleportation.

Multimode state of photon 1

$$|\psi_1\rangle = \int s(\omega)\big(a|h_1(\omega)\rangle + b|v_1(\omega)\rangle\big)d\omega$$

Polarization density matrix of particle 3

$$\tilde{\rho}_3 = \begin{pmatrix} |a|^2 & ab^* J \\ a^* bJ & |b|^2 \end{pmatrix}$$

$$J = \int d\overline{\omega}\left(\int s(\omega) f(\omega,\overline{\omega})^* d\omega\right)\left(\int s(\omega')^* g(\omega',\overline{\omega}) d\omega'\right)$$

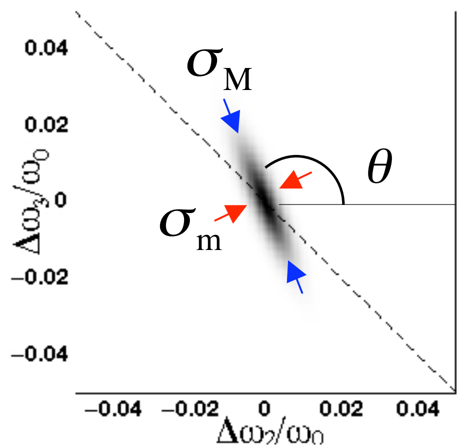
Teleportation fidelity

$$F = \mathrm{Tr}\left[\tilde{\rho}_1\tilde{\rho}_3\right] = 1 - 2|ab|^2(1-J) \qquad |a|^2 = |b|^2 = 1/2 \qquad F = \frac{1}{2}(1+J)$$

Analytical Example

Details of the spectral contribution to teleportation fidelity for the case of Gaussian spectra.

Approximate JSA



Spectrally correlated Gaussian

$$f \approx a\exp\left[-\sigma_M^2(cx+sy)^2/2 - \sigma_m^2(sx-cy)^2/2\right]$$

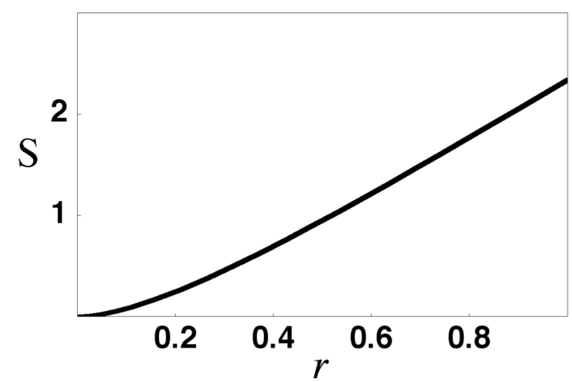
Major, minor widths $\sigma_{M,m}$

Correlation angle $c = \cos\theta, s = \sin\theta$

Difference freqs. x and y

Entanglement entropy

$$S = \cosh^2 r \log_2 \cosh^2 r - \sinh^2 r \log_2 \sinh^2 r$$



Just an alternative form

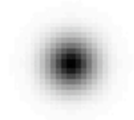
$$f \approx a\exp\left[-\left(e^{2r}(\alpha x+\beta y)^2 + e^{-2r}(\alpha x-\beta y)^2\right)/4\right]$$

