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**AUTOMATIC VARIANCE REDUCTION FOR MONTE CARLO SHIELDING
CALCULATIONS WITH THE DISCRETE ORDINATES ADJOINT FUNCTION**

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ABSTRACT

Although the Monte Carlo method is considered to be the most accurate method available for solving radiation transport problems, its applicability is limited by its exorbitant computational expense. Thus, variance reduction techniques, which require intuition, guess work, and iterations involving manual adjustments, are employed to make reactor calculations feasible. Responding to this difficulty, we have developed methodology for automatic variance reduction of Monte Carlo shielding calculations using the discrete ordinates adjoint function for source biasing and consistent transport biasing with the weight window technique. We describe the implementation of this method into the standard *production* Monte Carlo code MCNP, and its application to a realistic calculation; namely, the reactor cavity dosimetry calculation. The effectiveness of the method, as demonstrated through the increase in calculational efficiency, is quantified. Moreover, important issues associated with this method and its efficient use are discussed. The increases in performance associated with this method can substantially increase the practical applicability of the Monte Carlo method for large *real-world* applications.

I. INTRODUCTION

In the field of nuclear engineering, deterministic (Discrete Ordinates) and stochastic (Monte Carlo) methods are most often used to solve shielding type problems. As one might expect, each method has its own strengths and weaknesses. In general, Monte Carlo (MC) methods are more accurate, but require far greater computational resources. This is particularly true for deep-penetration shielding calculations involving several orders of magnitude attenuation. Despite the steady increase in available computational performance, unbiased or analog MC methods are not practical, or even possible, for real reactor applications. For these applications, either biased MC methods or the discrete ordinates (S_N) method, which contains uncertainties associated with the discretization of the independent variables of the transport equation, are used. In this work, we attempt to take advantage of the strengths of both methods and use them in a complementary manner. Specifically, deterministic adjoint solutions are employed for automatic variance reduction (VR) of MC shielding calculations. Herein, the expressions *variance reduction* and *biasing* refer to *fair-game* techniques used in MC calculations to reduce the computer time required to obtain results of sufficient precision.

To make a difficult MC shielding calculation computationally practical, or in most cases possible, we employ our basic knowledge of the physics of the problem and the available VR techniques to coerce *important* particles to contribute to the quantity of interest (e.g., reaction rate, dose, etc.). While the application of VR techniques is fairly straightforward for simple one-dimensional problems, it can be quite difficult for realistic problems that are often complex and three-dimensional. Thus, the shielding analyst typically engages in an iterative process to develop the VR parameters and subdivide the geometry to facilitate the parameters; converging at some acceptable level of calculational efficiency. Unfortunately, the appropriate VR parameters vary significantly with problem type and objective. Therefore, the iterative steps must be repeated to determine the VR parameters for calculations with different objectives. Automatic importance generators, such as the weight window generator in MCNP[1],

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are currently available, but are restricted by their statistical nature and are of limited use in multi-dimensional deep-penetration problems. In the absence of more sophisticated methods, however, the weight window generator is a very useful tool in determining the VR parameters[2].

A further difficulty lies in the statistical convergence of the MC results. For large complex applications, it is not uncommon to spend days (and possibly weeks) iterating and adjusting the VR parameters only to achieve reasonable efficiency with unstable statistical behavior. This unstable statistical behavior is caused by improper use of the VR methods (i.e., insufficient detail and/or inappropriate selection of the parameters) and is usually the result of undersampling some important region of the problem phase-space[3]. Further, this undersampling is often difficult to identify and correct.

It has long been recognized that the adjoint function (i.e., the solution to the adjoint Boltzmann transport equation) has physical significance as a measure of the importance of a particle to some objective function (e.g., the response of the detector)[4]. It is this physical interpretation that makes the adjoint function well suited for biasing MC calculations. Thus, the idea of using the adjoint function to accelerate MC calculations is not new; and a number of applications of deterministic adjoint solutions have been made[5, 6, 7].

In this work, we propose a method for using the S_N adjoint function for automatic VR of MC calculations through source biasing and consistent transport biasing with the weight window technique. We describe the implementation of this method into the standard *production* MC code MCNP, and its application to a realistic calculation, namely the reactor cavity dosimetry calculation. The effectiveness of the method, as demonstrated through the increase in calculational efficiency is quantified. Moreover, this paper addresses important issues associated with this method and its efficient use and implementation.

