APPENDIX F: 3-D Z-TRANSFER FUNCTION METHODOLOGY ERROR ANALYSIS

The z-transfer function coefficients for a sample of wall assemblies under consideration are shown below (Tables 1,2,3). What seems questionable at the first sight, are negative values of the coefficients b_n with higher indices, which appear for almost all lightweight wood- and steel-framed wall assemblies, and for the empty concrete blocks. It was necessary to admit them to satisfy, with sufficient accuracy, compatibility equations. For the steel-framed walls with heavy layers, brick or stucco, all b_n are positive. For the coefficients c_n and d_n the sign sequence is always + and - , alternately. One should take into account, however, that negative value of some b_n does not mean that the impact of a temperature value may be "negative", as temperatures enter into the expression for the current heat flux, see Equation (A9), not only through b_n and c_n, but also through the preceding values of the heat flux itself.

Table 1. Wood Stud System - Clear Walls And Details
Z-transfer Function Coefficients For Three-Dimensional Models

Wall assembly	R-value IP [SI]*)	*n*	0	1	2	3	4
2x4 clear wall	11.39127 [2.00486]	b_n	0.19337	0.24476	-0.15501	0.01954	0.00097
		c_n	7.64880	-12.33863	5.81307	-0.84437	0.02478
		d_n	1.00000	-0.91447	0.23694	-0.01887	0.00004
2x6 clear wall	17.48678 [3.07767]	b_n	0.15322	0.25518	-0.21002	0.00873	
		c_n	10.64983	-17.95135	8.12819	-0.60887	
		d_n	1.00000	-0.97292	0.19791	-0.00718	
2x4 corner	10.47675 [1.84391]	b_n	0.19209	0.20133	-0.14086	0.00992	
		c_n	6.43935	-9.92949	4.08246	-0.32742	
		d_n	1.00000	-0.93234	0.20719	-0.01098	
2x4 window Header	9.40385 [1.65508]	b_n	0.10141	0.16607	-0.01517	0.01302	
		c_n	5.05305	-7.80490	3.47950	-0.46234	
		d_n	1.00000	-0.99046	0.27744	-0.02166	
2x4 wall/floor	9.79471 [1.72387]	b_n	0.08635	0.14395	-0.00347	-0.00869	
		c_n	5.34251	-7.49327	2.44246	-0.05226	
		d_n	1.00000	-0.89680	0.13624		
2x4 wall/roof	9.40040 [1.65447]	b_n	0.09812	0.14358	-0.01307		
		c_n	7.09777	-9.93976	3.20196		
		d_n	1.00000	-0.92645	0.16872		

*) Btu/h ft^2 °F [m^2 K/W]

Table 2. Steel Stud System – Clear Walls And Details
Z-Transfer Function Coefficients For Three-Dimensional Models

Wall assembly	R-value IP [SI]*)	*n*	0	1	2	3	4	5
		b_n	0.27544	0.46780	-0.03740	-0.00348		
2x4 steel stud	8.79595	c_n	5.38342	-5.43867	0.77712	-0.01951		
Clear wall	[1.54809]	d_n	1.00000	-0.32450	0.02704	-0.00018		
		b_n	0.27182	0.23733	-0.11860	0.00796	0.00019	
2x4 corner	5.28141	c_n	3.32708	-4.25094	1.46144	-0.14221	0.00333	
	[0.92953]	d_n	1.00000	-0.73120	0.13649	-0.00664	0.00004	
		b_n	0.38340	0.40350	0.00274			
2x4 window	8.80252	c_n	2.00592	-1.25812	0.04204			
Header	[1.54924]	d_n	1.00000	-0.21374	0.00358			
		b_n	0.10985	0.12760	-0.14546	-0.01314		
2x4 wall/floor	3.79581	c_n	2.69068	-4.04151	1.51023	-0.07516		
	[0.66806]	d_n	1.00000	1.14685	0.23109			
		b_n	0.32878	0.33525	-0.08624	-0.00082		
2x4 wall/roof	2.45400	c_n	2.67848	-2.71613	0.65392	-0.03925		
	[0.43190]	d_n	1.00000	-0.48144	0.05880	-0.00035		
		b_n	0.01014	0.15042	0.11515	0.00060		
2x4 steel stud	12.79160	c_n	6.99557	-12.38192	7.07266	-1.50306	0.09593	
+1-in EPS+brick	[2.25132]	d_n	1.00000	-0.96989	0.26922	-0.02015		
		b_n	0.20008	0.48490	-0.02034	-0.00691		
2x6 steel stud	11.31363	c_n	6.90969	-7.48203	1.26283	-0.03276		
Clear wall	[1.99120]	d_n	1.00000	-0.37815	0.03605	-0.00017		
		b_n	0.10508	0.34909	0.01933			
2x6 steel stud	15.15703	c_n	8.12097	-11.41045	4.10657	-0.33939	0.00108	
+ EPS + stucco	[2.66764]	d_n	1.00000	-0.57984	0.05862			
		b_n	0.00820	0.14244	0.12483	0.00222		
2x6 steel stud	15.46532	c_n	8.28614	-14.88340	8.59382	-1.82738	0.10926	-0.00005
+ EPS + brick	[2.72190]	d_n	1.00000	-0.97089	0.26764	-0.01836		

*) Btu/h ft^2 °F [m^2 K/W]

