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1. Introduction

A companion paper¹ has described the development of sensitivity coefficients as a gauge of system similarity in validation studies using sensitivity and uncertainty (S/U) techniques for criticality safety applications. An alternative approach to exploring the similarity of systems is using uncertainty analyses. This procedure involves the propagation of estimated cross-section uncertainty information to the calculated k_{eff} value of a given system via the sensitivity coefficients. Mathematically, this is accomplished by a quadratic product of the sensitivity profile vectors by isotope and reaction type with the cross-section uncertainty matrices by nuclide and reaction type. The result of this procedure is not only an estimate of the uncertainty in the system k_{eff} due to cross sections, but also an estimate of the correlated uncertainty between systems. These correlated uncertainties can be represented by correlation coefficients, which effectively represent the degree of correlation in the uncertainties between the two systems. This parameter, denoted as c_k , has not only the desirability of a single quantity relating the two systems, but the similarity of the systems is measured in terms of uncertainty, not just sensitivity. These correlation coefficients are particularly useful when used in traditional trending analyses for criticality safety validation. When used as a trending parameter in these analyses, the correlation coefficient should relate to the degree in which the uncertainties in the

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critical benchmarks are coupled with the uncertainties in the application of interest. This coupling with the common uncertainties in the various systems is expected to closely mimic the coupling in predicted biases between the various systems, since they should both be related to the cross-section uncertainties. The underlying assumption in this approach is that the cross-section processing biases are either small or they are identified and included in the analysis.

2. Uncertainty Analysis Theory

The determination of uncertainties in the calculated values of the system multiplication factor is accomplished by two steps: the estimation/processing of uncertainties in the underlying cross-section data and the propagation of those uncertainties to the system k_{eff} value. The techniques for processing cross-section uncertainty data are well-known^{2,3} and will not be discussed here.

Once uncertainty information for the cross sections for all nuclides and reaction processes that are important to the systems become available for analysis, it is then possible to estimate the uncertainty in the system multiplication factor due to these data uncertainties. When the matrix of uncertainty information for all of the cross sections is denoted as $C_{\alpha\alpha}$ and the sensitivity matrix relating changes in each constituent material and process to the system k_{eff} is labeled as S_k , the uncertainty matrix for the system k_{eff} values, C_{kk} , is given as:

$$C_{kk} = S_k C_{\alpha\alpha} S_k^H, \text{ where } ^H \text{ indicates the transpose.} \quad (1)$$

The S_k matrix is $I \times N$, where I is the number of critical systems being considered, N is the number of nuclear data parameters in the problem. Typically, N is the number of nuclide/reaction processes times the number of energy groups. The $C_{\alpha\alpha}$ matrix is an $N \times N$ matrix, with the resulting $I \times I$ C_{kk} matrix. The C_{kk} matrix consists of variance values σ_i^2 for each of the critical systems under consideration (the diagonal elements), as well as the so-called ‘‘covariance’’ between systems σ_{ij}^2 (the off-diagonal elements).

These off-diagonal elements represent the shared or common variance, hence the term covariance, between any two systems. For presentation, these off-diagonal elements are typically divided by the square root of the corresponding (same row) diagonal elements (i.e., the respective standard deviations) to generate a correlation coefficient matrix. Thus, the c_k coefficients are defined as, $c_k = \sigma_{ij}^2 / \sigma_i \sigma_j$ such that each c_k value represents the correlation coefficient between uncertainties in system i and system j . These correlations arise due to the fact that the uncertainties in the k_{eff} values for two different systems are related, since they contain the same materials. Cross section uncertainties will propagate to all systems containing these materials. Systems with the same materials and similar spectra would be correlated, while systems with different materials or differing spectra would not be correlated. The physical interpretation of the correlation matrix is the following: a value of zero represents no correlation between the systems, a value of unity represents full correlation between the systems, and a value of -1 represents a full anticorrelation.