II. THEORY

Problems that can be solved by the MC method are essentially equivalent to integrations[8]. For example, the goal of most MC particle transport problems is to calculate the response (i.e., flux, dose, reaction rate, etc.) at some location. This is equivalent to solving the following integral

$$R = \int_{4\pi} \int_V \int_E \Psi(r, E, \hat{\Omega}) \sigma_d(r, E) dE dV d\hat{\Omega}, \quad (1)$$

where Ψ is the particle flux and σ_d is some objective function.

From the following identity[4]

$$\langle \Psi^\dagger H \Psi \rangle = \langle \Psi H^\dagger \Psi^\dagger \rangle, \quad (2)$$

where H^\dagger is the adjoint operator and $\langle \rangle$ signify integration over all the independent variables, one can show that the response R (for a vacuum boundary condition) is also given by

$$R = \int_{4\pi} \int_V \int_E \Psi^\dagger(r, E, \hat{\Omega}) q(r, E) dE dV d\hat{\Omega}, \quad (3)$$

where Ψ^\dagger and $q(r, E)$ are the adjoint function and source density, respectively, and (Eqs. 1 and 3) are equivalent expressions for R .

To solve this integral the independent variables are sampled from $q(r, E)$, which is not necessarily the best probability density function (*pdf*) from which to sample. An alternative *pdf*, $\hat{q}(r, E)$ can be introduced into the integral as follows:

$$R = \int_V \int_E \left[\frac{\phi^\dagger(r, E) q(r, E)}{\hat{q}(r, E)} \right] \hat{q}(r, E) dV dE, \quad (4)$$

where $\hat{q}(r, E) \geq 0$ and $\int_V \int_E \hat{q}(r, E) dV dE = 1$.

From importance sampling[9], the alternative *pdf* $\hat{q}(r, E)$ that will minimize the variance for R is then given by

$$\hat{q}(r, E) = \frac{\phi^\dagger(r, E) q(r, E)}{R}. \quad (5)$$

Thus, to increase the efficiency of the MC calculation the source energy and position are sampled from the biased source distribution $\hat{q}(r, E)$. Physically, the numerator is the detector response from space-energy element $dV dE$ at

(r, E) , and the denominator is the total detector response R . Therefore, the ratio is a measure of the contribution to the detector response.

Since the source variables are sampled from a biased *pdf*, the statistical weight of the source particles must be corrected such that

$$\hat{q}(r, E) W(r, E) = q(r, E). \quad (6)$$

Substituting (Eq. 5) into (Eq. 6) and rearranging, we obtain the following expression for the statistical weight of the particles

$$W(r, E) = \frac{R}{\phi^\dagger(r, E)}. \quad (7)$$

This equation shows an inverse relationship between the adjoint (importance) function and the statistical weight. Previous work[10] in this area has shown this relation to be near optimal, and others have verified this relationship through computational analysis[5, 11]. However in this work, this relationship was derived from importance sampling.

To use the weight window facilities within MCNP, we need to calculate weight window lower bounds W_l . The width of the weight window interval is controlled by the parameter C_u , which is the ratio of upper and lower weight window values ($C_u = \frac{W_u}{W_l}$). Therefore, the weight window lower bounds W_l are given by

$$W_l(r, E) = \frac{W}{\left(\frac{C_u+1}{2}\right)} = \frac{R}{\phi^\dagger(r, E)} \frac{1}{\left(\frac{C_u+1}{2}\right)}. \quad (8)$$

It is important to note that because the source biasing parameters and weight window lower bounds are consistent, the statistical weights of the source particles ($W(r, E) = \frac{q(r, E)}{\hat{q}(r, E)}$) are within the weight windows as desired. If the statistical weights of the source particles are not within the weight windows, the particles will immediately be split or rouletted in an effort to bring their weights into the weight windows[1]. This will result in unnecessary splitting/rouletting and a corresponding degradation in computational efficiency. For problems in which the adjoint function varies significantly within the source region (space and/or energy), this coupling between source and transport biasing is critical.

