RELATIONS BETWEEN STRUCTURAL AND DYNAMIC THERMAL CHARACTERISTICS OF BUILDING WALLS *

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Abstract: This study analyzes the effect of internal thermal structure on the dynamic characteristics of walls. It introduces the idea of structure factors and shows the conditions they impose on response factors. Simple examples of multilayer walls, representing different types of thermal resistance and capacity distribution, are analyzed to illustrate the general relations between structure factors and response factors. This study presents the idea of the "thermally equivalent wall," a plane multilayer structure with dynamic characteristics similar to those of a complex structure in which three-dimensional heat flow occurs.

1. RELATIONSHIPS BETWEEN RESPONSE FACTORS AND THERMAL STRUCTURE FACTORS FOR WALLS

The reason for studying relationships between the structural and dynamic thermal characteristics of building walls was the following problem: How should the response factors (or transfer function coefficients) for the plane walls used in computerized energy calculations be modified to account for the effects of thermal bridges.

The simplest method, of course, is simply to calculate the overall resistance of a wall with imperfections—solving the steady state heat transfer problem—and multiply response factors by the resulting correction factor. This approach is suitable for light walls for which storage effects are insignificant. However, calculating response factors separately for a variety of wall elements with thermal bridges and then including them in existing programs would be troublesome. A method is needed that is simple but sufficiently accurate.

Imperfections in plane walls not only change the resistance of the walls but also modify their dynamic properties, which in simulations may be represented by response factors. To account for this effect, the general conditions between structural and dynamic characteristics for walls must be noted. Such conditions follow from the asymptotic formulae for the heat flow across the surfaces of the separated wall element due to temperature difference on its two sides.

Consider the heat flow through an element of a building envelope of complex material and geometrical structure, embedded in a plane wall, homogeneous in every cross section, parallel to the wall surfaces. It is assumed that the thermophysical properties of the structure—thermal conductivity k, density ρ and specific heat c—are constant in time. The element, together with its nearest neighborhood, is represented by the region D. Region D is bounded by the inner surface facing room temperature T; the outer surface facing environmental temperature T; and the

adiabatic surface of the cut which separates it from those parts of the wall where the heat flow can be considered as one-dimensional.

Let $\theta(\mathbf{r})$ be the so-called reduced temperature for the steady-state heat conduction problem in D, that is, the dimensionless solution of the steady-state heat conduction equation for the ambient temperatures $T_i = 0$ and $T_e = 1$. For a plane wall, the role of function θ is played by the function R_{i-x}/R_T [1, 2, 3, and 4], where R_T is the total resistance for heat transmission through a wall and R_{i-x} is the resistance from the point x in a wall to the internal environment.

For T_i and T_e constant for time $t > 0$ and zero initial conditions, the asymptotic expressions for the total heat flow across the inner and outer surface in the direction of the outside normal, Q_{ni} and Q_{ne}, are as follows:

$$Q_{ni}(t) \Rightarrow \frac{t}{R_T}\left[T_e - T_i\right] - T_i\, C\varphi_{ii} - T_e\, C\varphi_{ie} \qquad (1)$$

$$Q_{ne}(t) \Rightarrow \frac{t}{R_T}\left[T_i - T_e\right] - T_i\, C\varphi_{ie} - T_e\, C\varphi_{ee} \qquad (2)$$

where R_T is the total thermal resistance of the element calculated for the steady state heat flow, C is the total thermal capacity. The quantities φ_{ii}, φ_{ee}, and φ_{ie} are given by

$$\varphi_{ii} = \frac{1}{C}\int_D dv\rho\, c\,(1-\theta)^2 \quad , \qquad \varphi_{ee} = \frac{1}{C}\int_D dv\rho\, c\,\theta^2 \quad , \qquad (3)$$

$$\varphi_{ie} = \frac{1}{C}\int_D dv\rho\, c\,\theta(1-\theta) \quad , \qquad (4)$$

The dimensionless quantities φ_{ii}, φ_{ie}, and φ_{ee} , together with total resistance R_T and capacity C, constitute the basic thermal characteristics of the specific wall element, which can be determined experimentally in the heat transfer processes with steady initial and final states of heat flow [1, 5]. These characteristics are called the thermal structure factors. In a plane wall, they depend directly on the wall's thermal structure, determined by the capacity and resistance profiles along its thickness. For an element in which three-dimensional heat flow occurs, this dependence is indirect, through the reduced temperature distribution. To calculate thermal structure factors effectively, as well as total resistance, the steady state heat transfer problem must be solved.

General rules concerning the magnitudes of the structure factors may be deduced immediately from the form of the integral expressions (3) and (4). Bear in mind that generally, given a steady state of heat flow through an element composed of different materials, significant temperature slope is observed in regions of small conductivity. In regions of high conductivity, temperature is rather stable.