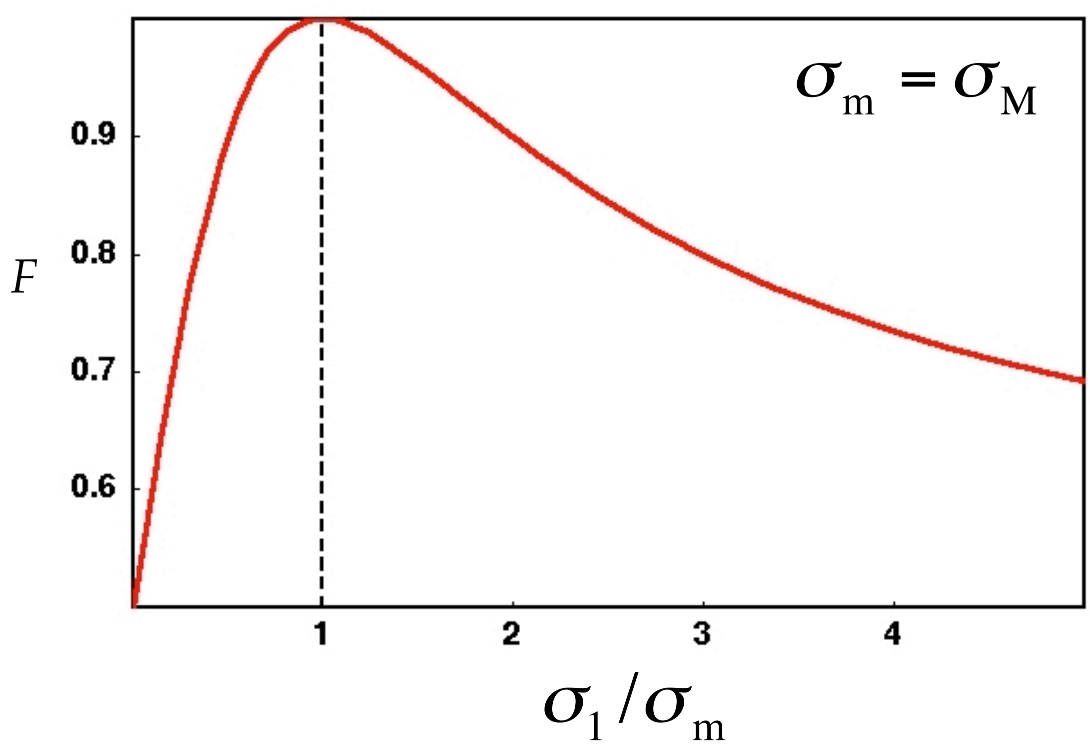
$$= \sum_{n=0}^{\infty} \lambda_n^{1/2} u_n(\alpha x) v_n(\beta y) \quad \lambda_n = \text{sech}^2 r \tanh^{2n} r$$

Teleportation Fidelity

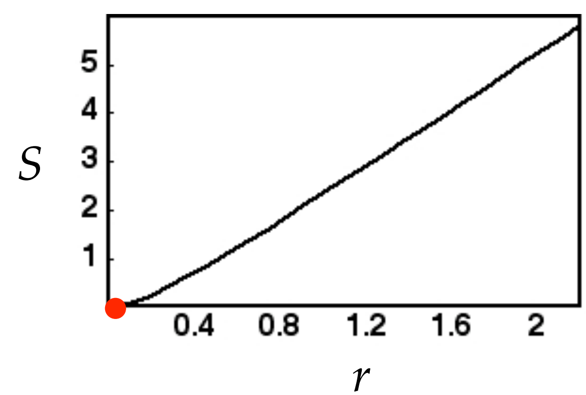
Calculated for Gaussian spectra, degenerate frequencies, and varying ratios of σ_M / σ_m <u>spectra correlates with polarization</u>

$$f(\omega,\omega') = g(\omega',\omega)$$





Spectral correlation



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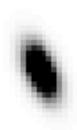