3. Uncertainty Analysis Examples

In a previous study,⁴ a number of critical benchmarks were studied to illustrate the application of S/U procedures to criticality safety. The results from that study indicated that a c_k value greater than 0.8 represents adequate similarity of a benchmark system for validation purposes (see also Ref. 5). A k_{eff} correlation matrix for ten low-enriched uranium critical benchmarks described in Ref. 4 (i.e., six U(2)F₄, two U(3)F₄, two U(5)₃O₈) is given in Table 1. Since the diagonal elements of a correlation matrix are unity, each diagonal element in Table 1 is replaced by the corresponding fractional standard deviation in k_{eff} for that system.

Table 1. Cross-section cross-correlation coefficients^a for experiments with U enrichments of 2%, 3%, 5%

Critical system	2%(195)	2%(294)	2%(406)	2%(496)	2%(614)	2%(972)	3%(133)	3%(277)	5%(147)	5%(757)
2% (195) ^b	0.0158									
2% (294)	0.9884	0.0139								
2% (406)	0.9729	0.9896	0.0129							
2% (496)	0.9556	0.9801	0.9908	0.0122						
2% (614)	0.9269	0.9605	0.9782	0.9880	0.0116					
2% (972)	0.8019	0.8560	0.8886	0.9177	0.9499	0.0110				
3% (133)	0.8850	0.8752	0.8489	0.8341	0.8133	0.7297	0.0157			
3% (277)	0.8485	0.8564	0.8453	0.8400	0.8294	0.7666	0.8396	0.0129		
5% (147)	0.8604	0.8624	0.8467	0.8383	0.8242	0.7521	0.8606	0.8426	0.0128	
5% (757)	0.7276	0.7713	0.7961	0.8153	0.8335	0.8373	0.7137	0.7733	0.7795	0.0093

^aNote the diagonal elements give the fraction standard deviation since the diagonal correlation coefficient is unity by definition.

^bValues in parentheses represent H/X values.

The standard deviation values shown in Table 1 range from 0.93 to 1.58%. The highest uncertainties correspond to the lowest H/X values due to the fact that a harder spectrum enhances the sensitivity to the higher-energy cross sections, which are usually less well known than the thermal values. Note that the correlation coefficients, denoted as c_k , are all 0.71 or higher, indicating that most of these systems are similar to each other. Another way of looking at the 0.71 coefficient is that 71% of the variance is common to all these systems. Thus, these systems are expected to behave in a very similar manner with respect to bias determinations for the SCALE 44-group cross-section library on which these results are based.

Shown in Table 2 is a correlation matrix for fourteen U(11)O₂ artificial systems which have a range of H/X values from 0 to 1000. The trends in standard deviation are replicated here with a peak uncertainty of 1.92% for an H/X of 0, going down to 0.87 for an H/X of 1000. Looking at these values with a c_k criterion of 0.8 or greater indicating similar systems leads to conclusions nearly identical to those

based on a comparison of the sensitivity profiles (see Ref. 4). For example, the H/X of 0 system is only similar to the H/X of 3 system and then only marginally, so c_k is 0.8328. We see that for H/X values between 5 and 40, the similar systems include only the two or three neighboring systems with higher or lower H/X values. For systems with H/X values of 80 to 1000, the systems are typically similar to the nearest three or four neighboring systems.

Table 2. Cross-section cross-correlation coefficients^a for UO₂ systems with 11% enrichment

Critical system	11%-0	11%-3	11%-5	11%-10	11%-20	11%-40	11%-80	11%-200	11%-300	11%-400	11%-500	11%-600	11%-800	11%-1000
11% (0) ^b	0.0191													
11% (3)	0.8328	0.0185												
11% (5)	0.7379	0.9818	0.0188											
11% (10)	0.6011	0.9205	0.9725	0.0188										
11% (20)	0.4887	0.8409	0.9161	0.9784	0.0176									
11% (40)	0.4067	0.7562	0.8403	0.9253	0.9763	0.0151								
11% (80)	0.3428	0.6585	0.7392	0.8327	0.9094	0.9698	0.0128							
11% (200)	0.2800	0.5240	0.5888	0.6760	0.7696	0.8705	0.9526	0.0106						
11% (300)	0.2633	0.4751	0.5315	0.6115	0.7058	0.8157	0.9148	0.9832	0.0099					
11% (400)	0.2557	0.4452	0.4953	0.5687	0.6604	0.7727	0.8798	0.9668	0.9846	0.0095				
11% (500)	0.2517	0.4329	0.4688	0.5359	0.6235	0.7349	0.8453	0.9448	0.9717	0.9845	0.0091			
11% (600)	0.2490	0.4076	0.4482	0.5097	0.5927	0.7014	0.8123	0.9200	0.9543	0.9742	0.9847	0.0089		
11% (800)	0.2432	0.3755	0.4076	0.4567	0.5278	0.6265	0.7331	0.8514	0.8991	0.9330	0.9576	0.9734	0.0087	
11% (1000)	0.2353	0.3452	0.3697	0.4071	0.4652	0.5509	0.6484	0.7702	0.8277	0.8731	0.9097	0.9367	0.9752	0.0087