The quantity φ_{ii} is comparatively large if most of the thermal mass is located near the interior surface of an element and most of the resistance belongs to the outer part located near the exterior surface; the opposite holds for φ_{ee}. The upper limit of φ_{ii} and φ_{ee} is 1, the lower limit is 0. For elements with internal symmetry planes, $\varphi_{ii} = \varphi_{ee}$. The quantity φ_{ie} is given as the integral over the volume of an element of the expression $\theta(1-\theta)$ multiplied by $\rho c/C$. $\theta(1-\theta)$ attains its maximum of 1/4 at $\theta = 1/2$. Therefore, φ_{ie} attains its maximum, equal to 1/4, for the case of “the whole thermal mass of negligible resistance in the center” and “the whole resistance distributed symmetrically on both sides of it.” For a homogeneous wall, $\varphi_{ii}=\varphi_{ee}=1/3$, $\varphi_{ie}=1/6$.

Structure factors of multilayer walls are affected by the differentiation of the thermal parameters of individual layers and their arrangement. This can be illustrated by a wall constructed in six different ways of two layers of heavyweight concrete (k = 1.73 W/mK, ρ = 2240 kg/m^3, c= 0.838 kJ/kgK) and two layers of insulation (k = 0.043 W/mK, ρ = 91 kg/m^3, c = 0.838 kJ/kgK) of the same thickness (Fig. 1).The structure factors for such a wall are shown in Table 1. The results for the homogeneous wall are added for comparison.

In computer simulations of energy usage in various building designs, heat flow rates across wall surfaces are modeled using the response factors. A response factor with the number m represents the response of a linear system to the unit triangular temperature pulse with the base width 2Δ, at the discrete time moment $m\Delta$.

Let $H_{ii}(m\Delta)$, $H_{ee}(m\Delta)$ and $H_{ie}(m\Delta)$ denote the normalized response factors corresponding to the three different heat transfer modes. The heat flow rates $\dot{Q}_{ni}(n\Delta)$ and $\dot{Q}_{ne}(n\Delta)$ across the internal and external surfaces of the element in the directions of the outside normals, as functions of the room and environmental temperature history, are represented in terms of the response factors as follows:

$$\dot{Q}_{ni}(n\Delta) = \frac{1}{R_T}\left\{\sum_{m=0}^{n-1} T_e\left[(n-m)\Delta\right]H_{ie}(m\Delta) - \sum_{m=0}^{n-1} T_i\left[(n-m)\Delta\right]H_{ii}(m\Delta)\right\} \tag{5}$$

$$\dot{Q}_{ne}(n\Delta) = \frac{1}{R_T}\left\{\sum_{m=0}^{n-1} T_i\left[(n-m)\Delta\right]H_{ie}(m\Delta) - \sum_{m=0}^{n-1} T_e\left[(n-m)\Delta\right]H_{ee}(m\Delta)\right\} \tag{6}$$

For the asymptotic compatibility of Eq. (5) and Eq. (6) with the steady state heat flow solution, response factors must satisfy the condition

$$\sum_{m=0}^{\infty} H_{ii}(m\Delta) = \sum_{m=0}^{\infty} H_{ie}(m\Delta) = \sum_{m=0}^{\infty} H_{ee}(m\Delta) = 1 \tag{7}$$

Another set of conditions, derived in Eqs. (1) and (2), follows from the compatibility of (5) and (6) with the asymptotic formulae (1) and (2):

$$-\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ii}(m\Delta) = \varphi_{ii} \quad , \qquad -\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ee}(m\Delta) = \varphi_{ee} \tag{8}$$

$$\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ie}(m\Delta) = \varphi_{ie} \tag{9}$$

Equations (8) and (9) represent the relationships between the dimensionless structural and dynamic thermal characteristics of the wall: φ_{ii}, φ_{ie}, and φ_{ee}, the ratio of the time interval Δ and time constant $R_T C$, and the dimensionless normalized response factors $H_{ii}(m\Delta)$, $H_{ee}(m\Delta)$, and $H_{ie}(m\Delta)$.

The conclusions that may be derived immediately from Eqs. (8) and (9) concerning the effect of structure factors on response factors are the following [1, 5]. Response factors $H(m\Delta)$, with $m \geq 1$, describe the storage effects—heat fluxes after the duration of the triangular temperature impulse. A large value for the structure factor, corresponding to a given heat flux response mode, indicates that response factors with the number $m \geq 1$ are comparatively large; a small value for the structure factor indicates that they are comparatively small. However, Eqs. (8)

and (9) must be satisfied simultaneously with Eq. (7), which states that the sum of all response factors must be equal to 1. Therefore, the larger the values of $H(m\Delta)$ for $m \geq 1$, the smaller the value of $H(0)$, and vice versa. Taking into account that $H(m\Delta)$ should be relatively smooth functions of the number m, response factors corresponding to small values of structure factors can be expected to decay relatively quickly, whereas those corresponding to large values of structure factors can be expected to decay relatively slowly.