III. IMPLEMENTATION INTO MCNP

A. Overlay of Importance Function

The general version of MCNP provides facilities for energy and cell dependent weight windows. This means that in order to use a fine spatial weight window grid (which is necessary in optically thick regions), the user must subdivide the cell based geometry such that the ratio of importances between adjacent geometric cells is not large. Because the importance ratios are not apriori known, this geometric discretization is not straightforward and typically requires iterations of adjustments. Further, the subdivision of the geometry into a very large number of cells is time consuming and can actually degrade the efficiency of the calculation. For these reasons, we use the deterministic S_N spatial mesh description to construct a separate, but related, geometric grid to facilitate the use of the adjoint distribution. This is done with a modified version of the MCNP code that is able to read the binary flux file (which contains the adjoint function and the spatial mesh and energy group information) from the standard S_N DORT code[12] and superimpose the variable spatial mesh and energy grid onto the MCNP problem. This grid facilitates the use of the spatial and energy dependent importance function, and does not directly affect the transport of particles. At various events in a particle history (e.g., collisions, surface crossings, and/or increments of mean free path), the grid is searched to determine the importance of the phase-space within which the particle resides. The importance is then compared to the statistical weight of the particle and the appropriate action is taken (e.g., splitting, Russian roulette, or no action). It should be noted that with the cell-based weight windows in the standard version of MCNP, additional spatial searching is not required.

B. Weight Checking

Various concepts for minimizing the amount of computational *overhead* associated with this process have been examined. The first issue of concern is the determination of the appropriate time (or event) to check the particle's

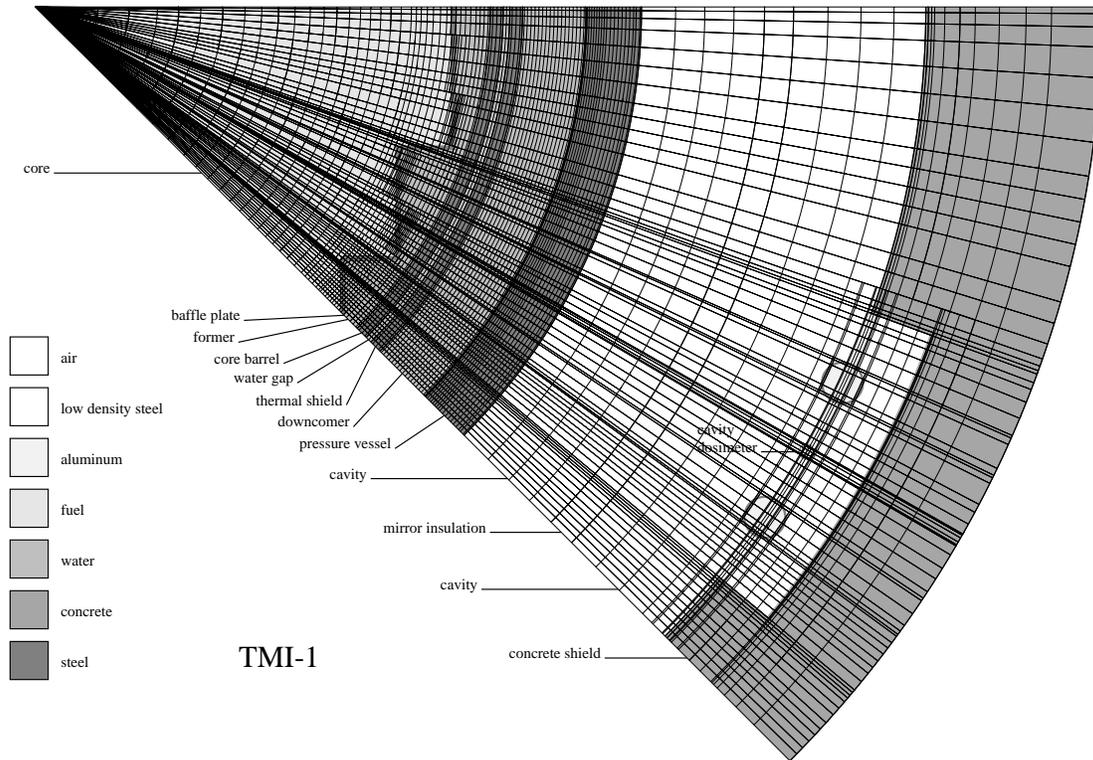


Figure 1: One Octant of the Three Mile Island unit 1 (TMI-1) Reactor

statistical weight. This is important for the following reasons: (1) because the MCNP geometry does not need to be manually subdivided to facilitate the spatial importances, the presently available weight checks (i.e., at collisions and surface crossings) are no longer sufficient to control particle weight and thus large differences in the weight scored by individual particles are possible, (2) there is a computational cost or penalty each time the weight is checked, and this penalty is the time required by the searching routines to determine the space and energy importance interval within which the particle resides, and (3) more frequent checking leads to more reliable results with well-behaved statistical convergence, but at some point the calculational efficiency is sacrificed. Therefore, it is clear that we need to determine an optimum or near-optimum criteria for checking particle statistical weights. Moreover, it is desirable that this criteria be problem independent.