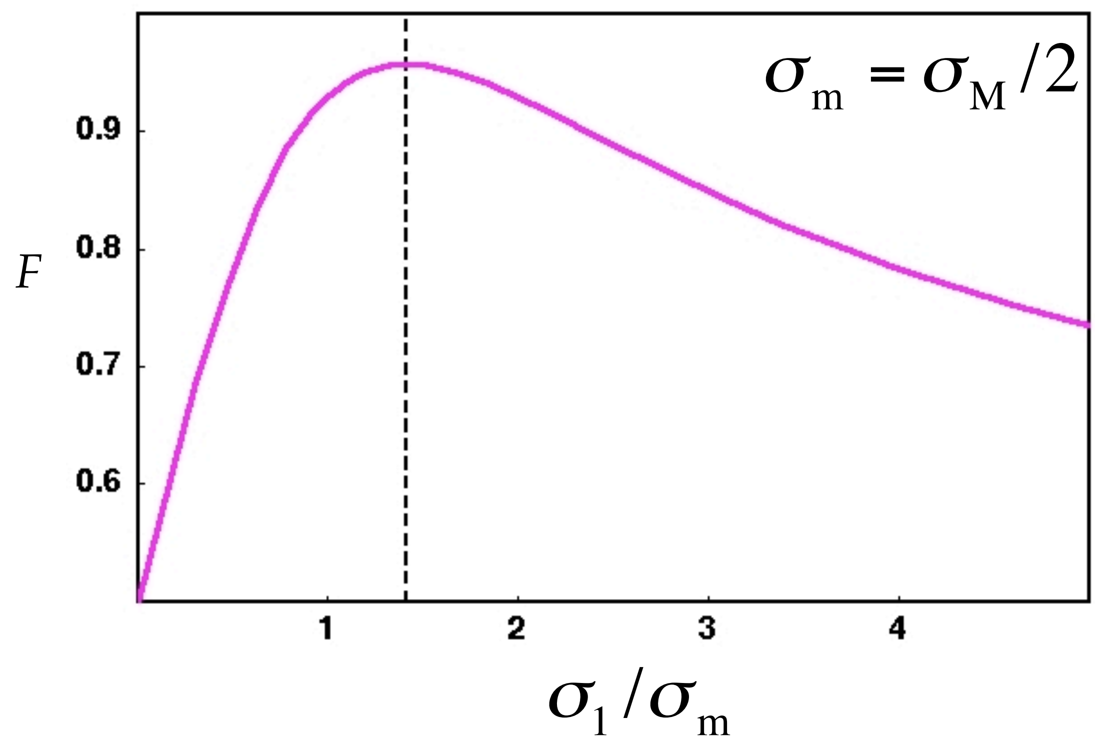

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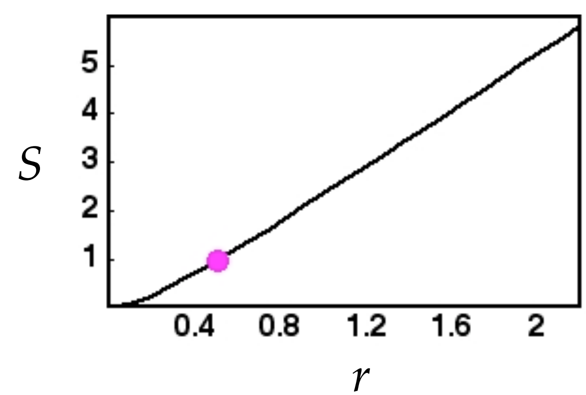

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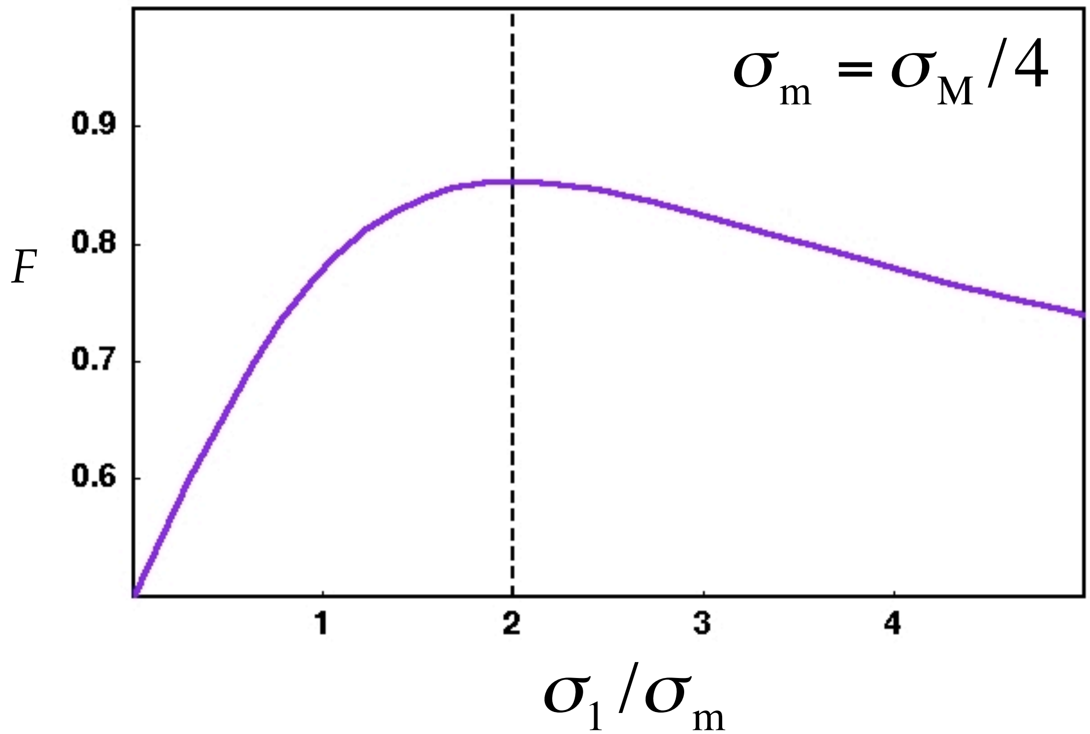