For exterior building walls, the values of φ_{ie}, which affect their thermal stability with re to ambgardient temperature variations, are most important.

Figure 1. *Different types of four-layer walls composed of concrete and insulation*

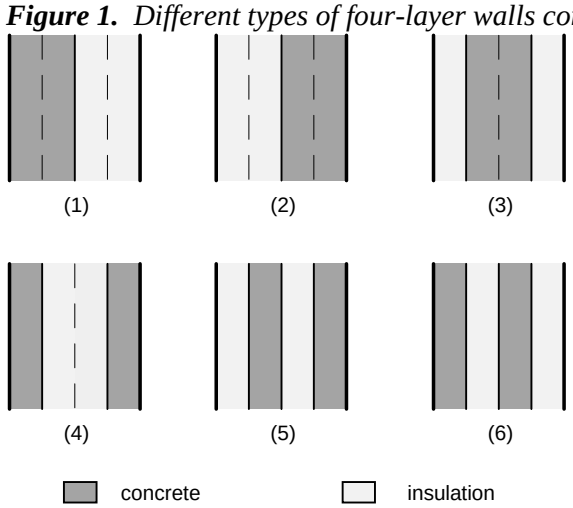

Table 1. *Thermal structure factors for different types of four-layer walls represented in Fig.1*

Wall	φ_{ie}	φ_{ii}	φ_{ee}
1	0,018	0,950	0,014
2	0,018	0.014	0,950
3	0,247	0,253	0,253
4	0,012	0,488	0,488
5	0,130	0,605	0,136
6	0,130	0,136	0,605
homog.	0,167	0,333	0,333

To demonstrate the effect of structure on the dynamic properties of a plane wall, normalized response factors $H_{ie}(n\Delta)$ were calculated for a set of symmetric three-layer walls with thicknesses in the ratio 1:2:1; with the same resistance R, capacity C, and thermal diffusivity common to all layers, but with a different ratio of resistance and capacity between the inner and outer layers (Fig.2). Time constant RC = 50 h, Δ=1h. The values of structure factor φ_{ie}, decrement factor df, and time lag τ of the heat flux for harmonic oscillations in a 24 h time period are collected in Table 2. (The quantity df is defined as the ratio of the amplitudes of the heat flux harmonic and quasistationary oscillations).

The plots of response factors in Figure 2 clearly illustrate that structure factors have an essential influence on the dynamic thermal behavior of a wall. Walls characterized by small

values of the structure factor φ_{ie} transfer thermal responses comparatively quickly, whereas those with large φ_{ie} values delay thermal responses. The response (in the form of the heat flux at the surface) to a thermal impulse at the opposite surface is comparatively large and increases and disappears comparatively quickly in the case of a wall with φ_{ie} close to zero. In the case of a wall with φ_{ie} close to the maximum possible value of 1/4, the response is smaller and decreases slowly. However, it increases more quickly for this case than for a homogeneous wall. At the same time, the damping effects of the harmonic heat flux oscillations increase with φ_{ie} (however, the time lag τ has a maximum below φ_{ie}=1/4).

Table 2. *Decrement factors and time lags for symmetric three-layer walls of different structure factors* φ_{ie}; *RC=50 h*

φ_{ie}	*df*	τ [h]
0.011	0.998	0.530
0.040	0.972	1.989
0.110	0.777	5.129
0.167	0.562	6.751
0.210	0.419	6.922
0.248	0.301	5.143

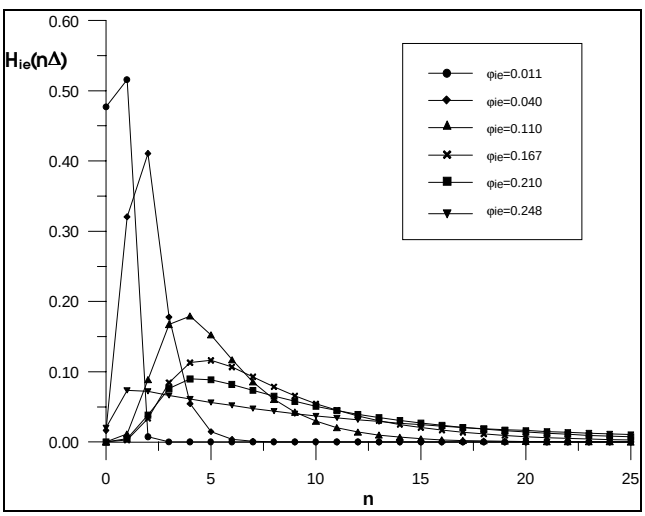

Figure 2. *Normalized response factors $H_{ie}(n\Delta)$ for three-layer walls of RC=50h, the same thermal diffusivity and different structure factors*

