Deterministic Adjoint Functions for Biasing Monte Carlo Reactor Cavity Dosimetry Calculations

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INTRODUCTION

Reactor cavity dosimetry is performed to benchmark models for pressure vessel fluence calculations. These calculations are used to estimate RPV integrity and provide a basis for plant life extension, and therefore, their accuracy is of great importance. Since the Monte Carlo method is considered to be the most accurate method available for solving radiation transport problems, it is only natural to attempt to use it for this application. However, due to its nature of simulating individual particles and inferring the average behavior of the particles in the system (using the Central Limit Theorem) from the average behavior of the individually simulated particles, it is extremely computationally expensive. In fact, for many reactor applications, as well as medical and nuclear-well logging applications, the computer time required by the Monte Carlo method is considered prohibitive and/or impractical. Therefore, for difficult problems such as the cavity dosimeter calculation, where the natural probability that a particle will contribute to the detector of interest is small, some effective means of variance reduction or biasing must be used.

It has long been recognized that the adjoint function (i.e., the solution to the adjoint Boltzmann transport equation) has physical significance as a measure of the importance of a particle to some objective function (e.g., the response of a detector)[1]. It is this physical interpretation that, in theory, makes the adjoint function well suited for use as an importance function for biasing certain types of Monte Carlo calculations. Specifically, problems involving a small detector region(s) at a large distance from the source, such as the reactor cavity dosimetry calculation which attempts to estimate reaction rates at a distance of approximately 350cm from the core centerline. The

Monte Carlo and discrete ordinates models for this calculation are described in Refs. [2, 3] and [4], respectively.

In this paper, we investigate the procedure and subsequent benefits of using the deterministic adjoint (importance) function to bias reactor cavity dosimetry calculations.

THEORY

From adjoint transport theory[1], the response R at a detector is given by $R = \langle \Psi^\dagger q \rangle$, where q is the source density, $\Psi^\dagger$ is the adjoint function, and $\langle\rangle$ signify integration over all the independent variables.

For the acceleration of this calculation, we use $\Psi^\dagger$ to bias the source of particles as well as the transport of the particles through the medium. To this end, we assume separability of the probability distributions (i.e., $q(r,E) = q(r)q(E)$), and calculate a biased source energy distribution $q_b(E)$ based on the group-wise detector response from the source[5],

$$q_b(E) = \frac{R_s(E)}{\int_E R_s(E)dE} = \frac{\Psi_s^\dagger(E)q(E)}{\int_E \Psi_s^\dagger(E)q(E)dE} \tag{1}$$

where $\Psi_s^\dagger$ represents the average adjoint over a reference spatial region s. Since we are biasing our source energy variable with respect to its response, and the particle statistical weights are inversely proportional to the adjoint function[6], we calculate the weight window lower bounds W_l from the the ratio of the two,

$$W_l(r,E) = \frac{\int_E R_s(E)dE}{\Psi^\dagger(r,E)} \frac{1}{(\frac{C_u+1}{2})} = \frac{\Psi_s^\dagger(E)q(E)}{\Psi^\dagger(r,E)q_b(E)} \frac{1}{(\frac{C_u+1}{2})} \tag{2}$$

where C_u is the ratio of upper and lower weight window values ($C_u = \frac{W_u}{W_l}$) and the last expression on the right makes use of $\int_E R_s(E)dE = \Psi_s^\dagger(E)q(E)/q_b(E)$ from (Eq. 1). It is important to note that for the various source regions, the statistical weight of the source particles after source biasing has been performed is $W(r,E) = \frac{q(r)q(E)}{q_b(r)q_b(E)}$. Using this expression along with (Eq. 2) we develop the following formulation for our spatial biasing parameters

$$q_b(r) = \frac{q(r)\Psi_E^\dagger(r)}{\Psi_E^\dagger(r')}. \tag{3}$$

where $\Psi_E^\dagger$ represents the average adjoint over energy. Note that the separability approximation will lead to some particles being started outside the weight window interval, and was used because of the extensive detail (amount of data) required by the source description in this problem.