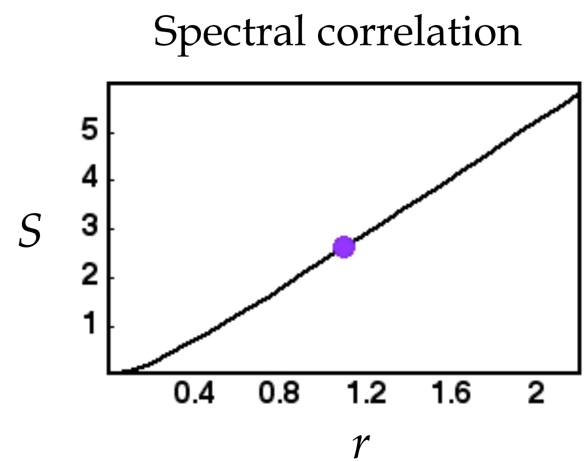

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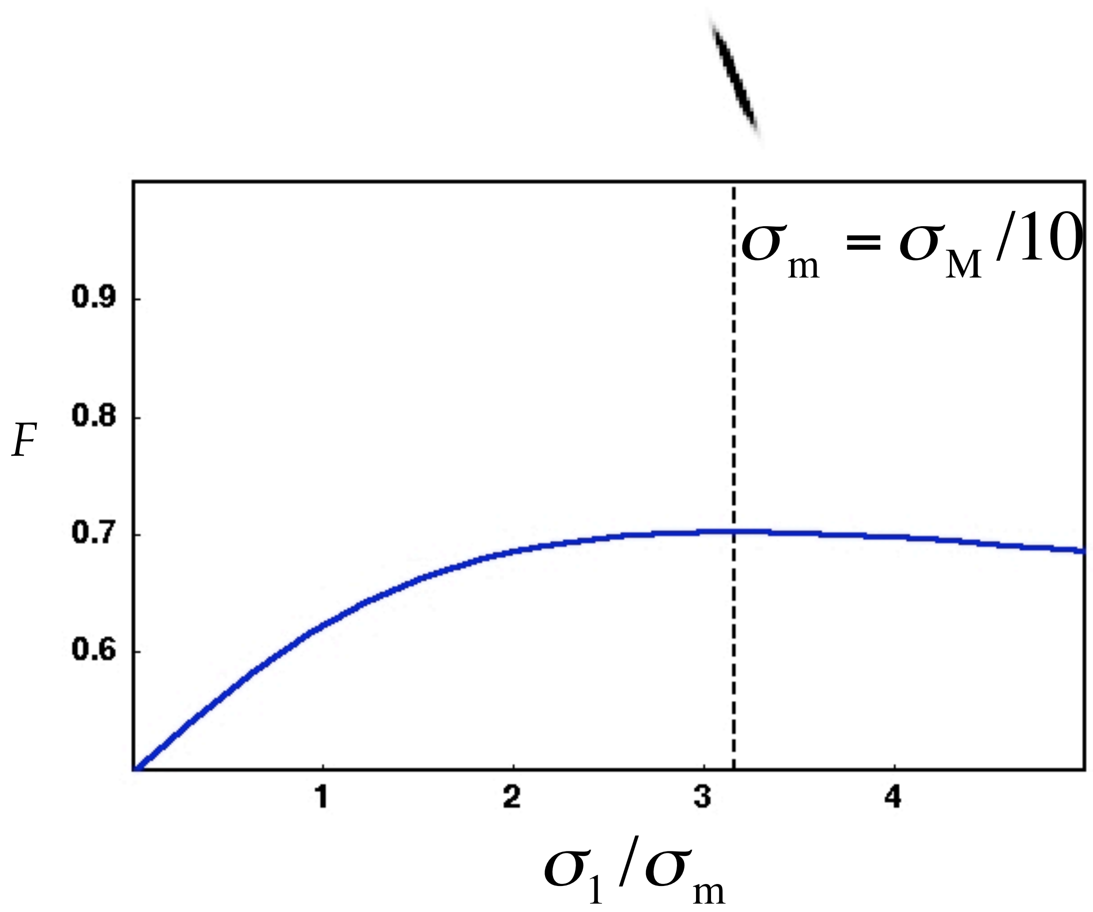
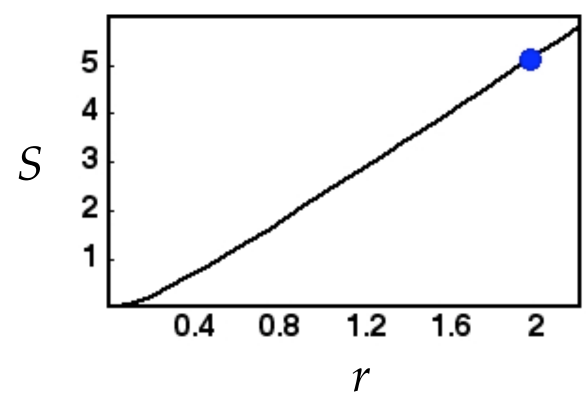