2. THE "THERMALLY EQUIVALENT WALL" CONCEPT

Thermal structure factors φ_{ii}, φ_{ie}, and φ_{ee} defined by the integrals Eqs. (3) and (4), together with total thermal resistance R_T and capacity C, determine to a great extent the dynamic thermal properties of a wall element through the conditions [Eqs. (8) and (9)] they impose on

response factors. However, those conditions do not determine the response factors in a unique way, but rather act as constraints. Nevertheless, it might be expected that walls with the same total thermal resistance, capacity and structure factors, would also have similar dynamic characteristics—response factors—even if they are quite different in details. This similarity leads to the concept of the "thermally equivalent wall" [5], a simple structure that has the same type of dynamic thermal behavior as a more complex one and thus may be substituted for it in building design energy use simulations.

To demonstrate the possibility of replacing a complex structure with a simple, thermally equivalent plane wall, an example was analyzed of a cuboid element, symmetrical with respect to its central plane, composed of heavyweight concrete and expanded polystyrene foam (Fig. 3). The central part of the element constitutes the thermal bridge in the three-layer wall, which is here called the "primary wall."

Table 3. *Static thermal characteristics of the wall element with a thermal bridge of the equivalent wall and primary wall (represented in Fig. 3)*

STRUCTURE	**R/A** [m^2K/W]	**C/A** [kJ/m^2K]	φ_{ie}	R_1/R	C_1/C
Thermal bridge	0.5902	269.909	0.10628	- - -	- - -
Equivalent wall	0.5902	269.909	0.10628	0.23676	0.47410
Primary wall	1.6216	307.409	0.02388	0.04776	0.49759

Static thermal characteristics of the element are given in Table 3; normalized response factors $H_{ie}(n\Delta)$, at $\Delta = 1$ h, are presented in Fig. 4. R_1 and C_1 in Table 3 denote the resistance and capacity of the outer parts of the structures, with a thickness of 4 in. A is the area of the transverse cross-section.

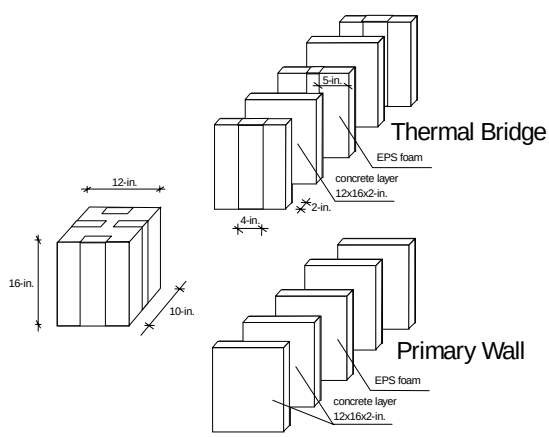

Figure 3. *The wall element representing a composition of two different types of concrete and insulation arrangement*

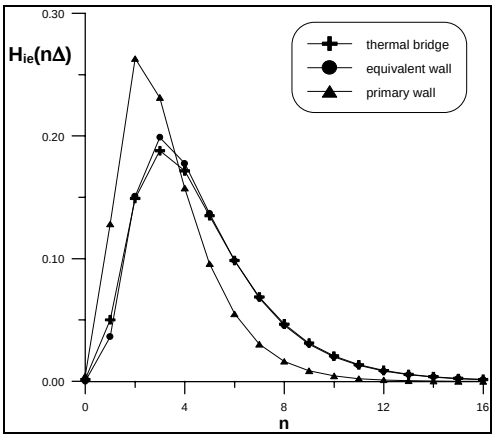

Figure 4. *Normalized response factors $H_{ie}(n\Delta)$ for the wall element with a thermal bridge represented in Fig. 3*

The essentially different arrangement of thermal mass and resistance in the thermal bridge region, compared with the primary wall, causes significant variation of the structure factor φ_{ie}. The structure factor is increased as a result of the translocation of thermal mass to the center and resistance to the outer parts of the wall. The course of the normalized response factors $H_{ie}(n\Delta)$ is also significantly modified; they decay more slowly with n. What is important is that the response factors for the equivalent wall, which is more nearly homogeneous than the primary wall, are almost identical with those for the wall element with the thermal bridge. Therefore, substituting the response factors of the equivalent wall for those of the element with the thermal bridge provides a much better approximation than does multiplying the response factors of the primary wall by the correction factor, to take into account just the change of its resistance.

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