Since the mean free path (mfp) is, by definition, the average distance a particle travels between collisions, it is a logical, problem independent parameter by which particle statistical weight can be controlled. Parametric studies analyzing the effect of the increment of mfp on problem efficiency and reliability[13] verify that a mfp increment of unity is a near-optimal problem-independent criteria for checking particle statistical weights.

III. APPLICATION AND ANALYSIS

A. Reactor Cavity Dosimetry Calculation

This problem addresses a major concern of nuclear utilities. The life of a reactor and its possible extension are directly dependent on the embrittlement of the reactor pressure vessel (RPV), a low carbon steel container surrounding the reactor core, under neutron irradiation. The embrittlement of the RPV material is due, primarily, to the bombardment of neutrons with energies greater than ~ 1 MeV, and cannot be directly determined from measured quantities. Radiation detectors are employed to provide data by which calculational methods can be benchmarked. Often the dosimeter is outside the RPV, in what is referred to as the cavity; hence it is called cavity dosimetry. The cavity dosimetry calculations attempt to estimate high-energy (≥ 1.0 MeV) reaction rates in a

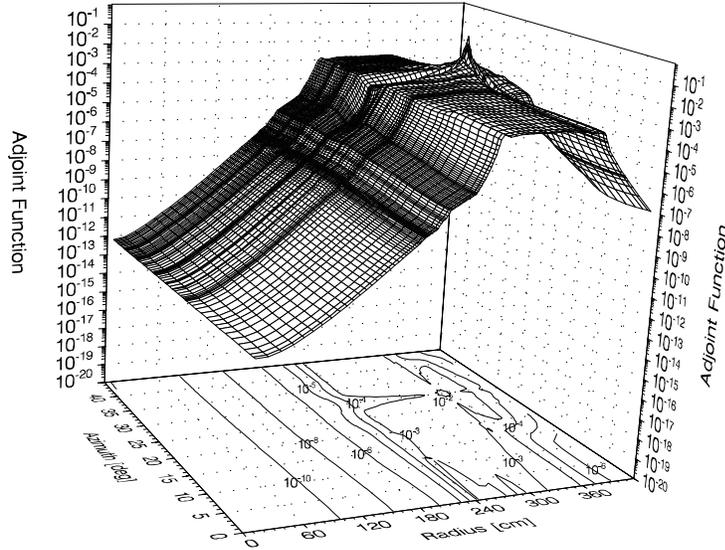


Figure 2: Adjoint Function Distribution for Energy Group 3 (10.00-12.14 MeV)

small volume at a distance of ~ 350 cm from the core centerline, and are thus used to estimate RPV integrity and provide a basis for plant life extension. The problem is illustrated in Fig. 1. In the past, the S_N method was used, almost exclusively, to perform these calculations. More recently, the MC method has been employed for this application in an effort to better understand the uncertainties associated with the S_N method and to attempt to benchmark S_N calculations.[14]

For this application, the $^{63}\text{Cu}(n,\alpha)$, $^{58}\text{Ni}(n,p)$, and $^{54}\text{Fe}(n,p)$ reaction rates (responses), which have thresholds energies of ~ 5.0 , ~ 1.0 , and ~ 1.0 MeV, respectively, are all of equal interest, and thus we calculate an effective response function as a normalized sum of each of the normalized response functions. Using this effective response function, we can generate an importance function that will simultaneously optimize the calculation for all three reaction rates, and thus avoid calculating an importance function for each individual response that would require three separate MC calculations.