Spectral correlation

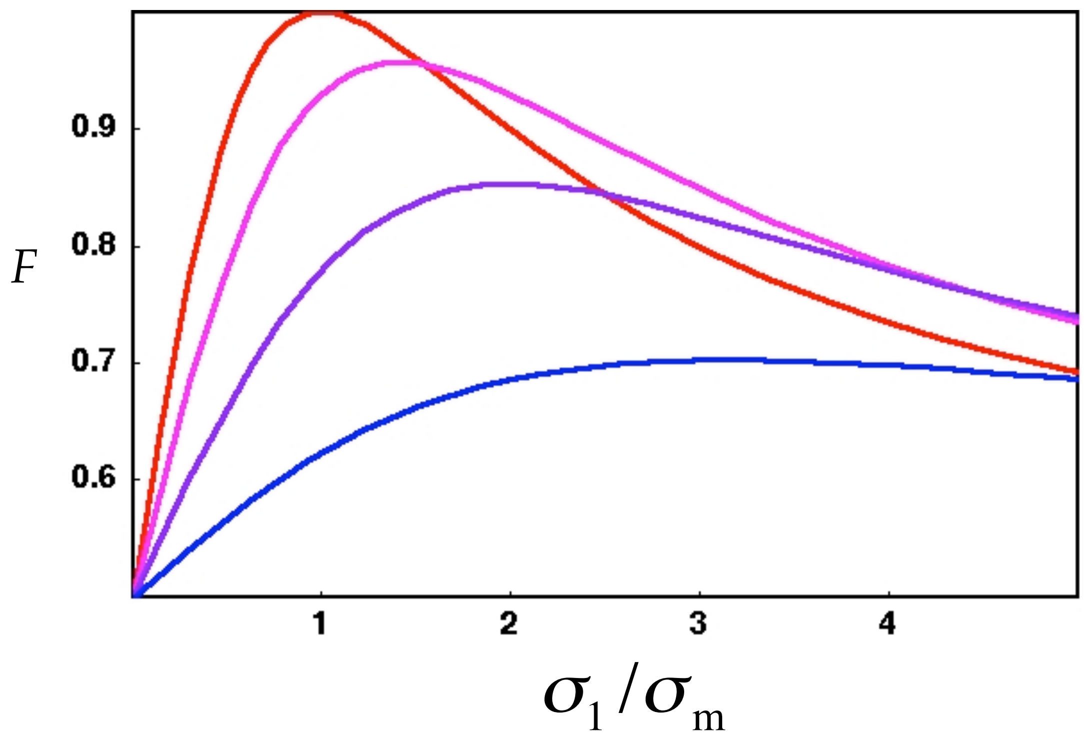


Teleportation Fidelity

Calculated for Gaussian spectra, degenerate frequencies, and varying ratios of σ_M / σ_m spectra correlates with polarization