^aNote the diagonal elements give the fraction standard deviation since the diagonal correlation coefficient is defined as unity.

^bValues in parentheses represent H/X values.

For applications with enrichments in the 10 wt % range, a measure of the applicability of systems in the 2–5 wt % enrichment range is useful, since few benchmarks exist for intermediate enrichments. Table 3 gives a comparison of c_k values for these systems. These results indicate that the 2 and 3 wt % systems are only marginally similar to the 11 wt % systems. For systems with similar H/X values, the 2 and 3 wt % systems typically have a c_k value of only 0.75 with respect to the 11 wt % systems. For the 5 wt % systems and H/X values near to those of the 11 wt % systems, c_k values are already above 0.80, indicating similarity. Of course such a comparison is not possible for H/X values below 100, where criticality is not possible for enrichments of 2–5 wt %.

These c_k values are judged to be most appropriate for correlation with error trends in a criticality safety validation analysis because they are essentially the sensitivities of k_{eff} to the individual cross sections weighted by the cross section uncertainties. Thus, the c_k values represent the systems' similarity with respect to materials with highest sensitivity/uncertainty combination.

Table 3. Comparison of correlation coefficients^a for experiments with U enrichments of 2%, 3%, 5%, and 11%

Critical system	2%-294	2%-406	2%-496	2%-614	2%-972	3%-133	3%-277	5%-757	11%-200	11%-300	11%-400	11%-500	11%-600	11%-800
2% (294) ^b	0.0139													
2% (406)	0.9896	0.0129												
2% (496)	0.9801	0.9908	0.0122											
2% (614)	0.9605	0.9782	0.9880	0.0116										
2% (972)	0.8560	0.8886	0.9177	0.9499	0.0110									
3% (133)	0.8752	0.8489	0.8341	0.8133	0.7297	0.0157								
3% (277)	0.8564	0.8453	0.8400	0.8294	0.7666	0.8396	0.0129							
5% (757)	0.7713	0.7961	0.8153	0.8335	0.8373	0.7137	0.7733	0.0093						
11% (200)	0.7575	0.7596	0.7621	0.7599	0.7142	0.7440	0.7633	0.7892	0.0106					
11% (300)	0.7391	0.7504	0.7589	0.7634	0.7330	0.7114	0.7496	0.8170	0.9832	0.0099				
11% (400)	0.7248	0.7429	0.7561	0.7663	0.7491	0.6870	0.7383	0.8383	0.9668	0.9846	0.0095			
11% (500)	0.7116	0.7354	0.7526	0.7675	0.7622	0.6661	0.7273	0.8544	0.9448	0.9717	0.9845	0.0091		
11% (600)	0.6992	0.7274	0.7478	0.7667	0.7716	0.6478	0.7164	0.8652	0.9200	0.9543	0.9742	0.9847	0.0089	
11% (800)	0.6667	0.7027	0.7290	0.7554	0.7805	0.6052	0.6858	0.8736	0.8514	0.8991	0.9330	0.9576	0.9734	0.0087
11% (1000)	0.6282	0.6699	0.7004	0.7326	0.7747	0.5601	0.6481	0.8639	0.7702	0.8277	0.8731	0.9097	0.9367	0.9752

^aNote the diagonal elements give the fraction standard deviation since the diagonal correlation coefficient is defined as unity.

^bValues in parentheses represent H/X values.

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