With this effective response function as the adjoint source, a two-dimensional $R-\theta$ adjoint function is calculated with the DORT code using the SAILOR 47-group library[15] and a symmetric S_8 quadrature set. Figure 2 shows this adjoint function distribution for energy group 3 (10.00-12.14 MeV). The modified version of MCNP reads the adjoint function from the standard DORT binary flux file, couples the original source distributions with the adjoint function to generate the source biasing parameters and weight window lower bounds, and then performs the transport calculation. The S_N spatial mesh which is used within MCNP to facilitate the spatial importance distribution is shown in Figure 1. Within MCNP the source spatial distribution is represented by a probability distribution function at 24 axial locations in each fuel pin of the last two (peripheral) layers of assemblies and the energy distribution is based on an equivalent fission spectrum for the U and Pu fissile isotopes.[16] The following approximations/assumptions are made in this process: (1) the axial behavior for the adjoint function is approximated with a cosine distribution, and (2) to represent the spatial dependence of the energy biasing parameters, the energy dependent adjoint function is averaged over each user defined spatial source cell, and a dependent source energy biasing distribution is calculated for each source cell. For this particular application, each assembly has an associated source energy biasing distribution. No modifications to the source routines are necessary, since the capabilities to handle source variable biasing and dependent source distributions are standard features of the MCNP code.

Since the focus of this work is on the automatic VR for the calculation and not on the calculation itself, the interested reader is referred to the references[14] for discussions regarding the accuracy of results with respect to measurements and S_N calculations, as well as sensitivity studies related to various aspects of this calculation. However, to provide some idea about the accuracy and to demonstrate that the automatic VR technique does not

Table I: Calculated-to-Experimental (C/E) Ratios at the Cavity Dosimeter for TMI-1

Reaction	Manually Optimized		Automatic VR	
	C/E	FOM	C/E	FOM
$^{63}\text{Cu}(n,\alpha)$	0.905(0.022) ^a	3.7	0.878(0.022) ^a	16 (4.3) ^b
$^{54}\text{Fe}(n,p)$	0.965(0.023)	3.5	0.964(0.020)	20 (5.7)
$^{58}\text{Ni}(n,p)$	0.947(0.020)	4.5	0.952(0.019)	22 (4.9)

^a 1σ uncertainties

^b ratio of *Automatic VR* and *Manually Optimized* FOMs

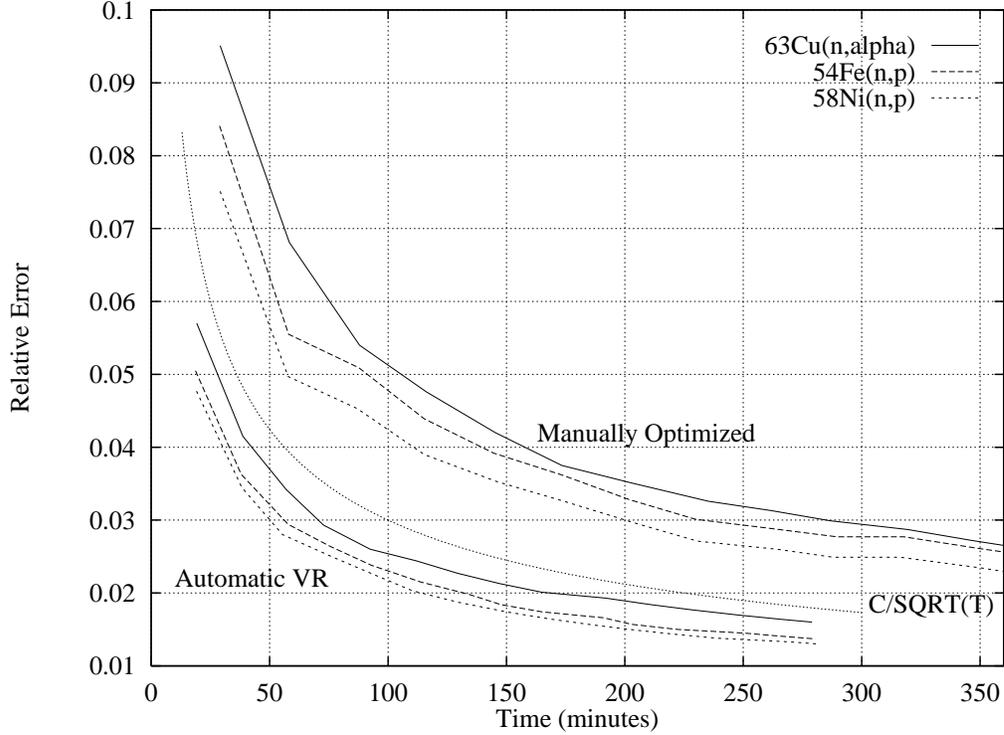


Figure 3: Relative Error vs Computer Time

bias the calculation, calculated-to-experimental (C/E) ratios, corresponding to ENDF/B-V material and SAILOR dosimetry cross-sections, are given in Table I. The differences between C/E ratios calculated with the manually optimized model and with the adjoint importance function are within the statistical uncertainties.