Fidelity is maximal at $\sigma_1 = \sqrt{\sigma_M \sigma_m}$

$$f(\omega,\omega') = g(\omega',\omega)$$



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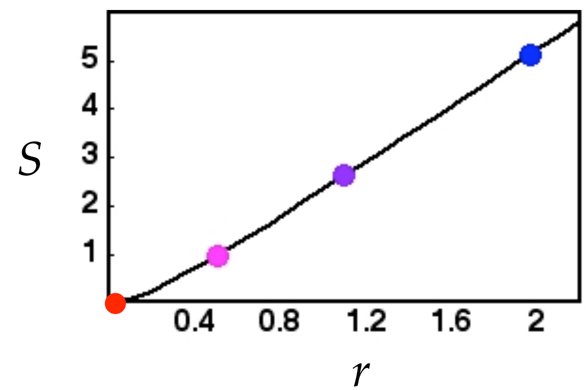
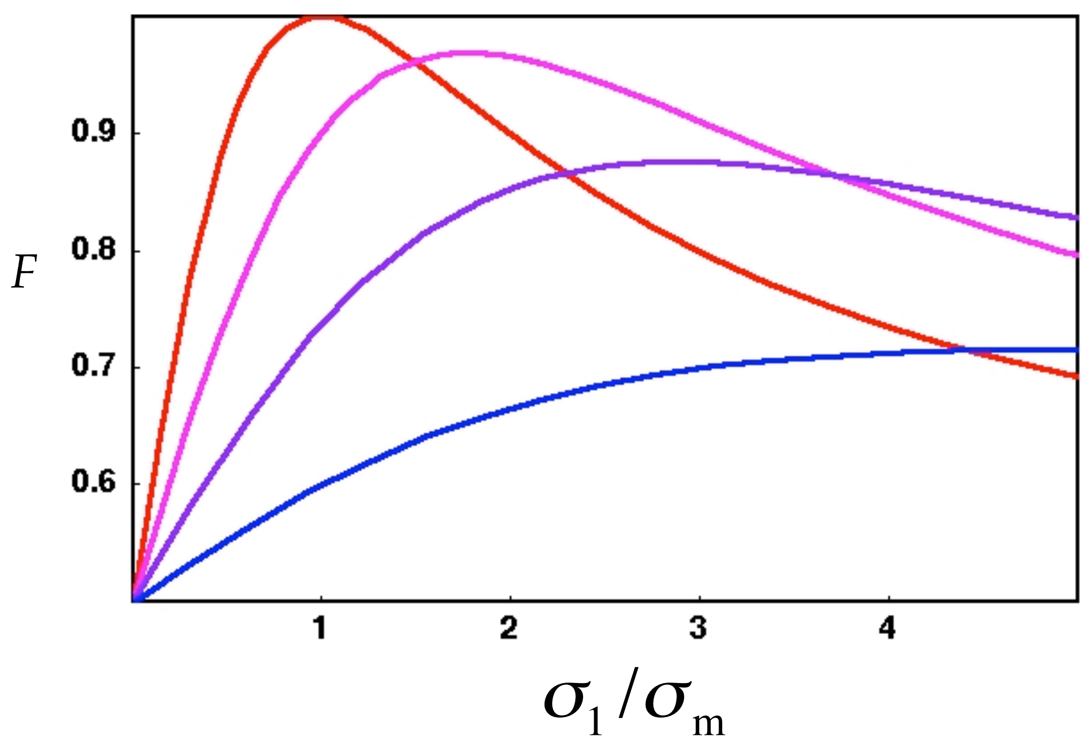


Teleportation Fidelity

Calculated for Gaussian spectra, degenerate frequencies, and varying ratios of σ_M / σ_m spectra correlates with path

$$f(\omega,\omega') = g(\omega,\omega')$$



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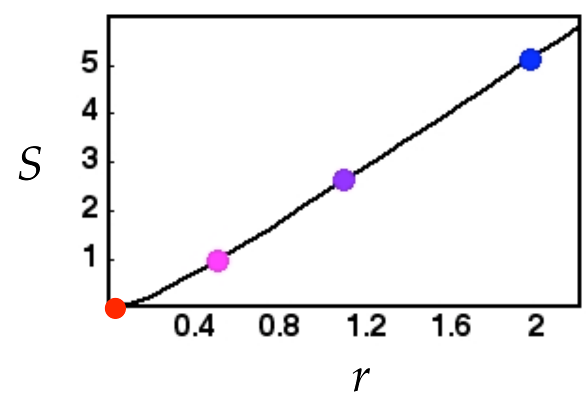
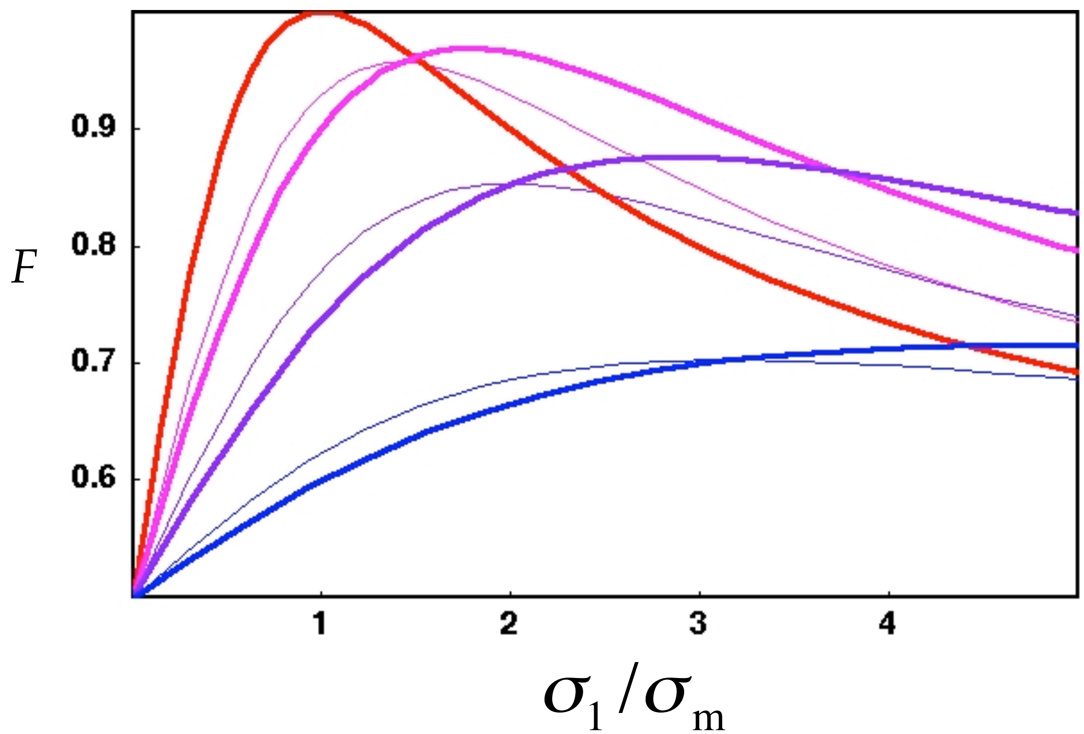