For this application, the ^{63}Cu(n,α), ^{58}Ni(n,p), and ^{54}Fe(n,p) reaction rates (responses), which have thresholds energies of ~5.0, ~1.0, and ~1.0 MeV, respectively, are all of equal interest, and thus we calculate a representative reaction rate as a weighted average of each of the normalized reaction rates based on the width of the energy range affected and the sensitivity to that energy range.

With this representative response function as the adjoint source, we calculate a 2-D R-θ adjoint function with the DORT[7] code using the SAILOR[8] P_3 47-group library and a symmetric S_8 quadrature set. (The axial behavior is approximated with a cosine distribution.) A modified version of MCNP[9] then reads the adjoint function and couples the original source distributions with the adjoint function to generate the source biasing parameters and spatially averaged weight window lower bounds. These source biasing parameters and weight window lower bounds are then inserted into the MCNP input file for a normal transport calculation.

DISCUSSION OF RESULTS

Without the use of variance reduction techniques, the calculation of reaction rates at the cavity dosimeter with sufficient precision is not possible. Hence, variance reduction techniques are essential for an application of this type.

The computer time required by the MCNP model to calculate the reaction rates at the ex-vessel cavity dosimeter with 1σ uncertainties of less than 3% is ~1 hour on an IBM RISC/6000 model 370. This behavior is demonstrated in Figure 1 which plots relative error and $\frac{C}{\sqrt{T}}$ (where C is a constant and T is computer time) versus computer time for the three reaction rates of interest. The two sets of curves in Figure 1 correspond to calculations performed with different importance functions; namely, an importance function generated with the weight window generator (2 energy groups) and an importance function derived from a 2-D adjoint function distribution using the representative response function. Note that the process of generating the weight window lower bounds (importance

function) from the weight window generator was extremely tedious and required many iterations of significant smoothing and adjusting, while the process of generating the weight window lower bounds from the adjoint function is described by the equations in the previous section.

It is important to note that the relative error (RE) follows the expected behavior predicted by the Central Limit Theorem which indicates the validity of the calculated relative errors. Moreover, the use of the adjoint importance function clearly leads to smoother statistical convergence, thereby producing more reliable error estimations.

Since the computer time T is directly proportional to the number of histories N, we expect the quantity $(RE)^2T$, and subsequently the FOM ($FOM = 1/(RE)^2T$) to be approximately constant. This behavior is clearly shown in Figure 2, which also demonstrates that the use of the adjoint importance function increases the calculational efficiency by more than a factor of four with respect to our *best* weight window generator importance function.

CONCLUSIONS

The usefulness of the adjoint function for biasing Monte Carlo reactor cavity dosimetry calculations has been demonstrated, and a procedure for doing so has been briefly discussed. The adjoint (importance) function has been shown to increase the efficiency of the reaction rate calculation by a factor of 4 and improve the statistical convergence. Further, the use of the adjoint function does not require the intuition, guess work, and/or manual intervention typical of current variance reduction techniques and importance function generators, thus significantly reducing the analyst's time for performing these calculations. Future work will involve the enhancement, refinement, and further automation of this process.