B. Comparison of Computational Efficiency

With the use of the automatic VR method, the computer time required by the MCNP model to calculate the reaction rates at the cavity dosimeter with 1σ uncertainties of less than 3% is ~ 1 hour on an IBM RISC/6000 model 370. To reach the same precision (3%) with the manually optimized model requires nearly 5 hours of CPU time. This behavior is demonstrated in Fig. 3 which plots relative error (RE) and $\frac{C}{\sqrt{T}}$ (where C is a constant and T is computer time) versus computer time for the three reaction rates of interest. The two sets of curves in Fig. 3 correspond to calculations performed with different VR approaches; namely, manually optimized VR (including, energy cutoff, source biasing, weight windows (2 energy groups), and the exponential transformation; see Ref. [14] for details) and the automatic VR derived from a 2-D adjoint function distribution (18 energy groups) using the effective response function.

Table I lists the Figure-Of-Merit (FOM) values [$FOM = 1/(R)^2T$] and reveals that the use of the automatic

Table II: Comparison of Estimated REs and Calculated Relative Standard Deviations at the Cavity Dosimeter

Reaction	Manually Optimized			Automatic VR		
	average estimated RE ^a	calculated relative standard deviation ^b	ratio	average estimated RE ^a	calculated relative standard deviation ^b	ratio
⁶³ Cu(n,α)	.0229	.0198	0.86	.0223	.0178	.799
⁵⁴ Fe(n,p)	.0203	.0243	1.20	.0197	.0132	.668
⁵⁸ Ni(n,p)	.0181	.0200	1.10	.0189	.0142	.750

^a refers to the average of the REs estimated by MCNP in the 10 runs

^b refers to the relative standard deviation of the calculated reaction rates in the 10 runs

VR increases the calculational efficiency by more than a factor of 4 with respect to our *best* manually optimized model. Further, the use of the adjoint function can be shown to increase the calculational efficiency by a factor of $\sim 10^4$ with respect to the unbiased case.

It should be noted that the aforementioned computer times do not include the S_N adjoint calculation. Also, for the purpose of comparison, the synthesized 3-D deterministic results (group fluxes for energies greater than ~ 1 MeV) can be produced by two 2-dimensional and one 1-dimensional calculations which require $\sim .25$, $.25$, and 0.005 hours of computer time, respectively, on an IBM RISC/6000 model 370. The computer time required for the synthesis process is essentially negligible. Thus, group fluxes over the entire spatial domain can be generated by DORT in < 1 CPU hour.

C. Statistical Convergence

One potential problem associated with the intense use of VR techniques is erratic or unreliable error estimations. Figure 3 shows that the RE follows the expected behavior predicted by the Central Limit Theorem ($RE \sim \frac{1}{\sqrt{N}} \sim \frac{1}{\sqrt{T}}$; where N is the number of particle histories), which provides some indication about the validity of the estimated RE. The use of the automatic VR method appears to lead to smoother statistical convergence. All three reaction rate tallies pass all 10 of the MCNP statistical checks[1], providing an additional indication of proper statistical convergence.

To further assess the estimated uncertainties, the manually optimized and automatic VR cases were each run 10 times with different starting random number seeds. This allows us to compare the RE as estimated by MCNP to the relative standard deviation of the calculated means (reaction rates) from the multiple runs. The number of histories were chosen such that both cases would yield similar REs; the manually optimized and automatic VR cases required $20E+6$ and $3E+6$ particle histories, respectively. Table II compares the calculated relative standard deviations to the average (MCNP) estimated REs, and reveals that the RE is underestimated by as much as 20% in the manually optimized case and overestimated by more than 20% in the automatic VR case. In other words, the estimated REs from the automatic VR case are much more reliable than those from the manually optimized case.

For a normal distribution, the calculated value should be within the 1σ RE 67% of the time. For our 10 runs (with 3 reaction rates per run), the calculated values are observed to be within the 1σ RE 57% of the time for the manually optimized case and 87% of the time for the automatic VR case. In addition, the standard deviation of the estimated REs and the estimated variance-of-the-variance (VOV) are a factor of ~ 3 less in the automatic VR case than in the manually optimized case. These results clearly demonstrate that the automatic VR method produces reliable error estimations.