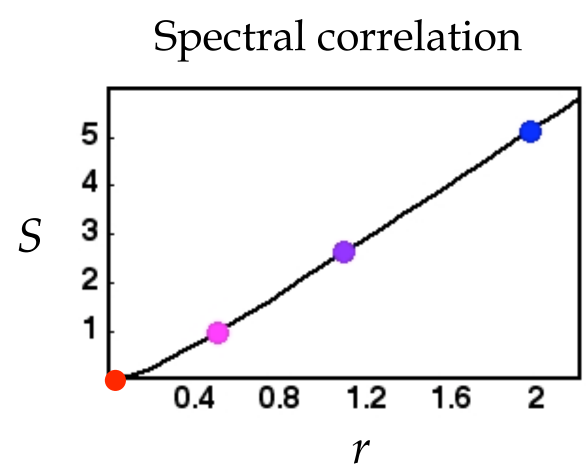
Teleportation Fidelity

Calculated for Gaussian spectra, degenerate frequencies, and varying ratios of σ_M / σ_m <u>spectra correlates with path</u>

Fidelity is maximal at $\sigma_1 = \sqrt[4]{\dfrac{\sigma_M \sigma_m \left(\sigma_M^2 c^2 + \sigma_m^2 s^2\right)}{\left(\sigma_M^2 s^2 + \sigma_m^2 c^2\right)}}$