Figure 1: Relative Error vs Computer Time

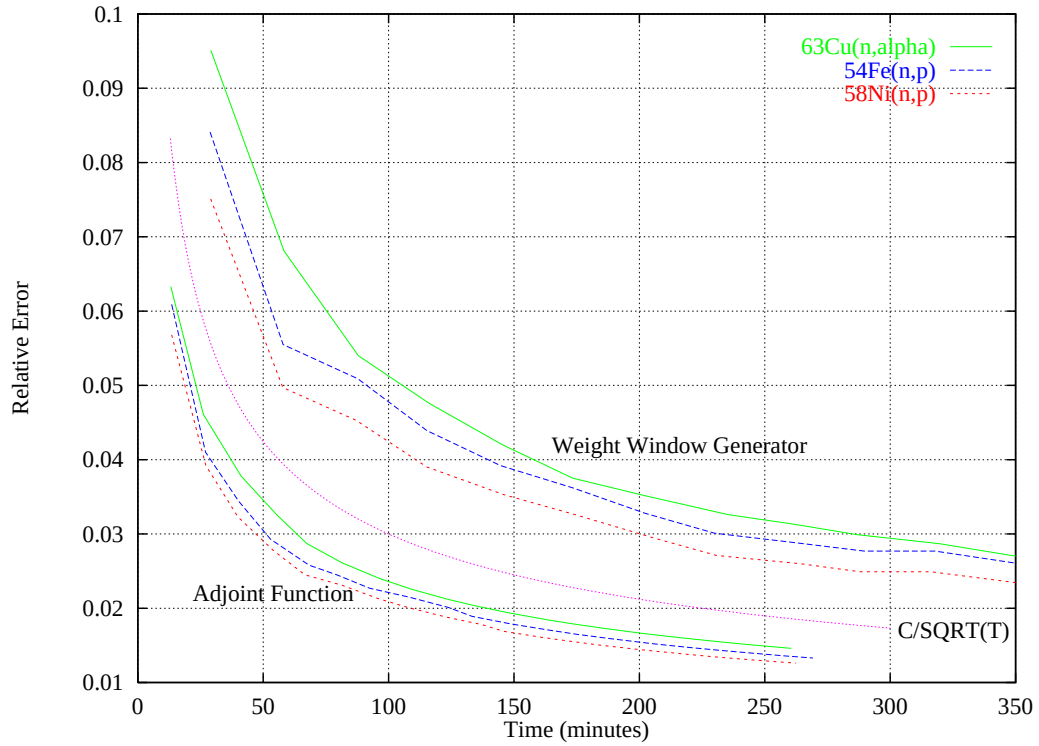
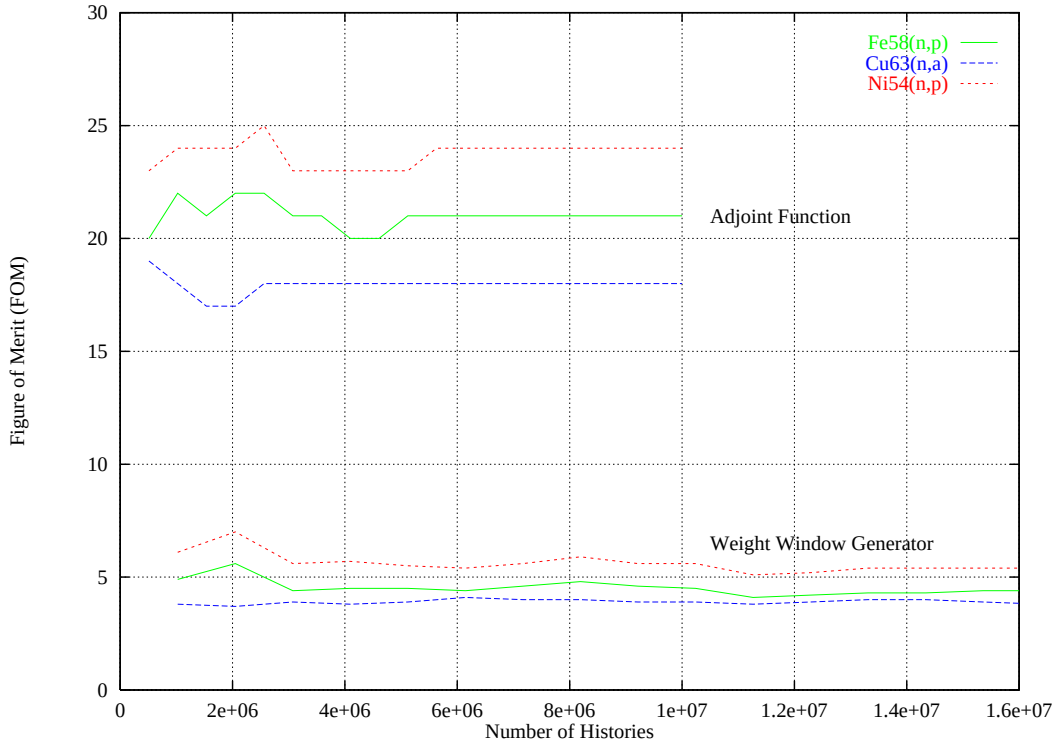


Figure 2: Figure of Merit (FOM) vs Number of Particle Histories



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