D. Effect of Adjoint Accuracy on Calculational Efficiency

The effectiveness of an adjoint (importance) function for VR of MC calculations is dictated by its accuracy. It is for this reason that we use an accurate method - the S_N method - to calculate the adjoint function. However, for extremely large problems and for the extension to three-dimensional adjoint functions, the memory and disk space requirements for the S_N adjoint calculations can become prohibitive. One way to alleviate this problem is to sacrifice some of the accuracy of the adjoint calculation through the use of fewer energy groups. However, the relationship between the accuracy of adjoint functions and their effectiveness for VR of MC calculations is

somewhat problem dependent and is not well known. Therefore, in this section we investigate the effectiveness of importance functions with varying degrees of energy-dependent accuracy.

To examine the effect of using an adjoint function with varying numbers of energy groups, one of the following two approaches may be employed: (1) collapse an appropriate multigroup library into various libraries with fewer groups and use these libraries to calculate adjoint functions or (2) calculate the adjoint function for a given number of (maximum) groups and collapse it into various coarse group structures. In this study, we have chosen the second approach because it does not require the generation of additional multigroup libraries and subsequent S_N calculations, and it uses an accurate importance function as its starting point. In other words, this method eliminates concerns related to the selection of multigroup boundaries for the cross section collapsing procedure; specifically, the effect of their selection on the accuracy of the S_N solution.

To collapse an adjoint function into fewer groups, it is necessary to introduce group quantities such that the total response R ,

$$R = \int_V \int_E \phi^\dagger(r, E) q(r, E) dE dV \quad (9)$$

is conserved. This is done by setting the integral equal to the product of the adjoint function and the source in broad group k :

$$\int_k \phi^\dagger(r, E) q(r, E) dE = \phi_k^\dagger(r) q_k(r), \quad (10)$$

where we define the coarse group source and adjoint by

$$\begin{aligned} q_k &= \int_k q(r, E) dE, \\ \phi_k^\dagger &= \frac{\int_k \phi^\dagger(r, E) q(r, E) dE}{q_k}. \end{aligned} \quad (11)$$

This procedure formally preserves the total response and leads to the definition of a coarse group adjoint through source-weighting. Because of the source-weighting, these relations are only applicable within the source region. However, for this study, it is desirable to collapse the adjoint throughout the problem (i.e., including regions outside the source region(s)). To do this, it is necessary to conserve the response from each spatial region. Since the forward flux in a spatial region can be considered to be the source of particles within that region, it may be used to weight the adjoint in the collapsing procedure. Replacing the source term with the forward flux in the preceding relations, yields:

$$\int_k \phi^\dagger(r, E) \phi(r, E) dE = \phi_k^\dagger(r) \phi_k(r), \quad (12)$$

where we define the coarse group flux and adjoint by

$$\begin{aligned} \phi_k &= \int_k \phi(r, E) dE, \\ \phi_k^\dagger &= \frac{\int_k \phi^\dagger(r, E) \phi(r, E) dE}{\phi_k}. \end{aligned} \quad (13)$$

A program was written to read the DORT binary flux files from forward and adjoint calculations, perform the collapse as discussed above, and generate a collapsed adjoint binary flux file in the same format. Thus, allowing the current modified version of MCNP to read and utilize the various collapsed adjoint functions without additional modifications.

The 18-group (energy groups above 1 MeV) adjoint is collapsed into 9-, 6-, 3-, 2-, and 1-group adjoint functions. The coarse group boundaries are a subset of the fine-group boundaries, and each coarse group contains the same number of fine groups. For example, in the 6-group structure, the highest energy group contains the highest three groups of the 18-group structure.

Figure 4 shows the relationship between the number of adjoint energy groups and the MC calculational efficiency, in terms of FOM. The FOM values are normalized such that the highest value is unity. The figure demonstrates that for the $^{54}\text{Fe}(n,p)$ and $^{58}\text{Ni}(n,p)$ reaction rate calculations there is no benefit to using more than ~ 9 groups, and relatively minor losses in efficiency are associated with using even fewer energy groups. On the other hand, because the $^{63}\text{Cu}(n,\alpha)$ reaction rate calculation is sensitive to a rather narrow energy range ($\sim 6-12$ MeV), large losses in