$$f(\omega,\omega') = g(\omega,\omega')$$



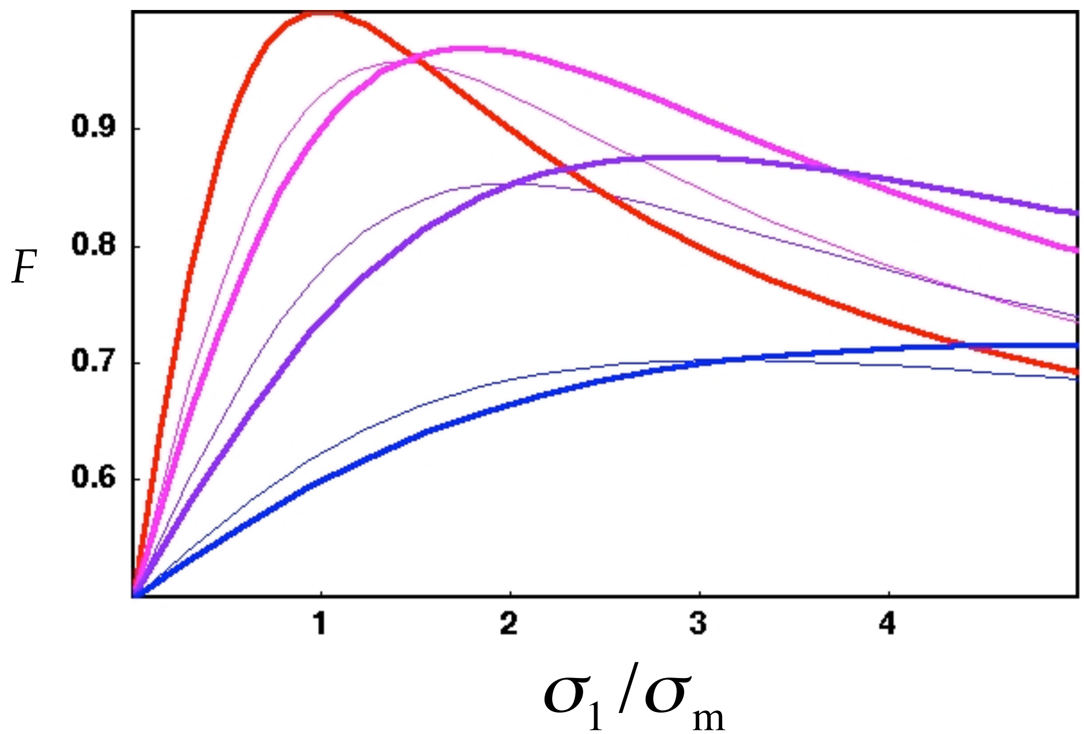


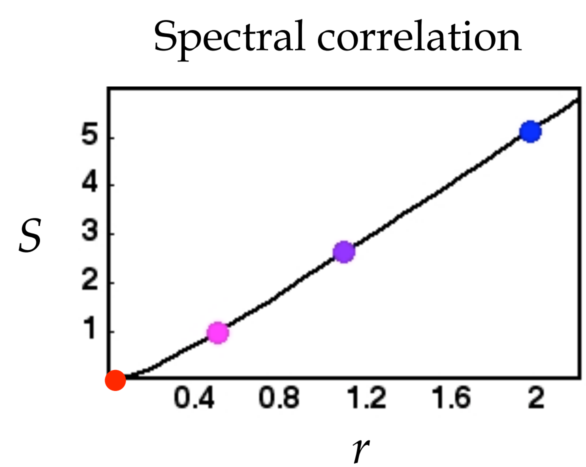
Teleportation Fidelity

Calculated for Gaussian spectra, degenerate frequencies, and varying ratios of σ_M / σ_m <u>spectra correlates with path</u>

Fidelity is maximal at $\sigma_1 = \alpha$

$$f(\omega,\omega') = g(\omega,\omega')$$





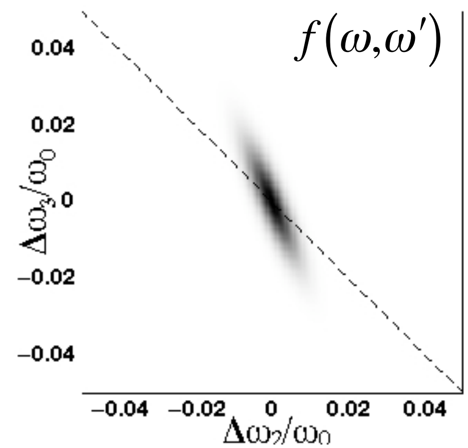
Summary

Broad bandwidth "qubits" make spectral effects important.

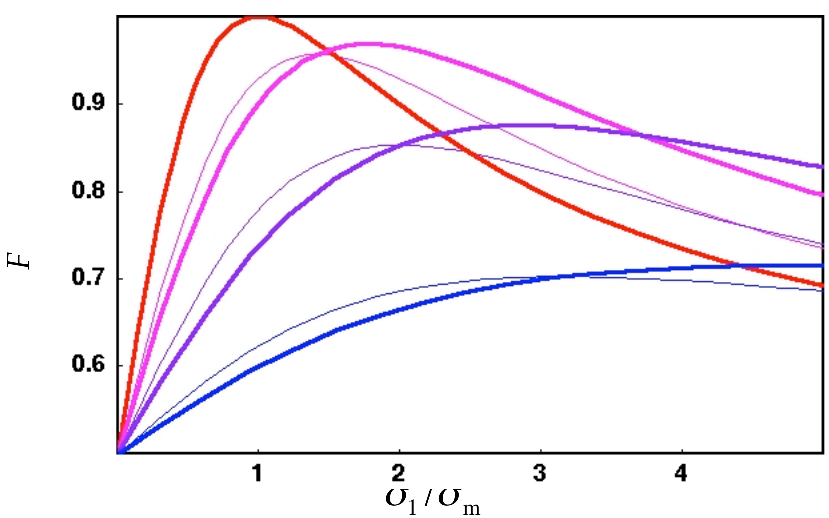
Spectral interference

$$J = \int d\overline{\omega}\left(\int s(\omega) f(\omega,\overline{\omega})^* d\omega\right)\left(\int s(\omega')^* g(\omega',\overline{\omega}) d\omega'\right)$$

Joint spectral amplitude



Teleportation Fidelity



Spectral correlation