Table 3. Concrete And Insulation Wall Assemblies
Z-Transfer Function Coefficients For Three-Dimensional Models

Wall assembly	R-value IP [SI]*)	*n*	0	1	2	3	4
ICF – wall	11.23044 [1.97768]	b_n	0.00333	0.01647	0.00388	0.00022	
		c_n	12.44518	-24.90756	14.40928	-1.92311	0.00012
		d_n	1.00000	-1.16082	0.18473		
Sandwich wall with metal ties	7.65912 [1.34800]	b_n	0.02377	0.24051	0.14861	0.00778	
		c_n	35.22708	-49.09367	15.08613	-0.79797	
		d_n	1.00000	-0.70113	0.12272	-0.00002	
Sandwich wall with plastic ties	10.58216 [1.86246]	b_n	0.01576	0.22377	0.16632	0.00908	
		c_n	50.74143	-71.97368	23.01010	-1.36291	
		d_n	1.00000	-0.71157	0.12653	-0.00002	
Empty concrete blocks	1.35549 [0.23857]	b_n	0.19225	0.15877	-0.15272	0.03684	
		c_n	5.98628	-9.65447	4.39335	-0.48275	
		d_n	1.00000	-0.91110	0.15573	-0.00234	
Insulated concrete blocks	2.29137 [0.40328]	b_n	0.04707	0.13063	0.01402	0.03217	0.00389
		c_n	9.72126	-16.99955	8.85120	-1.39633	0.05121
		d_n	1.00000	-0.98904	0.23027	-0.01356	0.00012

*) Btu/h ft^2 °F [m^2 K/W]

As an example, let's consider 2x4 wood stud clear wall (see Table 1). Summing the z-transfer function coefficients gives:

$$\sum_{n=0}^{4} b_n = 0.30364\,, \qquad \sum_{n=0}^{4} c_n = 0.30365\,, \qquad \sum_{n=0}^{4} d_n = 0.30365\,; \qquad E_r = -0.000017$$

The error E_1 is here very small. For most wall assemblies considered it's below 0.01, for several cases below 0.00001, however for two cases, wood stud wall/roof and steel stud wall/floor intersection, it reaches the value of 0.06.

Solving back equations (5) with respect to Y_n, X_n, gives the recurrence formulae, which may be used to additionally verify the solution obtained for the z-transfer function coefficients b_n, c_n, d_n; Equations (A17), (A18). Results of the "reversibility test", are presented in Table 4 and show very good compatibility of the response factors reproduced from the z-transfer function coefficients with those originally calculated for the three-dimensional model. This indicates the accuracy of 3-D the z-transfer function coefficients, derived using the method presented above, is determined mainly by accuracy of the response factors computations.

Table 4. Comparison of the original dimensionless 3D response factors with those reproduced from the z-transfer function coefficients – for 2x4 wood stud wall

	Dimensionless·X response factors			*Dimensionless Y response factors*		
n	***Reproduced from c_n, d_n***	***$R{\cdot}X_n$ original values***	***$R{\cdot}\Delta$ Xn***	***Reproduced from b_n, d_n***	***$R{\cdot}Y_n$ original values***	***$R{\cdot}\Delta$ Yn***
0	7.6488000	7.6488030	-0.0000027	0.1933700	0.1933737	-0.0000037
1	-5.3440320	-5.3440460	0.0000146	0.4215911	0.4215949	-0.0000039
2	-0.8861932	-0.8878742	0.0016811	0.1847053	0.1847107	-0.0000054
3	-0.2442194	-0.2442498	0.0000304	0.0922046	0.0922092	-0.0000046
4	-0.0897458	-0.0897582	0.0000124	0.0494752	0.0494763	-0.0000011
5	-0.0406983	-0.0406981	-0.0000002	0.0268640	0.0268632	0.0000008
6	-0.0205236	-0.0205199	-0.0000036	0.0145756	0.0145795	-0.0000038
7	-0.0108082	-0.0108069	-0.0000013	0.0078935	0.0078996	-0.0000061
8	-0.0057850	-0.0057861	0.0000010	0.0042696	0.0042756	-0.0000060
9	-0.0031149	-0.0031169	0.0000020	0.0023080	0.0023128	-0.0000048
10	-0.0016808	-0.0016829	0.0000020	0.0012473	0.0012508	-0.0000035
11	-0.0009077	-0.0009094	0.0000017	0.0006740	0.0006764	-0.0000024
12	-0.0004904	-0.0004916	0.0000012	0.0003642	0.0003657	-0.0000016
13	-0.0002649	-0.0002658	0.0000009	0.0001968	0.0001978	-0.0000010
14	-0.0001431	-0.0001437	0.0000006	0.0001063	0.0001069	-0.0000006
15	-0.0000773	-0.0000777	0.0000004	0.0000574	0.0000578	-0.0000004
16	-0.0000418	-0.0000420	0.0000002	0.0000310	0.0000313	-0.0000002
17	-0.0000226	-0.0000227	0.0000001	0.0000168	0.0000169	-0.0000001
18	-0.0000122	-0.0000123	0.0000001	0.0000091	0.0000091	-0.0000001
19	-0.0000066	-0.0000066	0.0000001	0.0000049	0.0000049	-0.0000001
20	-0.0000036	-0.0000036	0.0000000	0.0000026	0.0000027	-0.0000000
	Mean standard deviation σ_X = 0.000367			Mean standard deviation σ_Y = 0.0000032		