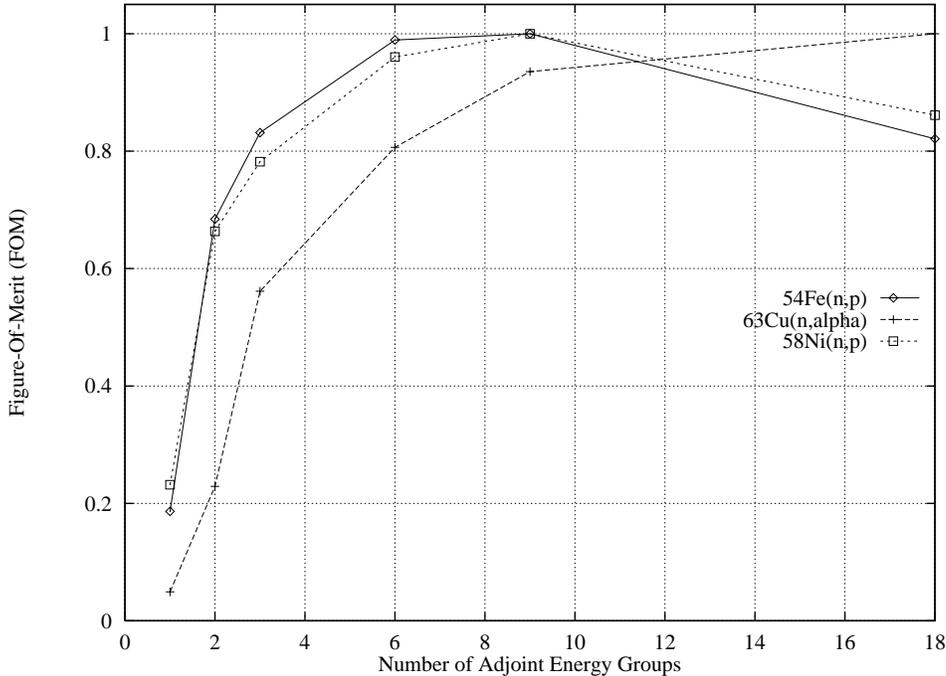


Figure 4: Effect of Number of Adjoint Energy Groups on FOM

efficiency are observed when fewer energy groups are used. Nevertheless, for this particular problem an adjoint with relatively few energy groups (~ 3 groups) is capable of increasing the calculational efficiency to approximately half of the observed maximum, which is a factor of $\sim 10^4$ more efficient than the analog case.

IV. SUMMARY

A general method for automatic VR of MC shielding calculations with the space- and energy-dependent S_N adjoint function has been presented. The theory supporting the use of the adjoint function to bias MC calculations is based on the physical interpretation of the adjoint function. The equations for determining the VR parameters (source biasing parameters and weight window values) were derived from basic importance sampling with the adjoint function as the weighting function. Physically, the equations for the source biasing parameters represent a measure of the contribution to the detector response from the space-energy elements.

This method is implemented into the general purpose MC code MCNP. Currently, this modified version of MCNP is able to (1) read the adjoint function and the variable spatial mesh and energy group information from a standard S_N code (DORT) binary flux file, (2) superimpose the variable spatial mesh and energy grid onto the MCNP problem, (3) couple the original source distributions with the adjoint function to generate dependent source biasing parameters and weight window lower bounds, and (4) perform the transport calculation using the constructed grids and calculated parameters. The grids facilitate the use of the space- and energy-dependent importance function, and do not directly affect the transport of particles.

The effectiveness of this method within the modified version of MCNP has been verified through its application to a realistic shielding calculation; namely, the reactor cavity dosimetry calculation. With the use of this method, the efficiency of the reaction rate calculation was shown to increase by more than a factor of 4 and the statistical convergence was improved with respect to our best manually optimized model. Further, the use of this method does not require the intuition, guess work, and/or manual intervention typical of current VR techniques (or importance function generators), thus significantly reducing the analyst's time for performing these calculations. Because it is based on the source biasing and weight window techniques, the method is practical and features statistical reliability and general applicability. In addition, the method does not in anyway restrict the accuracy of the explicit three-dimensional continuous energy and angular MC method

V. ON-GOING WORK

The limitation to this approach is the requirement of an S_N adjoint solution. The determination of the adjoint function requires a reasonably detailed S_N calculation, which can be a difficult task by itself. Therefore, we are currently developing strategies for generating input files for S_N calculations directly from MCNP input files. Coupling the work described in this paper with such strategies will lead to the automation of the generation and application of the adjoint function for VR in MCNP calculations.

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